

## **Physics of Turbulence**

This course (turbulence modeling) will present as much of the physics of turbulence as necessary to understand why existing modeling approximations have been made. It will involve:

- Understand the physics
- Correlation of measurements, Engineering judgment
- A healthy dose of mathematics and a lot of trial and error.

Many clever engineers (Prandtl, Taylor, von Karman, ... etc) spent their time devising engineering approximations and models describing complicated physical flows.

Computational Fluid Dynamics (CFD) contains three key elements:

1. Turbulence modeling
2. Grid generation and
3. Algorithm development

Mathematical model approximates the physical behavior of turbulent flows. According to using assumptions, far less precision has been achieved in turbulence modeling. This is not really a surprising event since our objective has been to approximate an extremely complicated phenomenon.

Two key questions have to be asked at the beginning:

1. What constitutes the ideal turbulence model? and
2. How complex must it be?

## **Definition of an Ideal Turbulence Model**

Simplicity combined with physical insight of clever engineers gave this description of an ideal model which serves as the keystone of this course.

*“An ideal model should introduce the minimum amount of complexity while capturing the essence of the relevant physics.”*

## **How Complex Must a Turbulence Model Be?**

According to physical considerations, turbulence is inherently three dimensional and time dependent. Thus, an enormous amount of information is required to completely describe a turbulent flow.

Fortunately, we usually require something less than a complete time history over all spatial coordinates for every flow property. Thus, for a given turbulent-flow application, we must pose the following question:

- Are a set of initial and/or boundary conditions given?
- How do we predict the relevant properties of the flow?
- What properties of a given flow are relevant?
- Is it generally dictated by the application?

For the simplest applications, only the skin-friction and heat-transfer coefficients may be required for an attached flow. A simple mixing-length model (Chapter 3) may suffice. Such models are well developed and can be implemented with very little specialized knowledge.

More mysterious applications may require detailed knowledge of energy spectra, turbulence fluctuation magnitudes and scales. Certainly, the complexity of the mathematics required for such applications increases as the amount of required flow field detail increases.

If a complete time history of every aspect of a turbulent flow is desired, only a solution to the complete Navier-Stokes equation will suffice. Such a solution requires an extremely accurate numerical solver and may require use of subtle transform techniques, not to mention vast computer resources.

Most engineering problems fall somewhere between these two extremes.

Thus, once the question of how much detail we need is answered, the level of complexity of the model follows, qualitatively speaking.

## **Comments on the Physics of Turbulence**

### **Importance of Turbulence in Practical Situations**

The motion is termed laminar and can be observed experimentally and in nature:

- For "small enough" scales
- "low enough" velocities
- Reynolds number is not too large
- The equations of motion for a viscous fluid have well-behaved
- Steady solutions.
- Such flows are controlled by viscous diffusion of vorticity and momentum.
- Flow moves in smooth layers

The motion is described as turbulent

- At larger Reynolds numbers
- The fluid's inertia overcomes the viscous stresses,
- The laminar motion becomes unstable.
- Rapid velocity and pressure fluctuations appear
- The motion becomes inherently three dimensional
- The motion becomes unsteady.

Turbulent flow always occurs when the Reynolds number is large. For slightly viscous fluids such as water and air, "large" Reynolds number corresponds to anything stronger than a tiny swirl, a small breeze or a puff of wind.

Virtually all flows of practical engineering interest are turbulent:

- Flow past vehicles such as rockets, airplanes, ships and automobiles.
- Turbulence dominates in geophysical applications such as river currents
- The planetary boundary layer

- The motion of clouds
- Turbulence even plays a role at the breakfast table, greatly enhancing the rate at which sugar and cream mix in a cup of coffee



Turbulence matters even in applications normally involve purely laminar flow. For example, blood flow is laminar in the arteries and veins of a healthy human. However, the presence of turbulence generally corresponds to a health problem such as a defective heart valve.

Thus, to analyze fluid motion for general applications, we must deal with turbulence. Although vigorous research has been conducted to help discover the mysteries of turbulence, it has been called the major unsolved problem of classical physics!

The most important aspects of turbulence will be explored in the following sections.

### **General Properties of Turbulence**

#### **Basic Definition:**

In 1937, von Karman defined turbulence as

*"Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighboring streams of the same fluid flow past or over one another."*

As the understanding of turbulence has progressed

- Researchers have found the term "irregular motion" to be too imprecise.
- Simply stated, an irregular motion is one that is typically aperiodic
- It cannot be described as a straightforward function of time and space coordinates.
- An irregular motion might also depend strongly and sensitively upon initial conditions.
- There are non-turbulent flows that can be described as irregular.
- Turbulent motion is indeed irregular in the sense that
- It can be described by the laws of probability.
- Even though instantaneous properties in a turbulent flow are extremely sensitive to initial conditions, statistical averages of the instantaneous properties are not.

To provide a sharper definition of turbulence, Hinze (1975) offers the following revised definition:

*"Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned."*

To complete the definition of turbulence, Bradshaw (1974) adds the statement that

*"Turbulence has a wide range of scales."*

Time and length scales of turbulence are represented by frequencies and wavelengths that are revealed by a Fourier analysis of a turbulent-flow time history.

The irregular nature of turbulence stands in contrast to laminar motion.

In describing turbulence, many researchers refer to eddy motion, which is a local swirling motion where the vorticity can often be very intense. Turbulent eddies of a wide range of sizes appear and give rise to strong mixing and effective turbulent stresses (a consequence of the "mixing" of momentum) that can be enormous compared to laminar values.

### **Instability and Nonlinearity:**

- Analysis of solutions to the Navier-Stokes equation, or more typically to its boundary-layer form, shows that turbulence develops as instability of laminar flow.
- To analyze the stability of laminar flows, classical methods begin by linearizing the equations of motion.
- For a real (i.e., viscous) fluid, mathematically speaking, the instabilities result mainly from interaction between the Navier-Stokes equation's nonlinear inertial terms and viscous terms.
- The interaction is very complex because it is rotational, fully three dimensional and time dependent.
- The nonlinearity of the Navier-Stokes equation leads to interactions between fluctuations of differing wavelengths and directions.
- The main physical process that spreads the motion over a wide range of wavelengths is **vortex stretching**. The turbulence gains energy if the vortex elements are primarily oriented in a direction in which the mean velocity gradients can stretch them. Most importantly, wavelengths that are not too small compared to the mean flow width interact most strongly with the mean flow.
- Consequently, the larger-scale turbulent motion carries most of the energy and is mainly responsible for the enhanced diffusivity and attending stresses.
- In turn, the larger eddies randomly stretch the vortex elements that comprise the smaller eddies, cascading energy to them. Energy is dissipated by viscosity in the shortest wavelengths, although the rate of dissipation of energy is set by the long-wavelength motion at the start of the cascade. The shortest wavelengths simply adjust accordingly.

### **Statistical Aspects:**

- The time-dependent nature of turbulence also contributes to its intractability.
- The additional complexity goes beyond the introduction of an additional dimension.
- Turbulence is characterized by random fluctuations thus mandating the use of statistical methods to analyze it.
- Even if we had a complete time history of a turbulent flow, we would usually integrate the flow properties of interest over time to extract time averages, or mean values.

### **Turbulence is a Continuum Phenomenon:**

- In principle, the time-dependent, three-dimensional continuity and Navier-Stokes equations contain all of the physics of a given turbulent flow.
- It is true and follows the fact that turbulence is a continuum phenomenon. As noted by Tennekes and Lumley (1983),

*"Even the smallest scales occurring in a turbulent flow are ordinarily far larger than any molecular length scale."*

- Nevertheless, the smallest scales of turbulence are still extremely small. Furthermore, the ratio of smallest to largest scales decreases rapidly as the Reynolds number increases.
- To make an accurate numerical simulation (i.e., a fully time-dependent three-dimensional solution) of a turbulent flow, all physically relevant scales must be resolved.
- While more and more progress is being made with such simulations, computers must have sufficient memory and speed to solve any turbulent-flow problem of practical interest. However, the results are very useful in developing and testing approximate methods.

### **Vortex Stretching:**

- The strongly rotational nature of turbulence goes hand-in-hand with its three-dimensionality. The vorticity in a turbulent flow is itself three dimensional so that vortex lines in the flow are nonparallel.
- The resulting strong stretching of vortex lines maintains the ever-present fluctuating vorticity in a turbulent flow.
- Vortex stretching is absent in two-dimensional flows so that turbulence must be three dimensional. There are no satisfactory two-dimensional approximations for determining fine details of turbulent flows even if the average motion is two dimensional.
- The induced velocity field attending these skewed vortex lines further increases three dimensionality and, at all but very low Reynolds numbers, the vorticity is drawn out into a tangle of thin tubes or sheets. Therefore, most of the vorticity in a turbulent flow resides in the smallest eddies.

### Turbulence Scales and the Cascade:

- Turbulence consists of a continuous spectrum of scales ranging from largest to smallest, as opposed to a discrete set of scales. In order to visualize a turbulent flow with a spectrum of scales we often cast the discussion in terms of eddies. A turbulent eddy can be thought of as a local swirling motion whose characteristic dimension is the local turbulence scale.

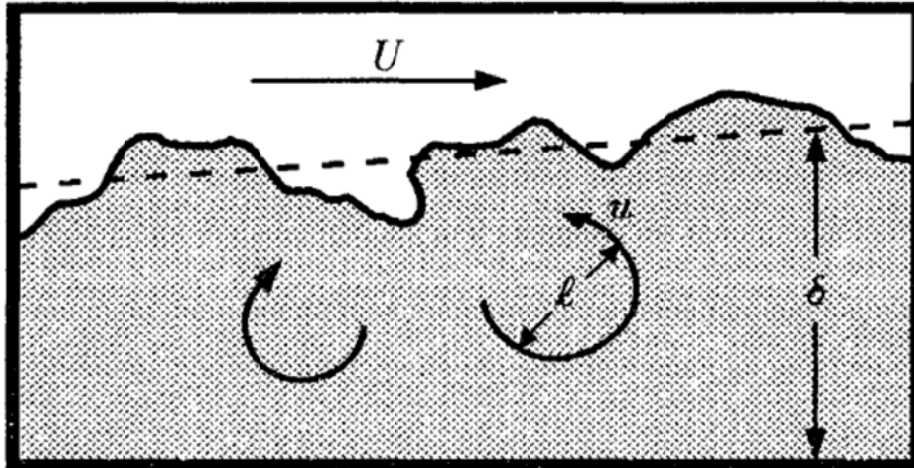


Figure: Schematic of large eddies in a turbulent boundary layer. The flow above the boundary layer has a steady velocity  $U$ ; these eddies move at randomly fluctuating velocities. The largest eddy size ( $\ell$ ) is comparable to the boundary-layer thickness ( $\delta$ ). The interface and the flow above the boundary is quite sharp [Corrsin and Kistler (1954)].

- Alternatively, from a more mathematical point of view, the discussion in terms of wavelengths uses a Fourier analysis of the fluctuating flow properties.
- It is observed that eddies overlap in space, large ones carrying smaller ones. Turbulence features a **cascade process**. As the turbulence decays, their kinetic energy transfer from larger eddies to smaller eddies. Ultimately, the smallest eddies dissipate into heat through the action of molecular viscosity. For any viscous flow, turbulent flows are always dissipative.

### Large Eddies and Turbulent Mixing:

- A special feature of a turbulent flow is the way large eddies migrate across the flow, carrying smaller-scale disturbances. The arrival of these large eddies near the interface between the turbulent region and non-turbulent fluid distorts the interface into a highly convoluted shape.
- Furthermore, they have a lifetime so long that they persist for distances as much as 30 times the width of the flow [Bradshaw (1972)].
- Hence, the state of a turbulent flow at a given position depends upon upstream history and cannot be uniquely specified in terms of the local strain-rate tensor as in laminar flow.

### **Enhanced Diffusivity:**

- This may be the most important feature of turbulence from an engineering point of view is its enhanced diffusivity.
- Turbulent diffusion greatly enhances the transfer of mass, momentum and energy. Apparent stresses in turbulent flows are often several orders of magnitude larger than in corresponding laminar flows.

### **In summary**

- Turbulence is dominated by the large, energy-bearing, eddies.
- The large eddies are primarily responsible for the enhanced diffusivity and stresses observed in turbulent flows.
- Because large eddies persist for long distances, the diffusivity and stresses are dependent upon flow history, and cannot necessarily be expressed as functions of local flow properties.
- Also, while the small eddies ultimately dissipate turbulence energy through viscous action, the rate at which they dissipate is controlled by the rate at which they receive energy from the largest eddies.
- These observations must play an important role in the formulation of any rational turbulence model.
- More specific details of turbulence properties for common flows will be introduced on an as-needed basis.