Problems

1.1 To appreciate why laminar flow is of minimal importance in many engineering applications, compute the percent of the vehicle over which laminar flow exists for the following situations. In each case, let x_t denote arclength measured from the leading stagnation point of the vehicle or wing and assume transition occurs at a (very high) Reynolds number of $Re_{x_t} = 5 \cdot 10^5$.

(a) A 14-foot automobile moving at 75 mph ($\nu = 1.62 \cdot 10^{-4}$ ft^2/sec).

(b) A 14-foot automobile moving at 25 mph ($\nu = 1.62 \cdot 10^{-4}$ ft^2/sec).

(c) A small aircraft with an average wing chord length of 8 feet moving at 150 mph ($\nu = 1.58 \cdot 10^{-4}$ ft^2/sec).

(d) A *Boeing 747* with an average wing chord length of 30 feet moving at 550 mph ($\nu = 4.25 \cdot 10^{-4}$ ft^2/sec).

1.2 To appreciate why laminar flow is of minimal importance in many engineering applications, compute the percent of the vehicle over which laminar flow exists for the following situations. In each case, let x_t denote arclength measured from the leading stagnation point of the vehicle or wing and assume transition occurs at a (very high) Reynolds number of $Re_{x_t} = 2 \cdot 10^6$. Note that 1 knot = 0.514 m/sec.

(a) A 10-meter sailboat moving at 3.5 knots ($\nu = 1.00 \cdot 10^{-6}$ m^2/sec).

(b) A 10-meter sailboat moving at 7.7 knots ($\nu = 1.00 \cdot 10^{-6}$ m^2/sec).

(c) A 30-meter yacht moving at 12 knots ($\nu = 0.90 \cdot 10^{-6}$ m^2/sec).

(d) A 100-meter tanker moving at 16 knots ($\nu = 1.50 \cdot 10^{-6}$ m^2/sec).

1.3 Using dimensional analysis, deduce the Kolmogorov length, time and velocity scales defined in Equation (1.1).

1.4 Using dimensional analysis, deduce the Kolmogorov $-5/3$ law, Equation (1.8), beginning with the assumption that the energy spectral density, $E(\kappa)$, depends only upon wavenumber, κ, and dissipation rate, ϵ.

1.5 As noted in Subsection 1.3.3, for an automobile moving at 65 mph, the Kolmogorov length scale near the driver's window is $\eta \approx 2 \cdot 10^{-4}$ inch. If $\nu = 1.60 \cdot 10^{-4}$ ft^2/sec, what are the Kolmogorov time and velocity scales? Repeat the computations for a point farther from the surface where $\eta = 0.02$ inch.

1.6 The viscous sublayer of a turbulent boundary layer extends from the surface up to $y^+ \approx 30$. To appreciate how thin this layer is, consider the boundary layer on the side of your freshly washed and waxed (and therefore smooth) automobile. When you are moving at $U = 55$ mph, the skin friction coefficient, c_f, just below your rear-view mirror is 0.0028. Using the fact that

$$U/u_\tau = \sqrt{2/c_f}$$

estimate the sublayer thickness and compare it to the diameter of the head of a pin, which is $d_{pin} = 0.05$ inch. Assume $\nu = 1.68 \cdot 10^{-4}$ ft^2/sec.

1.7 The viscous sublayer of a turbulent boundary layer extends from the surface up to $y^+ \approx 30$. To appreciate how thin this layer is, consider the boundary layer on the hull of a large tanker moving at speed U. Assuming the boundary layer has negligible pressure gradient over most of the hull, you can assume the boundary-layer thickness, δ, and skin friction, c_f, are

$$\delta \approx 0.37 x Re_x^{-1/5}$$

$$c_f \approx 0.0576 Re_x^{-1/5}$$

(a) Noting that $U/u_\tau = \sqrt{2/c_f}$, verify that the sublayer thickness, $\delta_{sl} = 30\nu/u_\tau$, is given by

$$\delta_{sl} \approx \frac{478}{Re_x^{7/10}}\,\delta$$

(b) Compute δ_{sl} at points on the hull where $Re_x = 2.8 \cdot 10^7$ and $\delta = 2.5$ in, and where $Re_x = 5.0 \cdot 10^8$ and $\delta = 25$ in. Express your answer in terms of h_δ/δ_{sl}, to the nearest integer, where $h_\delta = 1/10$ inch is the height of the symbol δ_{sl} on this page.

1.8 A surface is called *hydraulically smooth* when the surface roughness height, k_s, is such that

$$k_s^+ \equiv \frac{u_\tau k_s}{\nu} < 5$$

where u_τ is friction velocity and ν is kinematic viscosity. Consider the flow of air over a flat plate of length 1 m. For the following plate materials, what is the maximum freestream velocity, U, at which the surface will be hydraulically smooth? Assume skin friction is given by $c_f \approx 0.0576 Re_x^{-1/5}$ and that $\nu = 1.51 \cdot 10^{-5}$ m²/sec.

Plate Material	k_s (mm)
Copper	0.0015
Galvanized iron	0.15
Concrete	1.50

1.9 A surface is called *completely rough* when the surface roughness height, k_s, is such that

$$k_s^+ \equiv \frac{u_\tau k_s}{\nu} > 70$$

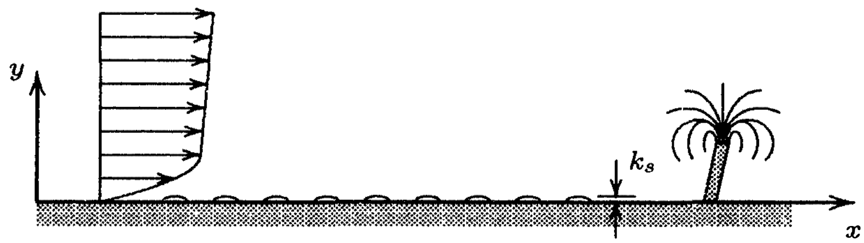
where u_τ is friction velocity and ν is kinematic viscosity. Consider the flow of water over a flat plate. For the following plate materials, what is the minimum freestream velocity, U, at which the surface will be completely rough at $x = 5$ ft? Assume skin friction is given by $c_f \approx 0.0576 Re_x^{-1/5}$ and that $\nu = 1.08 \cdot 10^{-5}$ ft²/sec.

Plate Material	k_s (ft)
Steel	$1.5 \cdot 10^{-4}$
Cast iron	$8.5 \cdot 10^{-4}$
Concrete	$5.0 \cdot 10^{-2}$

1.10 The *atmospheric boundary layer* over a smooth beach is a very large scale turbulent, flat-plate boundary layer, and its boundary-layer thickness and skin friction are accurately represented by

$$\delta \approx 0.37 x Re_x^{-1/5} \qquad \text{and} \qquad c_f \approx 0.0576 Re_x^{-1/5}$$

Suppose you are enjoying a day on the beach and the temperature is 85° F so that the kinematic molecular viscosity is $\nu = 1.72 \cdot 10^{-4}$ ft²/sec. The atmospheric boundary layer is 250 ft thick and the velocity at that altitude is 20 mph. Your forehead is about 6 inches above the ground level. Is your forehead in the sublayer, log layer or defect layer? What is the wind velocity over your forehead?



Problems 1.10, 1.11

1.11 Sunbathers are enjoying a day on the beach. They are lying on the sand with essentially uniform spacing, and their bodies resemble sandgrain roughness elements of height k_s = 30 cm to the *atmospheric boundary layer*. One of the sunbathers is an eager graduate student who decides to use what he learned in this chapter in a practical situation. First, just downstream of a cluster of sunbathers, he measures the wind velocity at head level, $y_1 \approx 1.8$ m, and finds $u_1 = 2.9$ m/sec. He then climbs a palm tree of height $y_2 \approx 5.0$ m and observes a wind velocity of $u_2 = 3.5$ m/sec. Assuming the beach surface is a *completely-rough* surface, what is the friction velocity according to his measurements? To verify the hypothesis that the surface is completely rough, check to see if $u_\tau k_s/\nu > 70$. Assume that $\nu = 1.60 \cdot 10^{-5}$ m²/sec.

1.12 Combining Equations (1.25) and (1.28), verify that the function $g(\eta)$ must be

$$g(\eta) = A - \frac{1}{\kappa}\ell n \eta$$

where A is a function of U_e, u_τ, Δ, ν, κ and C. To have a wide separation of scales, A must be a constant, i.e., it must be independent of Reynolds number. Noting that $U_e/u_\tau = \sqrt{2/c_f}$ and using Clauser's thickness, $\Delta = U_e\delta^*/u_\tau$, where δ^* is displacement thickness, determine the skin friction, c_f, as a function of A, C and $Re_{\delta^*} = U_e\delta^*/\nu$.

1.13 For a turbulent boundary layer, the velocity is given by $u^+ = y^+$ in the sublayer and by the law of the wall, Equations (1.20) and (1.21), in the log layer. Determine by trial and error (or Newton's iterations if you are familiar with the method) the value of y^+ (to the nearest 1/10) at which the sublayer and log-layer velocity profiles are equal.

1.14 We would like to determine the values of Reynolds number, Re, for which the Barenblatt exponent, α, is 1/6, 1/7 and 1/8. Compare the values inferred by using the Barenblatt correlation, Equation (1.39), and the Zagarola correlation, Equation (1.40).

1.15 According to Equation (1.32), at the boundary-layer edge we have

$$\frac{U_e}{u_\tau} = \frac{1}{\kappa}\ell n\frac{u_\tau \delta}{\nu} + C + \frac{2\Pi}{\kappa}$$

We would like to determine how skin friction, $c_f = 2u_\tau^2/U_e^2$, is affected by changes in the quantities C and Π.

(a) Assuming only u_τ varies with C, verify that

$$\frac{1}{c_f}\frac{dc_f}{dC} = -\frac{2\kappa\sqrt{c_f/2}}{\kappa + \sqrt{c_f/2}}$$

(b) Assuming $c_f = 0.002$, based on the result of Part (a), how much of a change in C is required to give a 3% change in c_f? Be sure to include a sign in your result.

(c) Derive a similar result for $(dc_f/d\Pi)/c_f$ and determine the approximate change in c_f for a decrease in Π of 1.0. Assume $c_f = 0.002$.