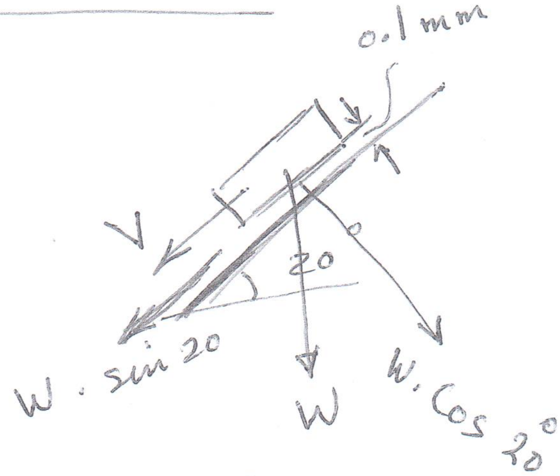
Model Answer
Hydraulics ME 362
25/5/2015

(1)



$$F = W \cdot \sin 20^\circ$$
$$= m * g * \sin 20^\circ$$
$$= 10 * 9.81 * \sin 20^\circ$$
$$= 33.552 \quad N$$

$$\mu = \frac{F * y}{A * V}$$
$$= \frac{33.552 * 0.1 * 10^{-3}}{0.2 * 0.22} = 0.07625 \ \frac{N \cdot s}{m^2}$$

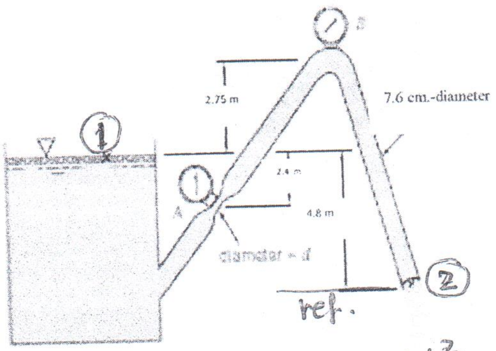
a — $\rho = \frac{\mu}{\nu} = \frac{0.07625}{4.2 * 10^{-4}} = \underline{181.558} \ Kg/m^3$

b — $S = \frac{\rho_{oil}}{\rho_{water}} = \underline{0.1815}$

c — $\gamma = \rho_{oil} * g = 1780.98 \ N/m^3$

d — $\tau = \frac{F}{A} = \frac{33.552}{0.2} = 167.76 \ N/m^2$

2- Water flows steadily from a large open tank and discharges into the atmosphere through a 7.62 cm.-diameter pipe as shown in Fig. Determine the diameter, d, in the narrowed section of the pipe at A if the pressure gages at A and B indicate the same pressure. ***[10 marks]***



$Z_2 = 0$ m
$Z_1 = 4.8$ m
$Z_A = 2.4$ m
$Z_B = 7.55$ m

B.L. 1–2

~~A&C~~
1&C

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$P_1 = P_2 = P_{atm} = 0$ $\quad Z_2 = 0$ $\quad Z_1 = 4.8$ m & $V_1 = 0$

$$V_2 = \sqrt{2gZ_1} = \sqrt{2 * 9.81 * 4.8} = 9.7 \text{ m/s}$$

Bernoulli between A & B

A&B

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B$$

$P_A = P_B$ $\quad Z_A = 2.4$ $\quad Z_B = 2.75 + 4.8 = 7.55$ m

$V_B = V_2 = 9.7$ m/s $\quad \frac{V_B^2}{2g} = 4.8$ m

$\frac{V_A^2}{2g} = 9.95$ m $\therefore V_A = 13.97$ m/s

$$A_2 V_2 = V_A A_A$$

$$\frac{\pi}{4} * (0.76)^2 * 9.7 = \frac{\pi}{4} * d_A^2 * 13.97$$

$d_A = 0.0633$ m $=$ 6.33 cm.

3 -

A closed cylindrical tank filled with water has a hemispherical dome and is connected to an inverted piping system as shown in Fig. P2.25. The liquid in the top part of the piping system has a specific gravity of 0.8, and the remaining parts of the system are filled with water. If the pressure gage reading at A is 60 kPa, determine: (a) the pressure in pipe B, and (b) the pressure head, in millimeters of mercury, at the top of the dome (point C).

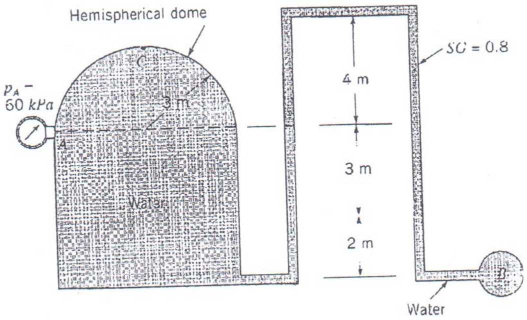


FIGURE P2.25

(a) $p_A + (SG)(\gamma_{H_2O})(3\,m) + \gamma_{H_2O}(2\,m) = p_B$

$p_B = 60\,kPa + (0.8)(9.81 \times 10^3 \frac{N}{m^3})(3\,m) + (9.80 \times 10^3 \frac{N}{m^3})(2\,m)$

$= \underline{\underline{103\,kPa}}$

(b) $p_C = p_A - \gamma_{H_2O}(3\,m)$

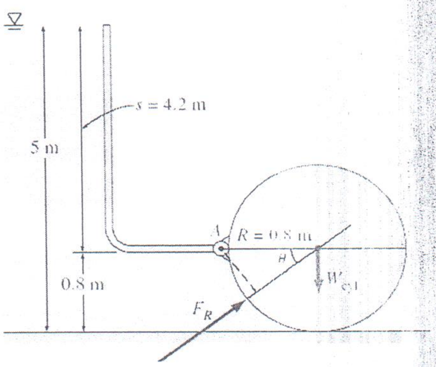
$= 60\,kPa - (9.80 \times 10^3 \frac{N}{m^3})(3\,m)$

$= 30.6 \times 10^3 \frac{N}{m^2}$

$h = \frac{p_C}{\gamma_{Hg}} = \frac{30.6 \times 10^3 \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}} = 0.230\,m$

$= 0.230\,m \left(\frac{10^3\,mm}{m}\right) = \underline{\underline{230\,mm}}$

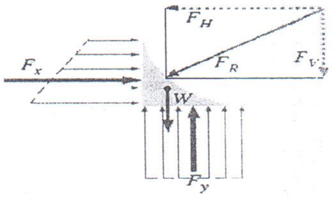
4- Long solid cylinder of radius 0.8 m hinged at point A is used as an automatic gate, as shown in fig. When the water level reaches 5 m, the gate opens by turning about the hinge at point A. Determine :



a- The hydrostatic force acting on the cylinder ant its line of action when the gate opens. ***[5 marks]***

b- The weight of the cylinder per m length of the cylinder. ***[2 marks]***

Horizontal force on vertical surface:



$$F_H = F_x = P_{avg}A = \rho g h_C A = \rho g(s + R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.2 + 0.8/2\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 36.1\ \text{kN}$$

Vertical force on horizontal surface (upward):

$$F_y = P_{avg}A = \rho g h_C A = \rho g h_{\text{bottom}} A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 39.2\ \text{kN}$$

Weight of fluid block for one m width into the page (downward):

$$W = mg = \rho g V = \rho g(R^2 - \pi R^2/4)(1\ \text{m})$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.8\ \text{m})^2(1 - \pi/4)(1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 1.3\ \text{kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.3 = 37.9\ \text{kN}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

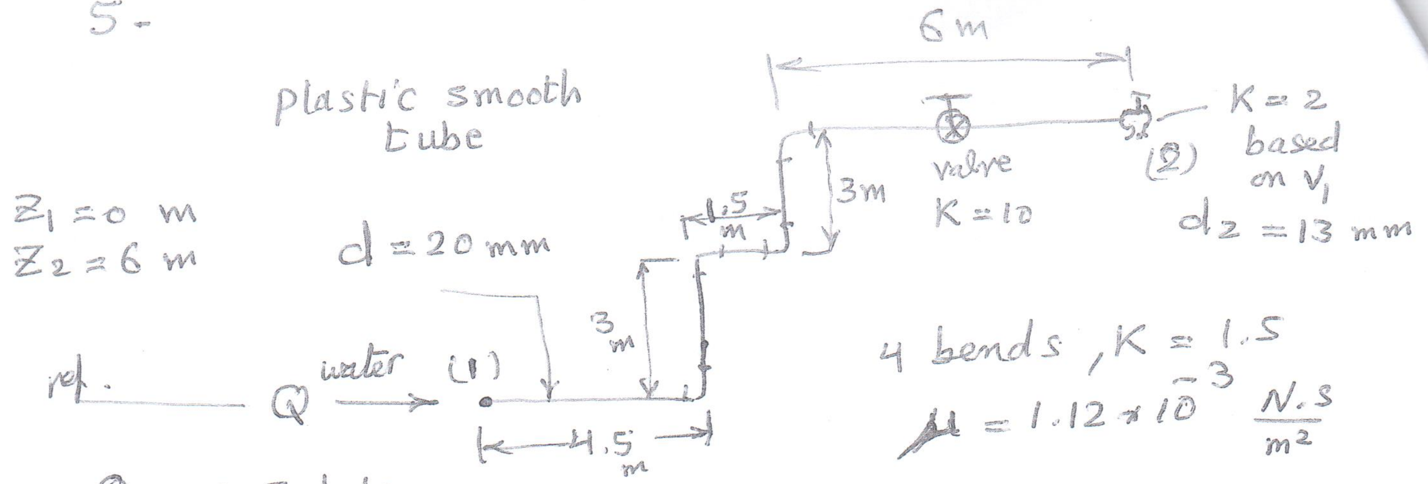
$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = 52.3\ \text{kN}$$

$$\tan\theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4°$$

$$F_R R \sin\theta - W_{cyl} R = 0 \rightarrow W_{cyl} = F_R \sin\theta = (52.3\ \text{kN})\sin 46.4° = 37.9\ \text{kN}$$

Discussion The weight of the cylinder per m length is determined to be 37.9 kN. It can be shown that this corresponds to a mass of 3863 kg per m length and to a density of 1921 kg/m³ for the material of the cylinder.

5-



Q = 1.5 L/s

B.L 1 → 2.

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} - H_L = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

$$V_1 = \frac{Q}{A_1} = \frac{1.5 \times 10^{-3}}{\frac{\pi}{4}(20 \times 10^{-3})^2} = 4.777 \text{ m/s}.$$

$$V_2 = \frac{Q}{A_2} = \frac{1.5 \times 10^{-3}}{\frac{\pi}{4}(13 \times 10^{-3})^2} = 11.3 \text{ m/s}.$$

$$Re = \frac{\rho V_1 d_1}{\mu} = \frac{10^3 \times 4.777 \times 20 \times 10^{-3}}{1.12 \times 10^{-3}} = 8.53 \times 10^4$$

Flow is Turbulent

From moody chart.

$$f = 0.018$$

$$h_f = f \cdot \frac{L}{d_1} \cdot \frac{V_1^2}{2g} = 0.018 \times \frac{18}{20 \times 10^{-3}} \times \frac{(4.777)^2}{2.981} = 18.82 \text{ m water}$$

$$h_m = \Sigma K \frac{V_1^2}{2g} = 18 \times \frac{(4.777)^2}{2 \times 9.81} = 20.94 \text{ m water}$$

$$H_L = h_f + h_m = 39.75 \text{ m water}$$

substituting in B.L equation. ($P_2 = 0$, $Z_1 = 0$, $Z_2 = 6$ m, $V_1 = 4.777$ m/s and $V_2 = 11.3$ m/s)

$$\therefore \frac{P_1}{\gamma} = 32.245 \text{ m water}$$

$$P_1 = 316.3 \text{ kPa}$$