The law of the wall

* It is one of the most famous empirically - determined relationship in turbulent flow near solid boundaries.

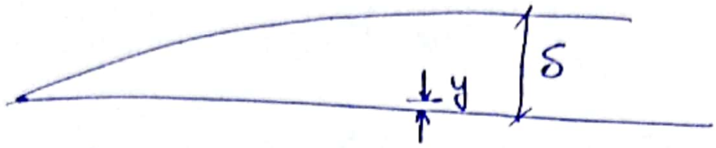
* From measurements, for both internal and external flows the streamwise velocity in the flow near the wall varies logarithmically with distance from surface. law of the wall

* High Re turbulent boundary layers $\xrightarrow{\text{gives}}$ approximate description of the near-surface turbulence statistics

* near the surface in turbulent boundary layer
  - The effect of the fluid's inertia & the pressure gradient are small.
  - statistics of the flow are established by two primary mechanisms :-
    1. rate of transferred momentum to surface per unit area per unit time $\xrightarrow{\text{equal to}}$ local shear stress τ
    2. molecular diffusion of momentum $\longrightarrow$ plays an important role very close to surface.
  - The details of the eddies farther from the surface are of little importance to the near-wall flow statistics.
  - $\frac{y}{\delta}$ decreasing $\longrightarrow$ improve the validity of

this approximate description.
δ is the bounday layer thickness

proof.



- $\dfrac{\text{typical eddy size far from surface}}{\text{eddy size close to surface}} \uparrow$ as $\dfrac{y}{\delta} \downarrow$

- $\delta \uparrow$ $Re \uparrow$
  at high Re a wide separation of scales appears

- τ varies near surface, although y is fairly slow.
  $\longrightarrow$ τ_w : surface shear stress is used in dimensional analysis in place of local shear stress τ

- Turbulence behaves the same in gases and liquids $\longrightarrow$ start with $\frac{\tau_w}{\rho}$ & $\nu = \frac{\mu}{\rho}$

- $\dfrac{\tau_w}{\rho} : \dfrac{m^2}{s^2}$

  $\dfrac{\mu}{\rho} : \dfrac{m^2}{s}$

- length scale $\dfrac{\nu}{u_\tau}$ as $u_\tau \equiv \sqrt{\dfrac{\tau_w}{\rho}}$

as u_τ = friction velocity : velocity scale represent velocities close to solid boundary

$$\frac{\partial U}{\partial y} = \frac{u_\tau}{y} F\left(\frac{u_\tau y}{\nu}\right)$$

Kármán's Constant

$F\left(\frac{u_\tau y}{\nu}\right)$: universal function $\longrightarrow \frac{1}{\kappa}$ as $\frac{u_\tau y}{\nu} \to \infty$

It approaches a constant value

means $\longrightarrow$ viscous effects cease to matter far from surface.

integrate over y

$C \approx 5$ for smooth surfaces

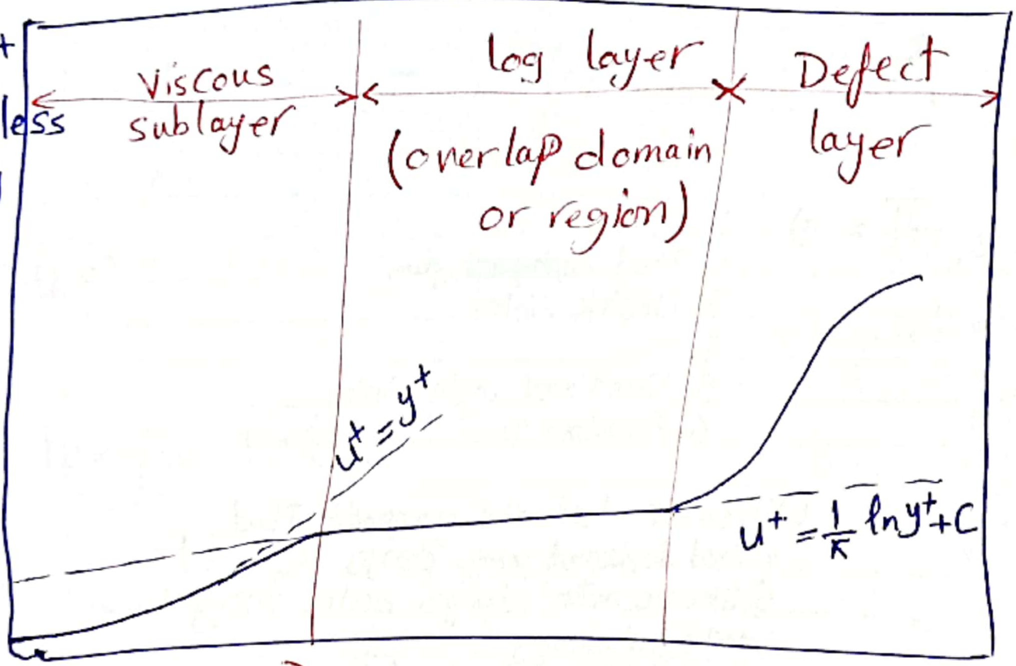
$$\frac{U}{u_\tau} = \frac{1}{\kappa} \ln \frac{u_\tau y}{\nu} + C$$

$$u^+ = \frac{1}{\kappa} \ln y^+ + C$$

$\kappa \approx 0.41$ for smooth & rough surfaces

u^+ dimensionless velocity

$u^+ \equiv \frac{U}{u_\tau}$



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log scale

y^+ dimensionless distance

$y^+ \equiv \frac{u_\tau y}{\nu}$

Definition of log layer

It is the portion of the boundary layer where sub layer and defect layer merge and the law of the wall accurately represents the velocity

* log layer is not a distinct layer.
  It is an overlap region between the inner and outer parts of the boundary layer.

* The velocity in the viscous sublayer

$$U = u_\tau \underbrace{f(y^+)}_{\text{dimensionless function}} \longrightarrow \text{law of the wall}$$

* In the defect layer, numerous experiment found that velocity data correlate reasonably well with the so-called velocity-defect law (clauser defect law)

$$U = U_e - u_\tau \underbrace{g(\eta)}_{\text{dimensionless function}} \qquad \text{as } \eta = \frac{y}{\Delta}$$

U_e: velocity at the boundary-layer edge

Δ: thickness characteristic of the outer portion of the boundary layer

* inner length scale $\dfrac{\nu}{u_\tau}$

  outer length scale Δ

* If a wide separation of scales exists

$$\frac{\nu}{u_\tau} \ll \Delta \qquad \frac{m^2/s}{m/s} = m$$

then an overlap domain exists such that

$$u_\tau f(y^+) = U_e - u_\tau g(\eta) \qquad \text{for } y^+ \gg 1 \ \& \ \eta \ll 1$$

differentiate with respect to y

$$\frac{u_\tau^2}{\nu} f'(y^+) = -\frac{u_\tau}{\Delta} g'(\eta)$$

$* \frac{y}{u_\tau}$

$$y^+ f'(y^+) = -\eta\, g'(\eta)$$

$y^+ = \frac{u_\tau y}{\nu}$ &

$\eta = \frac{y}{\Delta}$

a wide separation of scales means y^+ and η are independent variables

$$y^+ f'(y^+) = \text{Const} = \frac{1}{\kappa}$$

$$f(y^+) = \frac{1}{\kappa} \ln y^+ + C$$

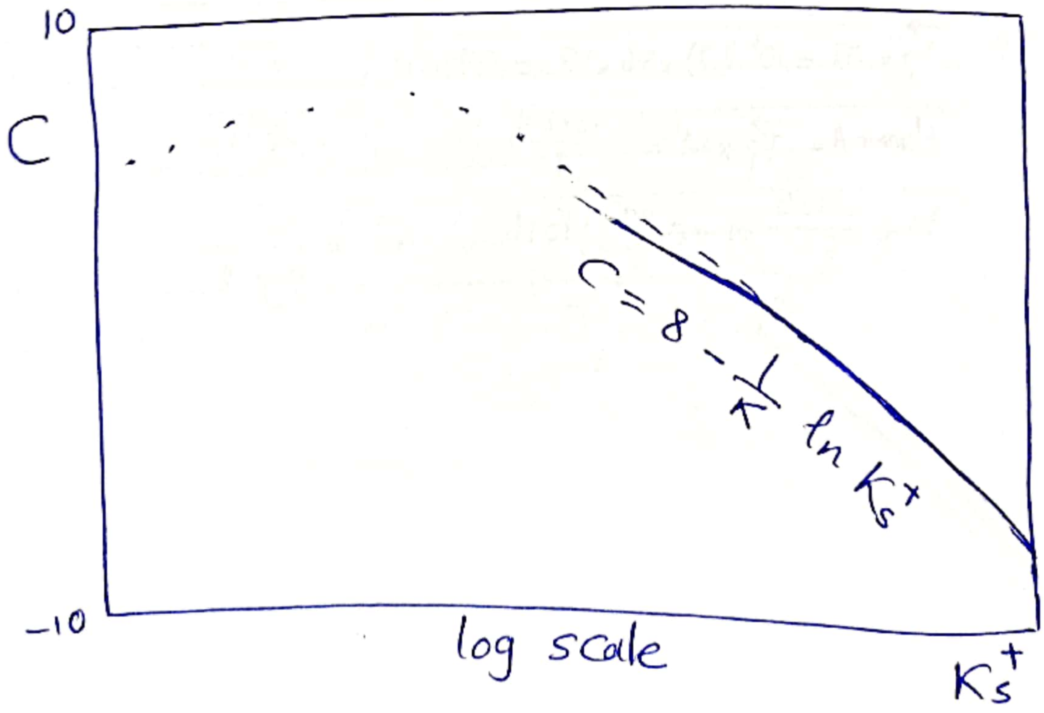
$C \approx 5$ for perfectly-smooth surfaces

* For surfaces with roughness elements of average height K_s

dimensionless roughness $K_s^+ \equiv \frac{u_\tau K_s}{\nu}$

for a large roughnes height $K_s^+ >> 1$

$$C \rightarrow 8 - \frac{1}{\kappa} \ln K_s^+$$



as $K_s \uparrow$ $C \downarrow$

$$\frac{U}{u_\tau} = \frac{1}{\kappa}\ln\frac{u_\tau y}{\nu} + C$$

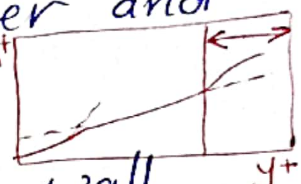
sub. with C

$$\boxed{\frac{U}{u_\tau} = \frac{1}{\kappa}\ln\left(\frac{y}{K_s}\right) + 8}$$

for Completely-rough wall

* The absence of viscosity in this equation
because
the surface "shear stress" is due to pressure drag on the roughness elements.

* Defect layer lies between log layer and the edge of the boundary layer.



the velocity follows the law of the wall as $\frac{y}{\delta} \longrightarrow 0$

Coles' wake-strength parameter ≈ 0.6 from measurements

$$U^+ = \frac{1}{\kappa}\ln y^+ + C + \frac{2\Pi}{\kappa}\sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

boundary layer thickness

Composite law of the wall and law of the wake profile

* The velocity in the defect layer varies like the equilibrium parameter $\longrightarrow \beta_T \equiv \frac{\delta^*}{\tau_w}\frac{dP}{dx}$

mean pressure

displacement thickness

* equilibrium turbulent boundary layer
boundary layer at which β_T is Constant

$$\Pi = 0.6 + 0.51\beta_T - 0.01\beta_T^2$$