A Brief History of Turbulence Modeling

At the nineteenth century, Reynolds published his research's results on turbulence (1895). His pioneering work proved to have such profound importance for all future developments referred to the standard time-averaging process as one type of Reynolds averaging.
A mathematical description of turbulent stresses is developed to mimic (imitate) the molecular gradient-diffusion process. In this spirit, Boussinesq (1877) introduced the concept of a so-called eddy viscosity. The Boussinesq eddy-viscosity approximation is so widely known that few authors find a need to reference his original paper.
Neither Reynolds nor Boussinesq attempted a solution of the Reynolds averaged Navier-Stokes equation in any systematic manner.

Much of the physics of viscous flows was a mystery in the nineteenth century, and further progress awaited Prandtl's discovery of the boundary layer in (1904). Focusing upon turbulent flows, Prandtl (1925) introduced the mixing length (an analog of the mean free path of a gas) and a straightforward prescription for computing the eddy viscosity in terms of the mixing length.

The mixing-length hypothesis, closely related to the eddy-viscosity concept, formed the basis of virtually all turbulence modeling research for the next twenty years. Important early contributions were made by several researchers, most notably by von Karman (1930). In modern terminology, model based on the mixing-length hypothesis referred to as an **algebraic model** or **a zero-equation model of turbulence**. By definition, an n-equation model signifies a model that requires solution of n additional differential transport equations in addition to expressing conservation of mass, momentum and energy for the mean flow.

To improve the ability to predict properties of turbulent flows and to develop a more realistic mathematical description of the turbulent stresses, Prandtl (1945) assumed a model in which the eddy viscosity depends upon the kinetic energy of the turbulent fluctuations, k. He proposed a modeled partial-differential equation approximating the exact equation for k. This improvement, on a conceptual level, takes account of the fact that the turbulent stresses, and thus the eddy viscosity, are affected by where the flow has been, i.e., upon flow history. Thus was born the concept of the so-called **one-equation model of turbulence**.

While having an eddy viscosity that depends upon flow history provides a more physically realistic model, the need to specify a turbulence length scale remains. That is, on dimensional grounds, viscosity has dimensions of velocity x length. Since the length scale can be thought of as a characteristic eddy size and since such scales are different for each flow, turbulence models that do not provide a length scale are **incomplete**. In order to obtain a solution, something about the flow must be known, other than initial and boundary conditions. Incomplete models are not without worth and, in fact, have proven to be of great value in many engineering applications.

To elaborate a bit further, an incomplete model generally defines a turbulence length scale in a prescribed manner from the mean flow, e.g., the displacement thickness (δ^*) for an attached boundary layer. However, a different length scale in this example would be needed when the boundary layer separates since δ^* may be negative. Yet another length might be needed for free shear flows, etc. In essence, incomplete models usually define quantities that may vary more simply or more slowly than the Reynolds stresses (e.g., eddy viscosity and mixing length). Likely, such quantities would prove to be easier to correlate than the actual stresses.

A particularly desirable type of turbulence model would be one that can be applied to a given turbulent flow by prescribing at most the appropriate boundary and/or initial conditions. Ideally, no advance knowledge of any property of the turbulence should be required to obtain a solution. We define such a model as being **complete**. Note that our definition implies nothing regarding the accuracy or universality of the model, only that it can be used to determine a flow with no prior knowledge of any flow details.

Kolmogorov (1942) introduced the first complete model of turbulence. In addition to having a modeled equation for k, he introduced a second parameter ω that he referred to as "the rate of dissipation of energy in unit volume and time" The reciprocal of ω serves as a turbulence time scale, while $k^{1/2}/\omega$ serves as the analog of the mixing length and $k\omega$ is the analog of the dissipation rate, ε. In this model, known as a k-ω model, ω satisfies a differential equation somewhat similar to the equation for k. The model is thus termed a **two-equation model of turbulence**. While this model offered great promise, it went with virtually no applications for the next quarter century because of the unavailability of computers to solve its nonlinear differential equations.

Chou (1945) and Rotta (1951) laid the foundation for turbulence models that avoid the use of the Boussinesq approximation. Rotta invented a plausible (reasonable) model for the differential equation governing evolution of the tensor that represents the turbulent stresses, i.e., the Reynolds-stress tensor. Such models are most appropriately described as stress-transport models. Many authors refer to this approach as second-order closure or second-moment closure. The primary conceptual advantage of a stress-transport model is the natural manner in which nonlocal and history effects are incorporated.

Although quantitative accuracy often remains difficult to achieve, such models automatically accommodate complicating effects such as sudden changes in strain rate, streamline curvature, rigid-body rotation, and body forces. *This stands in distinct contrast to eddy-viscosity models that account for these effects only if empirical terms are added.*

For a three-dimensional flow, a stress-transport model introduces seven equations, one for the turbulence (length or equivalent) scale and six for the components of the Reynolds-stress tensor. As with Kolmogorov's k-ω model, stress-transport models awaited satisfactory computer resources.

Thus, by the early 1950's, four main categories of turbulence models had evolved:

1. Algebraic (Zero-Equation) Models
2. One-Equation Models
3. Two-Equation Models
4. Stress-Transport Models

Algebraic Models.

Van Driest (1956) developed a viscous damping correction for the mixing-length model that is included in virtually all algebraic models in use today.

Cebeci and Smith (1974) refined the eddy-viscosity/mixing-length model to a point that it can be used with great confidence for most attached boundary layers.

To remove some of the difficulties in defining the turbulence length scale from the shear-layer thickness, Baldwin and Lomax (1978) proposed an alternative algebraic model that enjoyed widespread use for many years.

One-Equation Models.

Of the four types of turbulence models described above, the one-equation model has enjoyed the least popularity and success. Perhaps the most successful early model of this type was formulated by Bradshaw, Ferriss and Atwell (1967).

In the 1968 Stanford Conference on Computation of Turbulent Boundary Layers (Coles and Hirst (1969)], the best turbulence models of the day were tested against the best experimental data of the day. In this author's opinion, of all the models used, the Bradshaw-Ferriss-Atwell model most faithfully reproduced measured flow properties.

There has been renewed interest in one-equation models based on a assumed equation for eddy viscosity [c.f. Sekundov (1971), Baldwin and Barth (1990), Goldberg (1991), Spalart and Allmaras (1992) and Menter (1994)]. This work has been motivated primarily by the ease with which such model equations can be solved numerically, relative to two-equation models and stress-transport models.

Of these recent one-equation models, that of Spalart and Allmaras appears to be the most accurate for practical turbulent-flow applications.

Two-Equation Models.

While Kolmogorov’s k-ω model was the first of this type, it remained in insignificance until the coming of the computer.

The most extensive work on two-equation models has been done by Launder and Spalding (1972) and a continuing succession of students and colleagues.

Launder's k-ε model is as well-known as the mixing-length model and, until the last decade of the twentieth century, was the most widely used two-equation model.

Even the model’s demonstrable inadequacy for flows with adverse pressure gradient [c.f. Rodi and Scheuerer (1986), Wilcox (1988a, 1993b) and Henkes (1998a)] initially did little to discourage its widespread use.

With no prior knowledge of Kolmogorov's work, Saffman (1970) formulated a k-ω model that enjoys advantages over the k-ε model, especially for integrating through the viscous sublayer and for predicting effects of adverse pressure gradient.

Wilcox and Alber (1972), Saffman and Wilcox (1974), Wilcox and Traci (1976), Wilcox and Rubesin (1980), Wilcox (1988a, 1998), Menter (1992a), Kok (2000) and Hellsten (2005), for example, have pursued further development and application of k-ω models.

Lakshminarayana (1986) observed that k-ω models had become the second most widely used type of two-equation turbulence model even before the k-ε model’s numerous insufficiencies were widely known.

Stress-Transport Models.

By the 1970’s, sufficient computer resources became available to permit serious development of this class of model. The most noteworthy efforts were those of Donaldson [Donaldson and Rosenbaum (1968)], Daly and Harlow (1970) and Launder, Reece and Rodi (1975). The latter evolved as the baseline stress-transport model despite its dependence on essentially the same flawed (damaged) equation for ε that plagues the k-ε model.

As a concluding comment, turbulence models have been created that fall beyond the bounds of the four categories cited above. This is true because model developers have tried unconventional approaches in an attempt to remove deficiencies of existing models of the four basic classes. Given the erratic track record of most turbulence models, new ideas are always welcome.