Fluid Mechanics I

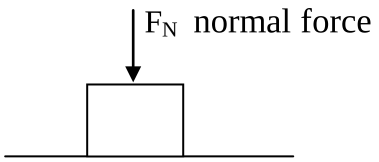
References

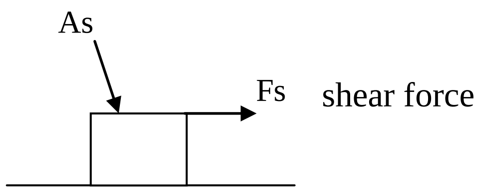
1. "Fluid Mechanics and applications" by Frank M. White.
2. "Fluid Mechanics", By J.F. Douglas, J.M. Grasiore, and J.A. Swaffield.
3. "Fluid Mechanics Fundamentals and Applications" By Yunus A. Cengel, and John M. Cimbala.
4. "Fluid Mechanics and fluid power engineering" By D.S. Kumar.

Fluid:

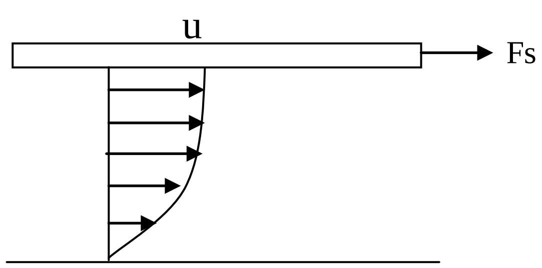
The science that deals with the behavior of fluids at rest (fluid statics) or in motion (fluid dynamics) and the interaction of fluids with solids or other fluids at the boundaries.

It is a substance which deforms continuously under the action of shearing forces, however small they are. This deformation is permanent even if the force is removed.





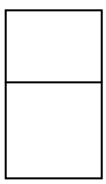
$$\tau = \frac{F_s}{A_s} = \text{shear stress}$$

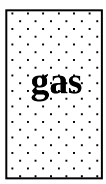


* Difference between solid & fluid?

* Difference between liquid & gas?

Incompressible
&
Free surface





compressible
&
Has no free surface

Units

	B.S units	C.GS	S.I
Force	lb_f	dyne	N
mass	lb	gm	Kg
distance	ft	cm	m
time	S	S	S

SI units

M , L , T , θ Dimensions

		Dim	SI unit
الوحدات الاساسية	Mass Length Time temperature	M L T θ	Kg m S K
الوحدات المشتقة	Velocity acceleration Force pressure	L/T L/T^2 MLt^{-2} $ML^{-1}T^{-2}$	m/s m/s^2 $kg.m/s^2$ =N N/m^2=Pa =kg/ms^2

(work , energy , torque) = N.m = Joule

$$\text{Power} = \frac{N.m}{S} = \frac{J}{S} = \text{watt}$$

Units Conversion

1 ft = 0.3049 m

$1 lb_f$ = 4.448 N & 1 N = 10^5 dyne

1 kw = 1.36 HP

1 kg = 2.2 lb & 1 bar = 10^5 Pa

1 kg_f = 9.81 N

TABLE 1–1

The seven fundamental (or primary) dimensions and their units in SI

Dimension	Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Temperature	kelvin (K)
Electric current	ampere (A)
Amount of light	candela (cd)
Amount of matter	mole (mol)

TABLE 1–2

Standard prefixes in SI units

Multiple	Prefix
10^{12}	tera, T
10^{9}	giga, G
10^{6}	mega, M
10^{3}	kilo, k
10^{2}	hecto, h
10^{1}	deka, da
10^{-1}	deci, d
10^{-2}	centi, c
10^{-3}	milli, m
10^{-6}	micro, μ
10^{-9}	nano, n
10^{-12}	pico, p

Primary dimension	SI unit	BG unit	Conversion factor
Mass $\{M\}$	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length $\{L\}$	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time $\{T\}$	Second (s)	Second (s)	1 s = 1 s
Temperature $\{\Theta\}$	Kelvin (K)	Rankine (°R)	1 K = 1.8°R

$$\text{Force} = (\text{Mass})(\text{Acceleration})$$
$$F = ma$$
$$W = mg \quad (\text{N})$$

Dimensional Homogeneity

Example

A useful theoretical equation for computing the relation between pressure, velocity, and altitude in a steady flow of a nearly inviscid, nearly incompressible fluid with negligible heat transfer and shaft work is:

$$p_0 = p + \tfrac{1}{2}\rho V^2 + \rho g Z$$

where p_0 = stagnation pressure
p = pressure in moving fluid
V = velocity
ρ = density
Z = altitude
g = gravitational acceleration

(a) Show that Eq. (1) satisfies the principle of dimensional homogeneity, which states that all additive terms in a physical equation must have the same dimensions. (b) Show that consistent units result without additional conversion factors in SI units.

Solution

Part (a) We can express Eq. (1) dimensionally, using braces by entering the dimensions of each term from Table 1.2:

$$\{ML^{-1}T^{-2}\} = \{ML^{-1}T^{-2}\} + \{ML^{-3}\}\{L^2T^{-2}\} + \{ML^{-3}\}\{LT^{-2}\}\{L\}$$
$$= \{ML^{-1}T^{-2}\} \text{ for all terms} \qquad \textit{Ans. (a)}$$

Part (b) Enter the SI units for each quantity from Table 1.2:

$$\{\text{N/m}^2\} = \{\text{N/m}^2\} + \{\text{kg/m}^3\}\{\text{m}^2/\text{s}^2\} + \{\text{kg/m}^3\}\{\text{m/s}^2\}\{\text{m}\}$$
$$= \{\text{N/m}^2\} + \{\text{kg/(m}\cdot\text{s}^2)\}$$

Example

1.9 According to information found in an old hydraulics book, the energy loss per unit weight of fluid flowing through a nozzle connected to a hose can be estimated by the formula

$$h = (0.04 \text{ to } 0.09)(D/d)^4 V^2/2g$$

where h is the energy loss per unit weight, D the hose diameter, d the nozzle tip diameter, V the fluid velocity in the hose, and g the acceleration of gravity. Do you think this equation is valid in any system of units? Explain.

$$h = (0.04 \text{ to } 0.09)\left(\frac{D}{d}\right)^4 \frac{V^2}{2g}$$

$$\left[\frac{FL}{F}\right] \doteq [0.04 \text{ to } 0.09]\left[\frac{L^4}{L^4}\right]\left[\frac{1}{2}\right]\left[\frac{L^2}{T^2}\right]\left[\frac{T^2}{L}\right]$$

$$[L] \doteq [0.04 \text{ to } 0.09][L]$$

Since each term in the equation must have the same dimensions, the constant term (0.04 to 0.09) must be dimensionless. Thus, the equation is a general homogeneous equation that is valid in any system of units. Yes.

Example

1.12 A formula to estimate the volume rate of flow, Q, flowing over a dam of length, B, is given by the equation

$$Q = 3.09BH^{3/2}$$

where H is the depth of the water above the top of the dam (called the head). This formula gives Q in ft^3/s when B and H are in feet. Is the constant, 3.09, dimensionless? Would this equation be valid if units other than feet and seconds were used?

$$Q = 3.09\, B\, H^{3/2}$$

$$[L^3T^{-1}] \doteq [3.09][L][L]^{3/2}$$

$$[L^3T^{-1}] \doteq [3.09][L]^{5/2}$$

Since each term in the equation must have the same dimensions the constant 3.09 must have dimensions of $L^{1/2}T^{-1}$ and is therefore not dimensionless. No.

Since the constant has dimensions its value will change with a change in units. No.

Properties of fluids

* **Density :** mass per unit volume $\rho = \dfrac{m}{V}$

Dim. $\dfrac{M}{L^3}$ for water $\rho = 1000\ kg/m^3$

* **Specific weight :** weight per unit volume

$$\gamma = \frac{weight}{volume} = \frac{m * g}{V} = \rho g$$

Dim. $\dfrac{ML}{T^2} * \dfrac{1}{L^3}$, for water $\gamma = 1000 * 9.8 \quad \dfrac{N}{m^3}$

* **Specific volume :** volume per unit mass

$$v = \frac{volume}{mass} = \frac{1}{\rho} \quad m^3/kg$$

For water $v = 0.001 \quad m^3/kg$

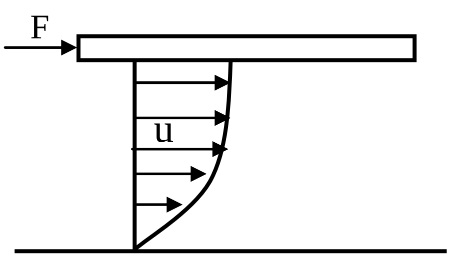
* **Specific gravity** : $SG = \dfrac{\text{Sp. Weight of fluid}}{\text{Sp. Weight of water}}$

$$= \frac{\gamma_f}{\gamma_w} = \frac{\rho_f\ g}{\rho_w\ g} = \frac{\rho_f}{\rho_w} \quad \text{dimensionless}$$

For water $SG_W = 1$

*** Viscosity** (μ): The property which causes friction between fluid and boundary or between fluid layers if they is velocity difference.

It's a property that represents the internal resistance of a fluid to motion or the "fluidity". The viscosity of a fluid is a measure of its "resistance to deformation."



μ = viscosity

= Absolute viscosity

= Dynamic viscosity

= Coefficient of viscosity

$F_{viscous} = F$

$$F_{viscous} \;\alpha\; A_{friction} \frac{du}{dy}$$

$$\boxed{F_{vis} = Const\ .\ A_{friction} \frac{du}{dy}}$$

$$\boxed{F_{vis} = \mu\ A_{friction\ moving} \frac{du}{dy}} \longleftarrow$$ Newton's law of viscosity

μ = coefficient of viscosity depends on type of fluid and its temperature

$$\mu = \frac{Fvis}{Afriction} * \frac{dy}{du}$$

for water $\mu = 0.001 \frac{N.s}{m^2}$

$= 0.01 \frac{dyne\ .\ s}{cm^2}$

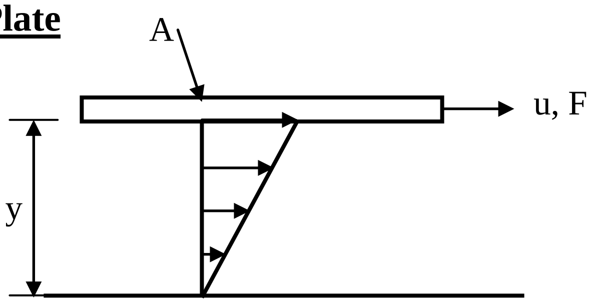
$= 0.01$ poise

$= 1$ centi poise

Units of $\mu = \frac{N.}{m^2} * \frac{m}{m/s} = P_a.S$

For a small thickness of fluid layer , velocity distribution car be assumed straight line. $\frac{du}{dy} = \frac{\Delta U}{\Delta y}$

1- Flat Plate



$$\tau \;\alpha\; \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

$$F = \mu\ A \frac{du}{dy}$$

as : τ : shear stress

μ : viscosity

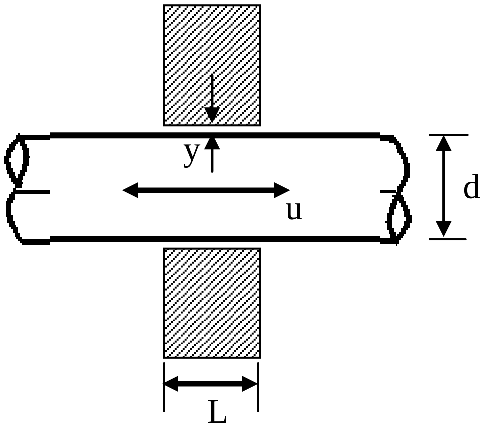
$\frac{du}{dy}$: rate of shear strain

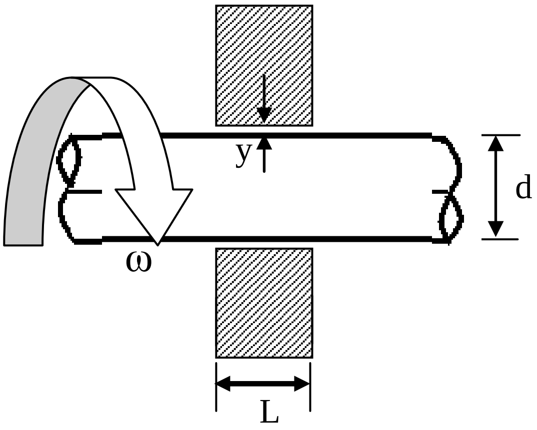
F : viscous force

2- Moving Shaft

$F = \mu A \dfrac{V}{y}$

Where $A = \pi\ d\ L$





3- Rotating Shaft

$F = \mu A \dfrac{u}{y}$

$u = \omega\ r$

$r = \dfrac{d}{2}$

If N is given $\therefore\ \omega = \dfrac{2\pi N}{60}$

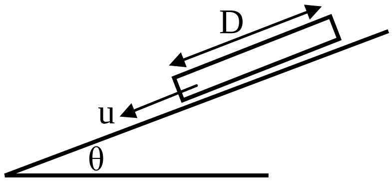
as N : rpm & ω : rad /sec

If rps $\xrightarrow{convert}$ ω rad /sec

$$\omega = 2\pi(rps)$$

4 - Sliding Disk

$A = \frac{\pi}{4} D^2$



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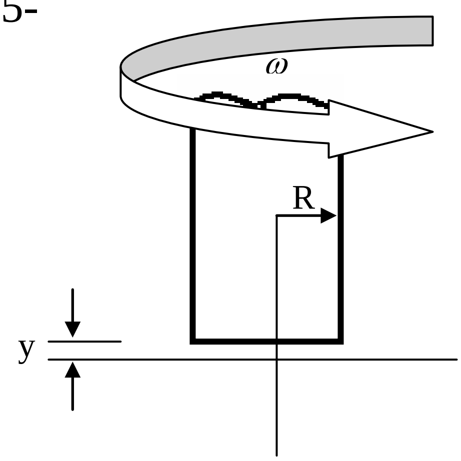
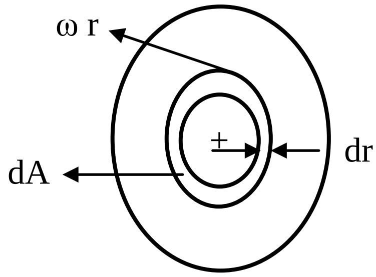




$$dA = 2\pi\ rdr \quad \& \quad u = \omega r$$

$$dF = \mu(2\pi\ rdr) * \frac{\omega r}{y}$$

$$F = \frac{\mu.2\pi\omega}{y} \int_0^R r^2 dr$$

$$\boxed{F = \frac{2\pi\omega\mu}{3y} R^3}$$

dT = r.dF

$$= r * \frac{\mu\ 2\pi\ rdr\ \omega r}{y}$$

$$\int_0^T dT = \frac{2\pi\mu\omega}{y} \int_0^R r^3 dr$$

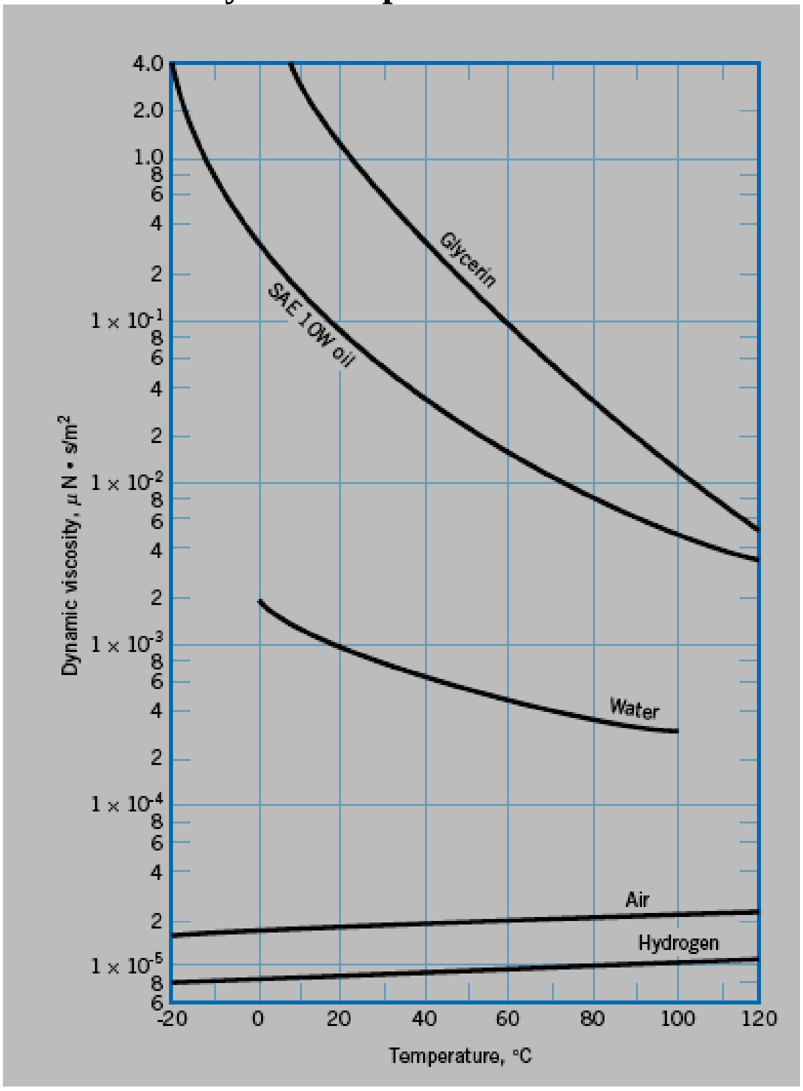
$$\boxed{T = \frac{2\pi\mu\omega}{4y} R^4}$$

* power = T * ω

= F * r * ω

= F * v

*** Relation between viscosity and temperature for a certain fluid**

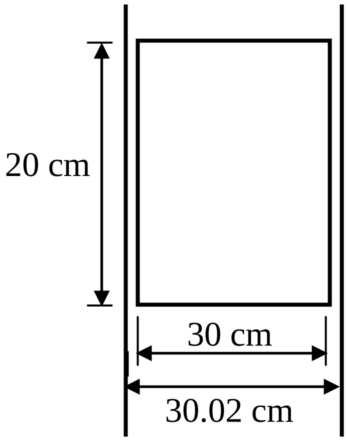


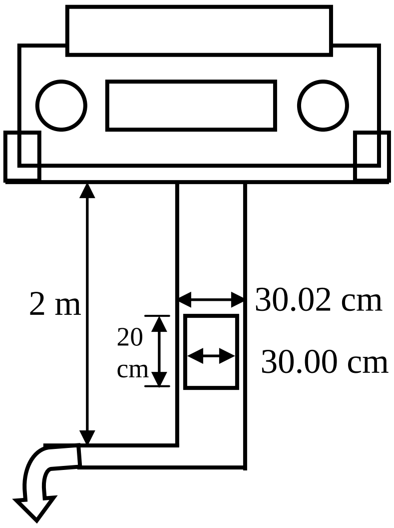
<u>Example</u>

For the shown position of car lifting system

<u>Calculate:</u> the force required to overcome friction when the piston moves at 2m/s

take μ_{oil} = 0.02 Pa.s





$$F_{vis} = \mu A \frac{du}{dy}$$

$$= 0.02 * (\pi d L) * \frac{u}{\left(\frac{D-d}{2}\right)}$$

$$= 0.02 * \left(\pi * \frac{30}{100} * \frac{20}{100}\right) * \frac{2}{\left(\frac{30.02-30}{200}\right)}$$

= Newton

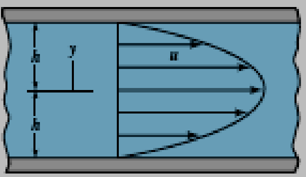
Example

EXAMPLE 1.5

The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates (see Fig. E1.5) is given by the equation

$$u = \frac{3V}{2}\left[1 - \left(\frac{y}{h}\right)^2\right]$$

where V is the mean velocity. The fluid has a viscosity of 0.04 lb · s/ft². When V = 2 ft/s and h = 0.2 in. determine: (a) the shearing stress acting on the bottom wall, and (b) the shearing stress acting on a plane parallel to the walls and passing through the centerline (midplane).



■ FIGURE E1.5

SOLUTION

For this type of parallel flow the shearing stress is obtained from Eq. 1.9,

$$\tau = \mu \frac{du}{dy} \qquad (1)$$

Thus, if the velocity distribution $u = u(y)$ is known, the shearing stress can be determined at all points by evaluating the velocity gradient, du/dy. For the distribution given

$$\frac{du}{dy} = -\frac{3Vy}{h^2} \qquad (2)$$

(a) Along the bottom wall $y = -h$ so that (from Eq. 2)

$$\frac{du}{dy} = \frac{3V}{h}$$

and therefore the shearing stress is

$$\tau_{\substack{\text{bottom}\\\text{wall}}} = \mu\left(\frac{3V}{h}\right) = \frac{(0.04\ \text{lb}\cdot\text{s/ft}^2)(3)(2\ \text{ft/s})}{(0.2\ \text{in.})(1\ \text{ft}/12\ \text{in.})}$$

$$= 14.4\ \text{lb/ft}^2\ \text{(in direction of flow)} \qquad \textbf{(Ans)}$$

This stress creates a drag on the wall. Since the velocity distribution is symmetrical, the shearing stress along the upper wall would have the same magnitude and direction.

(b) Along the midplane where $y = 0$ it follows from Eq. 2 that

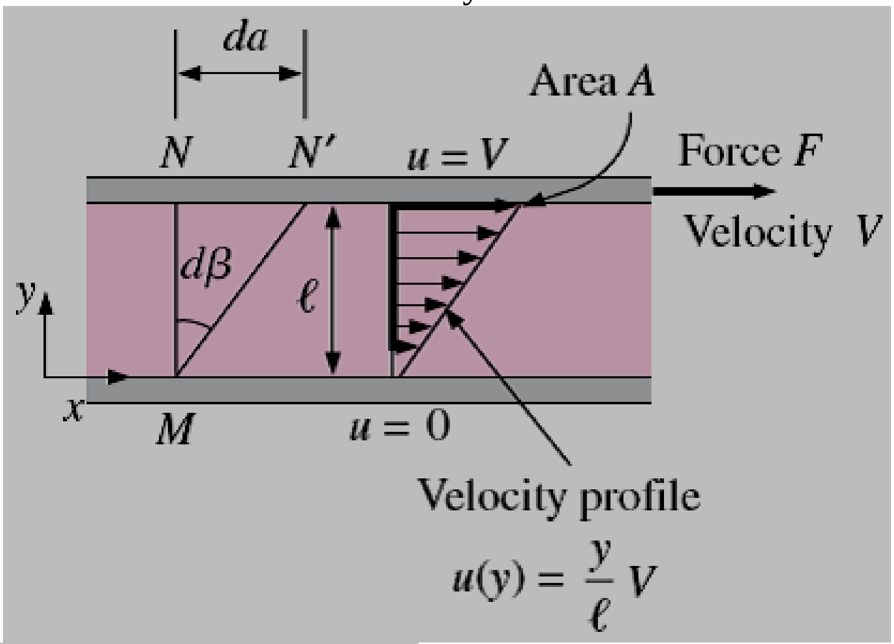
$$\frac{du}{dy} = 0$$

and thus the shearing stress is

$$\tau_{\text{midplane}} = 0 \qquad \textbf{(Ans)}$$

From Eq. 2 we see that the velocity gradient (and therefore the shearing stress) varies linearly with y and in this particular example varies from 0 at the center of the channel to 14.4 lb/ft^2 at the walls. For the more general case the actual variation will, of course, depend on the nature of the velocity distribution.

The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.



$$u(y) = \frac{y}{\ell}V \qquad \text{and} \qquad \frac{du}{dy} = \frac{V}{\ell}$$

$$d\beta \approx \tan d\beta = \frac{da}{\ell} = \frac{V\,dt}{\ell} = \frac{du}{dy}\,dt$$ **then** $$\frac{d\beta}{dt} = \frac{du}{dy}$$

Fluids for which the rate of deformation is proportional to the shear stress

$$\tau \propto \frac{d(d\beta)}{dt} \qquad \text{or} \qquad \tau \propto \frac{du}{dy}$$

<u>* Kinematic viscosity</u> (υ):

It is defined as the ratio of dynamic viscosity to density

$$\upsilon = \frac{\mu}{\rho} = \frac{Pa.S}{kg/m^3} = \frac{kg.m.s\ m^3}{s^2 m^2\ kg} = (m^2/s)$$

For water υ = 0.01 cm^2/s

= 0.01 stoke as stoke = cm^2/s

= 1 centi stokes

<u>* Newtonian & Non - Newtonian:</u>

$$\tau = \frac{F_{vis}}{A} = \mu \frac{du}{dy}$$

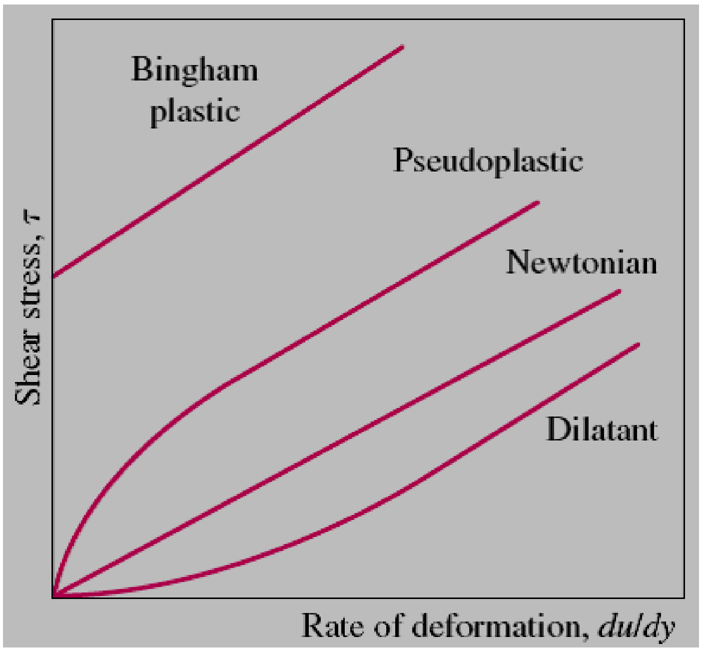
as τ : Shear stress

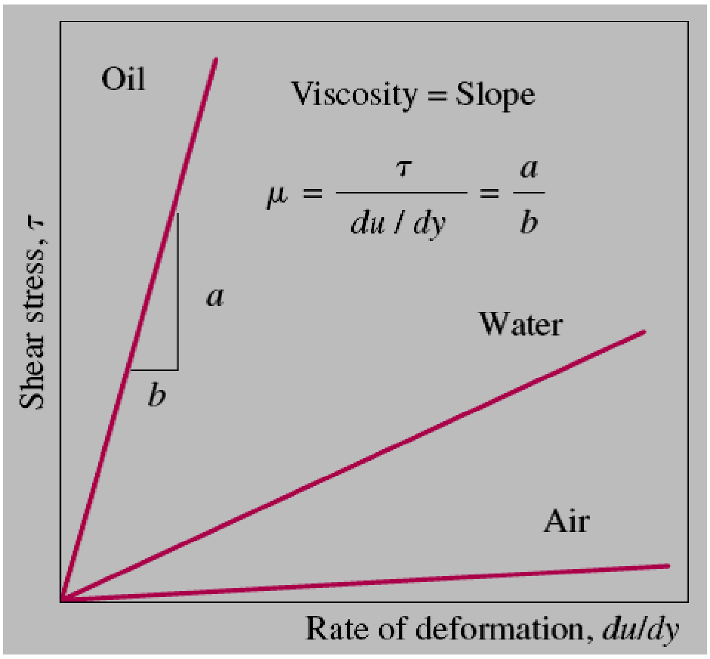
$\frac{du}{dy}$: rate of shear strain

If $\tau \ \alpha \ \frac{du}{dy}$ $\therefore$ μ = const. $\therefore$ It is a Newtonian fluid

μ = const. ⟶ Newtonian fluid

μ ↑↓ ⟶ Non Newtonian

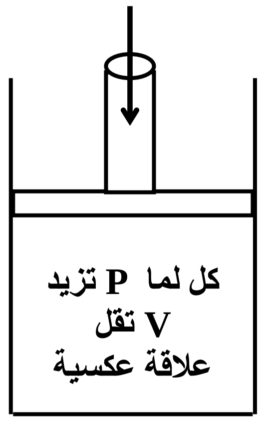




* **Bulk modulus of elasticity (k)**

It's the rate at which the pressure changes with volumetric strain ($\Delta V / V$)

$$K = \frac{-\Delta P}{\Delta V / V}$$



Large value of k

means that big change of pressure Causes small change of volume

$\therefore$ The fluid is incompressible

$k = \ldots\ldots\ldots * 10^{9}\ (P_a)$

Small value of k

means that small change of pressure causes a large change of volume

$\therefore$ The fluid is compressible

$k = \ldots\ldots\ldots * 10^{6}\ (P_a)$

The quantity $\beta = 1/k$ is called the compressibility of the fluid.

$$1/\beta = k = \frac{-\Delta P}{\Delta V / V}$$

Notice that the greater the value of the bulk modulus of elasticity (k), the smaller the value of compressibility (β).

* **Vapour pressure of liquids** (P_{vap}):

It is the pressure at which a liquid start to boil at working temperature.

Boling temp increases by increasing pressure on liquid surface.

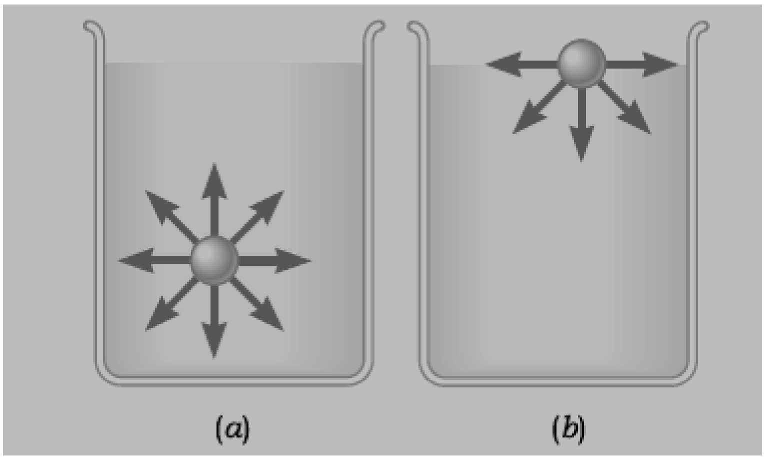
Boling temp decreases by decreasing pressure on liquid surface.

P	0.3	0.5	1	4	10	atm.
Boling temp	40	70	100	120	180	ºC

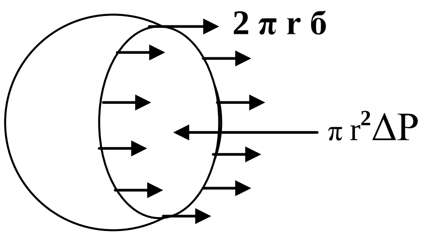
To avoid vapor formation $P_{min} > P_{vap}$

* Surface tension

- Surface tension is a property of liquids which is felt at the interface between the liquid and another fluid (typically a gas).
- Surface tension has dimensions of force per unit length.
- Surface molecules are subject to an attractive force from nearby surface molecules so that the surface is in a state of tension.



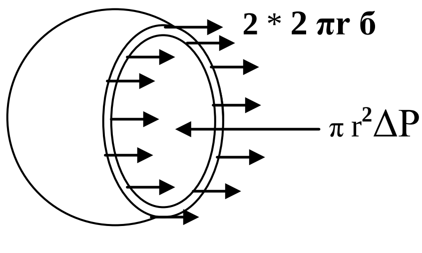
- A molecule within the bulk liquid is surrounded on all sides by other molecules, which attract it equally in all directions, leading to a zero net force.
- A molecule in the surface experiences a net attractive force pointing toward the liquid interior, because there are no molecules of the liquid above the surface.



Half of droplet

$2 \pi r \text{б} = \pi r^2 \Delta P$

$$\Delta P = \frac{2\pi r \text{б}}{\pi r^2} = \frac{2\text{б}}{r}$$



half of a soap bubble

$4 \pi r \sigma = \pi r^2 \Delta P$

$$\Delta P = \frac{4\text{б}}{r}$$

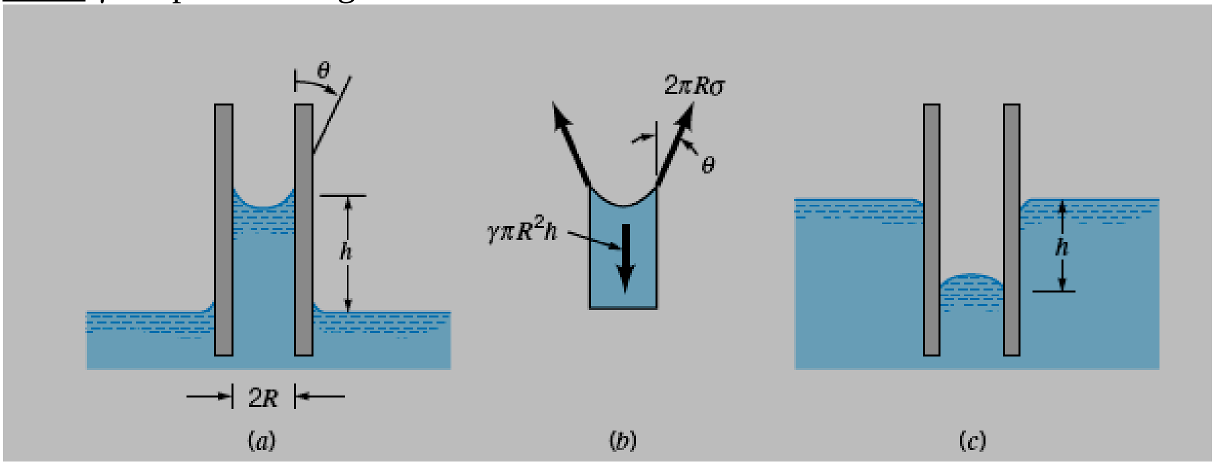
* Capillary effect

Capillary effect: The rise or fall of a liquid in a small-diameter tube inserted into the liquid.

Capillaries: Such narrow tubes or confined flow channels.

The capillary effect is partially responsible for the rise of water to the top of tall trees.

Note: γ = Specific weight



Effect of capillary action in small tubes. (a) raise of column for a liquid that wet the tube (b) Free-body diagram for calculating column height (c) Depression of column for non-wetting liquid.

If adhesion of molecules to the solid surface is weak compared to the cohesion between molecules, the liquid will not wet the surface and the level in a tube placed in a non-wetting liquid will actually be depressed as shown in Fig. c. Mercury is a good example of a non-wetting liquid when it is in contact with a glass tube.

$$\gamma \pi R^2 h = 2\pi R \sigma \cos\theta$$

so that the height is given by the relationship

$$h = \frac{2\sigma \cos\theta}{\gamma R}$$

Example:

1.86 Two vertical, parallel, clean, glass plates are spaced a distance of 2 mm apart. If the plates are placed in water how high will the water rise between the plates due to capillary action?

For equilibrium in the vertical direction,

$$W = 2(\sigma \ell \cos\theta)$$

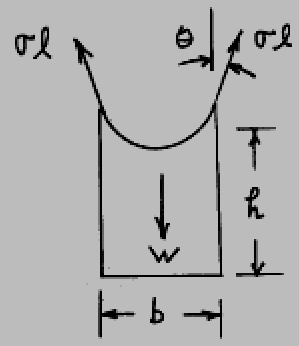
Since,

$$W = \gamma h b \ell$$

$$\gamma h b \ell = 2 \sigma \ell \cos\theta$$

or,

$$h = \frac{2\sigma \cos\theta}{\gamma b}$$



($\ell \sim$ width of plates)

Thus, (for $\theta = 0$)

$$h = \frac{2\,(7.34\times10^{-2}\,\frac{N}{m})(1)}{(9.80\times10^{3}\,\frac{N}{m^3})(0.002\,m)} = 7.49\times10^{-3}\,m = \underline{\underline{7.49\,mm}}$$