The smallest scales of Turbulence

* Turbulence is regarded as a Continuum phenomenon because The smallest scales of turbulence are much larger than any molecular length (l_m)

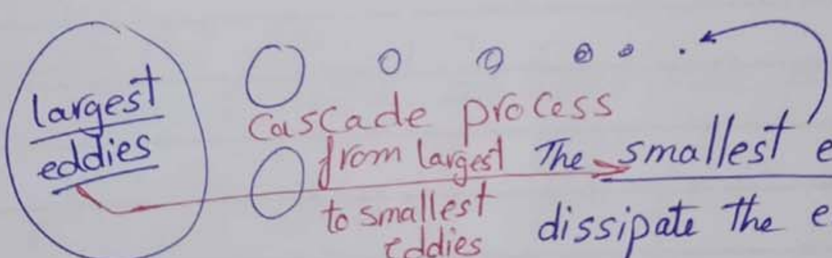
Ex. Car moving at 65 Km/hr, $\eta = 1.8 * 10^{-4}$ m, $l_m = 2.5 * 10^{-6}$ m $\Rightarrow \frac{\eta}{l_m} \approx 7$

* The cascade process presented in all turbulent flows involves of turbulent Kinetic energy per unit mass from larger to smaller eddies

$$K \Rightarrow \frac{N.m}{Kg} = \frac{\cancel{Kg}.m.m}{\cancel{Kg} * s^2} = \frac{m^2}{s^2}$$

* Dissipation of Kinetic energy to heat occurs at the scale of the smallest eddies.

because small scale motion tends to occur on a short time scale.



The smallest eddies dissipate the energy to heat.

* Universal equilibrium theory of Kolmogrov

the smallest scales should only depends upon

a) $\epsilon = -\frac{dK}{dt}$ dissipation $\frac{K}{t} = \frac{m^2}{s^2 * s} = m^2/s^3$

b) ν Kinematic viscosity m^2/s

Kolmogrov scales

* length $\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$ $\left[\left(\frac{m^2}{s}\right)^3 * \frac{s^3}{m^2}\right]^{1/4} = m$

* time $\tau = \left(\frac{\nu}{\epsilon}\right)^{1/2}$ $\left[\frac{m^2}{s} * \frac{s^3}{m^2}\right]^{1/2} = s$

* velocity $v = (\nu\epsilon)^{1/4}$ $\left(\frac{m^2}{s} * \frac{m^2}{s^3}\right)^{1/4} = m/s$

* spectral representation and the Kolmogrov $\frac{-5}{3}$ law

* spectral representation is a Fourier decomposition into wavenumber K or equivently, wavelengths $\lambda = \frac{2\pi}{K}$

$E(K)$ energy spectral density
or energy spectrum function
& is related to Forier transform of K

$$E(K) \propto \nu, \epsilon, K$$

$$\epsilon \sim \frac{K^{3/2}}{l} \Longrightarrow K \sim (\epsilon l)^{2/3} \qquad \frac{m^2}{s^2} \sim \left(\frac{m^2 m}{s^3}\right)^{2/3}$$

length of large scale $\to$ $l \gg \eta \Longrightarrow \frac{l}{\eta} \gg 1$

$$\frac{l}{\eta} = \frac{l}{(\nu^3/\epsilon)^{1/4}} = \frac{l\,\epsilon^{1/4}}{\nu^{3/4}} \sim \frac{l\left(\frac{K^{3/2}}{l}\right)^{1/4}}{\nu^{3/4}} \sim \frac{l^{3/4}\,K^{3/8}}{\nu^{3/4}}$$

$$\sim Re_T^{3/4}$$

$$Re = \frac{\rho v d}{\mu} = \frac{v d}{\nu} = \frac{K^{1/2} l}{\nu}$$

because units of $K = \frac{m^2}{s^2}$

$$\frac{l}{\eta} \gg 1$$

$$\therefore \; Re_T \gg 1$$

$$K = \int_0^\infty E(K)\, dK$$

$$\boxed{E(K) = C_K \, \epsilon^{2/3} K^{-5/3}}$$ $\frac{1}{\ell} \ll K \leq \frac{1}{\eta}$

$C_K \rightarrow$ Kolmogrov Constant

or

$$\boxed{E(K) = \epsilon^{1/4} \nu^{5/4} f(K\eta)}$$

$f(K\eta) \rightarrow$ undetermined function

or

Afzal and Narasimha assumed

* to use the previous equation for small scales & it represents inner solution
* to use the next equation for largest eddies & it represents outer solution

$$E(K) = K\ell \, g(K\ell)$$

$g(K\ell) \rightarrow$ undetermined function

* They insist that the inner & outer solutions are identical

$$\epsilon^{1/4} \nu^{5/4} f(K\eta) = K\ell \, g(K\ell)$$ for $K\eta \ll 1$ & $K\ell \gg 1$

Differentiate both sides

$$\epsilon^{1/4} \nu^{5/4} \eta f'(K\eta) = K\ell \, \ell g'(K\ell)$$

use the values of η & K

$$\nu^2 f'(K\eta) = \epsilon^{2/3} \ell^{8/3} g'(K\ell)$$ for $K\eta \ll 1$ & $K\ell \gg 1$

use the value of $\eta^{8/3} = \nu^2 \epsilon^{-2/3}$

multiply both sides by $K^{8/3}$

$$(K\eta)^{8/3} f'(K\eta) = (Kl)^{8/3} g'(Kl)$$

$K\eta$ & Kl are separate independent variables

from this eqn

a function of independent variable $K\eta$ = a function of independent variable Kl

$\therefore$ both functions are Constant

$$(K\eta)^{8/3} f'(K\eta) = \text{Const.}$$

$$\boxed{f(K\eta) = C_K (K\eta)^{-5/3}}$$

after integration

$$\therefore \boxed{E(K) = C_K \epsilon^{2/3} K^{-5/3}}$$

Kolmogrov $\frac{-5}{3}$ law

and this is the same first eqn of Kolmogrov.

K turbulent Kinetic energy per unit mass

m^2/s^2 $\qquad K = \epsilon^{2/3} l^{2/3} = (\epsilon l)^{2/3}$

$\epsilon = \frac{-dK}{dt}$ dissipation m^2/s^3

ν Kinematic viscosity m^2/s

η length scale m

τ time scale s

u velocity scale m/s

κ wave number

λ wavelength $\lambda = \frac{2\pi}{\kappa}$

$E(\kappa)$ energy spectral density
or enery spectrum function

l length characteristic of larger scale
or integral length scale

Re_T Turbulent Reynolds number $Re_T = \frac{K^{1/2} l}{\nu}$