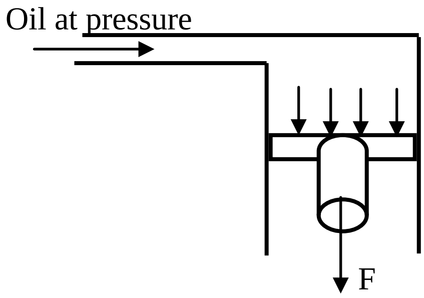
Fluid Statics

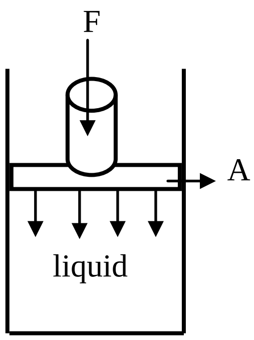
- Fluid Statics deals with problems associated with fluids at rest.
- In fluid statics, there is no relative motion between adjacent fluid layers.
- Therefore, there is no shear stress in the fluid trying to deform it.
- The only stress in fluid statics is *normal stress*
  - ✓ Normal stress is due to pressure
  - ✓ Variation of pressure is due only to the weight of the fluid → fluid statics is only relevant in presence of gravity fields.
- Applications: Floating or submerged bodies, water dams and gates, liquid storage tanks, etc.

- Pressure : is the Normal force per unit area

$$p = \frac{F}{A}$$

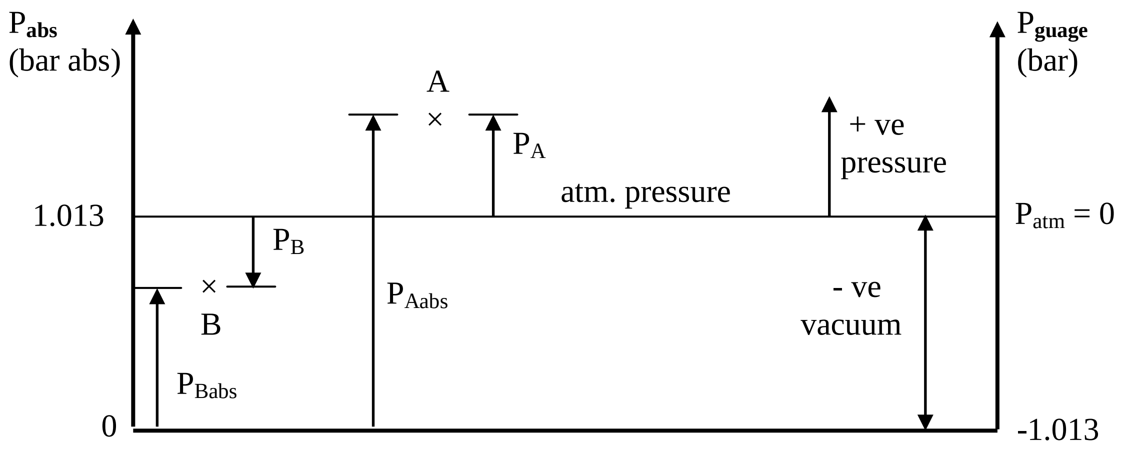
$$Pa = \frac{N}{m^2}$$





* Absolute, atmospheric and guage pressure

Absolute pressure **guage pressure**



Absolute pressure = true pressure

$$\boxed{P_{abs} = P_{guage} + P_{atm}}$$

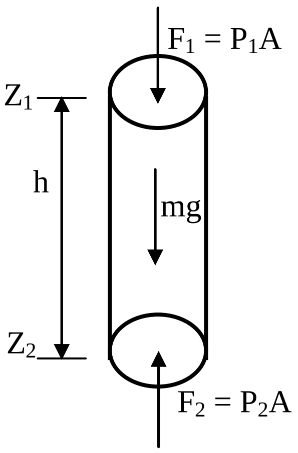
* All given values for pressure are guage except if :

1. (abs) is mentioned beside the unit.
2. Dealing with atmospheric pressure.
3. Dealing with vapour pressure.

* No pressure guage value less than -1.013 bar
* 1 bar = 10^5 pascal
* -ve pressure is called vacuum

* In a static liquid :

1. The pressure, at a certain point, is the same in all directions.
2. The pressure is constant in the same horizontal plane.
3. The pressure changes in the vertical direction.



$$\rho = \frac{m}{V} \qquad \therefore m = \rho V = \rho A h$$

$$\sum F_y = 0 \quad \downarrow +$$

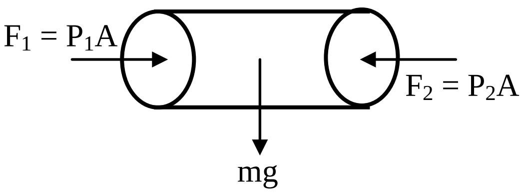
$$F_1 + mg - F_2 = o$$

$$P_1 A + \rho A (Z_2 - Z_1) g - P_2 A = o \qquad \div A$$

$$P_1 + \rho h g - P_2 = o$$

$$\boxed{P_2 - P_1 = \rho\ g\ h} \quad \text{or} \quad \boxed{P_2 - P_1 = \gamma\ h}$$

$$\rightarrow + \sum F_x = o$$

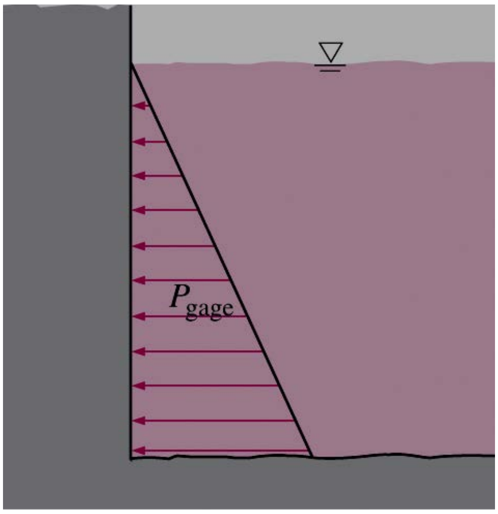


$$F_1 - F_2 = o$$

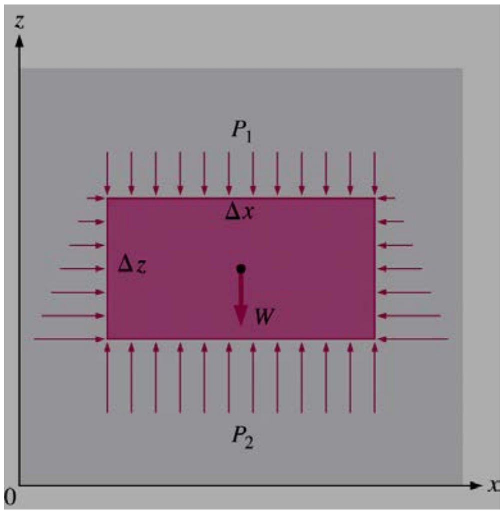
$$P_1 A - P_2 A = o \qquad \div A$$

$$P_1 - P_2 = o$$

$$\boxed{P_1 = P_2}$$



The pressure of a fluid at rest increases with depth (as a result of added weight).



Free-body diagram of a rectangular fluid element in equilibrium.

* Pressure and head

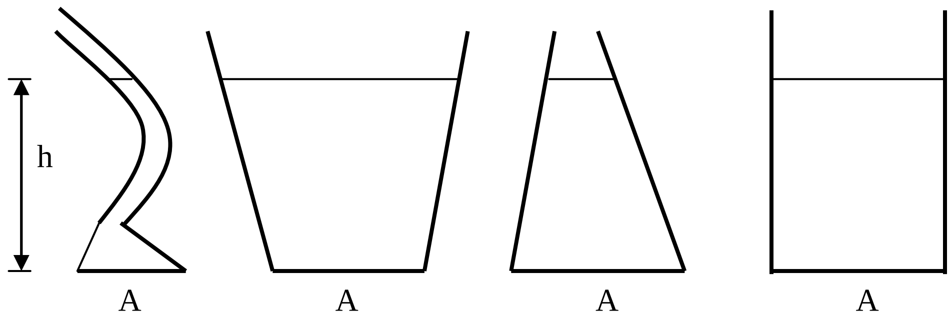
$$P = \gamma\, h$$

Pressure — head

N/m^2 — m of liquid

* head : is the vertical length that can define the pressure.

*The hydrostatic paradox :



$P_{\mathbf{bottom}} = \gamma\, h$ then $F_{\mathbf{bottom}} = PA = \gamma\, hA$

Although the weight of fluid is different, the force in the base of the four vessels is the same. This force depends on the depth (h) and the base area A.

Example :

A cylinder contains a fluid at pressure of 350 KN/m^2

- Express the pressure in terms of a head of :-

a) Water $\rho_\omega = 1000\ kg/m^3$ b) Mercury $SG_m = 13.6$

- Determine the absolute pressure if $P_{atm} = 101.3\ KN/m^2$?

Solution

$$P = \gamma h = \rho g h$$

a) $350 * 10^3 = 1000 * 9.81 * h_w$ $\therefore\ h_w = 35.68$ m of water

b) $P = SG_m \rho_\omega g h$ $SG_m = \frac{\rho_m}{\rho_\omega}$ $\rho_m = SG_m\ \rho_\omega$

$350 * 10^3 = 13.6 * 1000 * 9.81 * h_m$ $\therefore\ h_m = 2.62$ m of mercury

$$P_{abs} = P_{gage} + P_{atm}$$

$$= 350 * 10^3 + 101.3 * 10^3$$

$$= 451300\ N/m^2 * 10^{-3}$$

$$= 451.3\ KN/m^2$$

Example :

If $h_{atm} = 76$ cm Hg, determine P_{atm}?

Solution

$$P_{atm} = \gamma_m h \qquad SG_m = \frac{\gamma_m}{\gamma_w}$$

$$= SG_m\ \gamma_w\ h \qquad \gamma_m = SG_m\ \gamma_w$$

$$= 13.6 * 9800 * (76 * 10^{-2})$$

$$= 1.013 * 10^5\ N/m^2$$

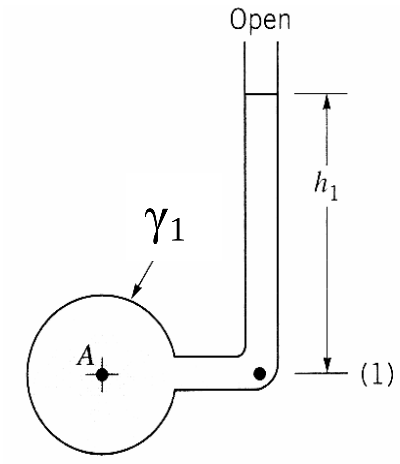
* pressure measurements by manometers

* Piezometer

Pressure tube or piezometer

Consists of a single vertical tube

$$P_A = \gamma_1 \, h_1$$



* U- tube manometer

Statics

Same horizontal plane

* to make pressure equivalence

1 – Still liquid

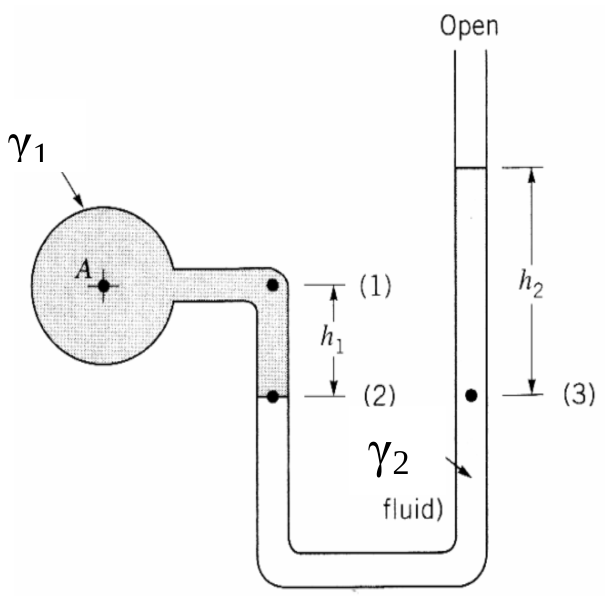
2 – Continues liquid

3 – Same liquid

$$P_I = P_{II}$$

$$P_A + \rho_1 g h_1 = \rho_2 g h_2$$

$$P_A + \gamma_1 h_1 = \gamma_2 h_2$$



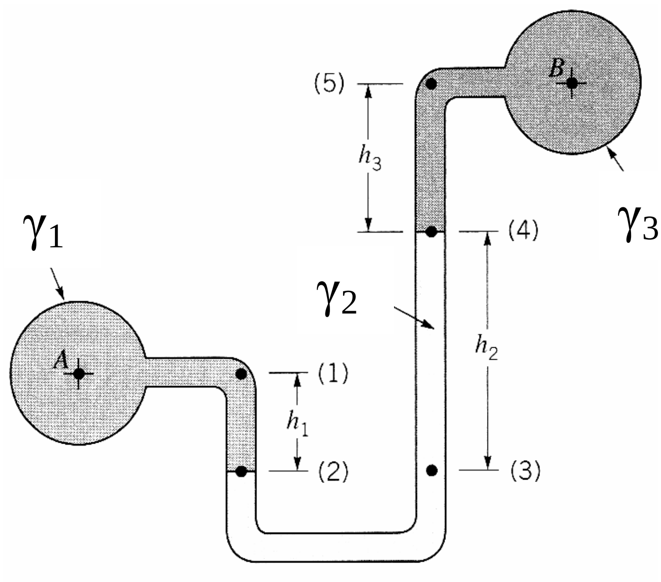
U-tube manometer

$$P_I = P_{II}$$

$$P_A + \rho_1 g h_1 = P_B + \rho_2 g h_2 + \rho_3 g h_3$$

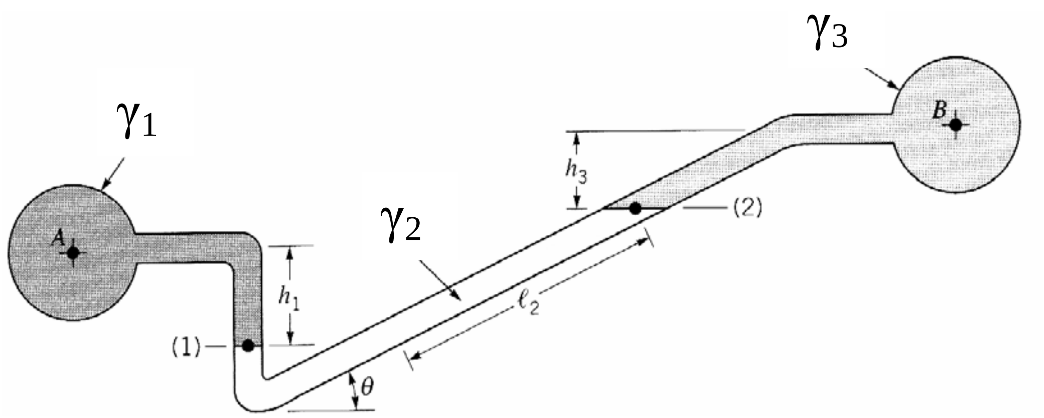
$$P_A - P_B = \rho_2 g h_2 + \rho_3 g h_3 - \rho_1 g h_1$$

$$P_A - P_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$



Differential U-tube manometer.

* Inclined-Tube Manometer



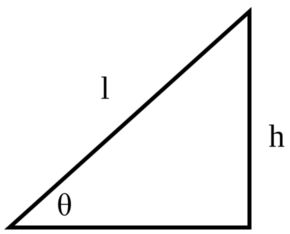
Inclined-tube Manometer.

$$P_I = P_{II}$$

$$P_A + \rho_1 g h_1 = P_B + \rho_2 g l_2 \sin\theta + \rho_3 g h_3$$

$$P_A - P_B = \rho_2 g l_2 \sin\theta + \rho_3 g h_3 - \rho_1 g h_1$$

$$P_A - P_B = \gamma_2 l_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$



$$\sin\theta = \frac{h}{l}$$

$$h = l \sin\theta$$

* U-tube with one enlarged

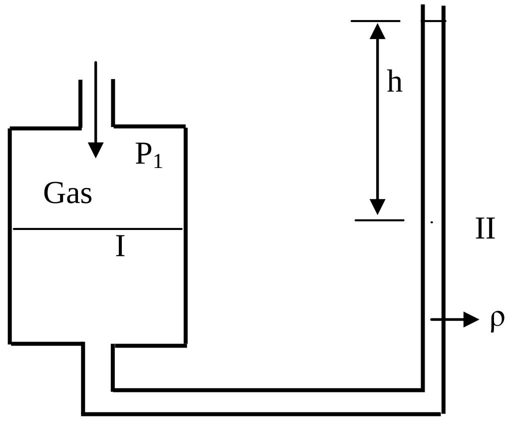
volume = volume

$$A * \ell\ell = a * h$$

$$\ell\ell = \frac{a}{A} * h$$

$$= \frac{\frac{\pi}{4} d^2}{\frac{\pi}{4} D^2} * h$$

$$\ell\ell = \frac{d^2}{D^2} * h$$

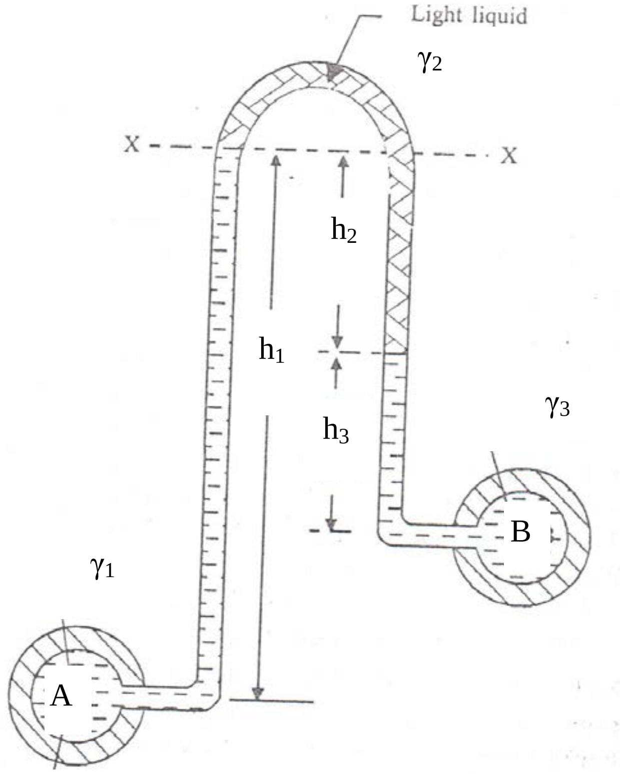


$P_I = P_{II}$

$$P_1 = \rho g \ell\ell + \rho g h$$

$$= \rho g * \frac{d^2}{D^2} h + \rho g h$$

$$= \rho g h \left(\frac{d^2}{D^2} + 1 \right)$$



* Inverted U-tube

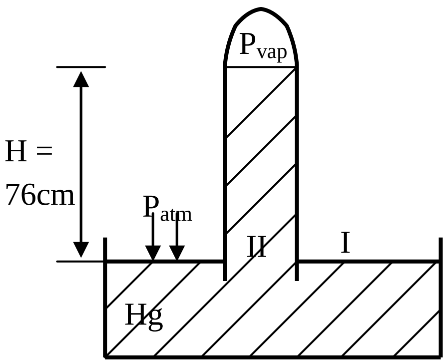
$P_I = P_{II}$

$$P_A - \rho_1 g h_1 = P_B - \rho_3 g h_3 - \rho_2 g h_2$$

$$P_A - P_B = \rho_1 g h_1 - \rho_3 g h_3 - \rho_2 g h_2$$

$\Delta P =$

*Atmospheric pressure (Barometric pressure)



$$P_{\substack{vap\\Hg}} = 1.7 * 10^{-5} \quad \text{bar}$$

$$= 1.7 \frac{N}{m^2} \approx 0 \qquad \text{neglected}$$

$$P_I = P_{II}$$

$$P_{atm} = P_{\substack{vap\\hg}} + \rho_m g H$$

$$= 13600 * 9.8 * 0.76$$

$$= 1.013 * 10^5 \quad N/m^2$$

$$= 1.013 \quad \text{bar}$$

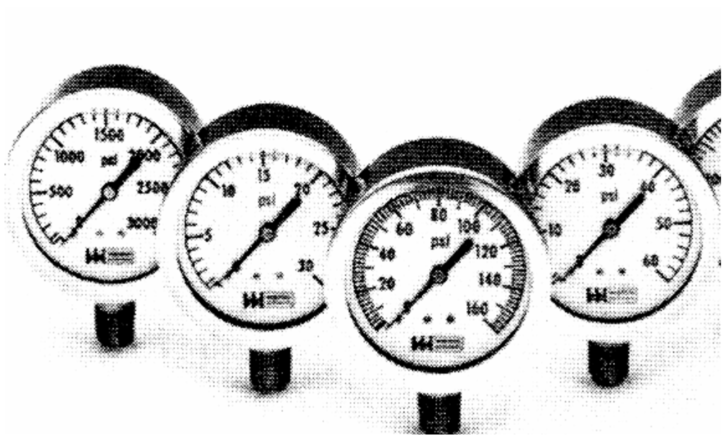
* Bourdon tube gauge

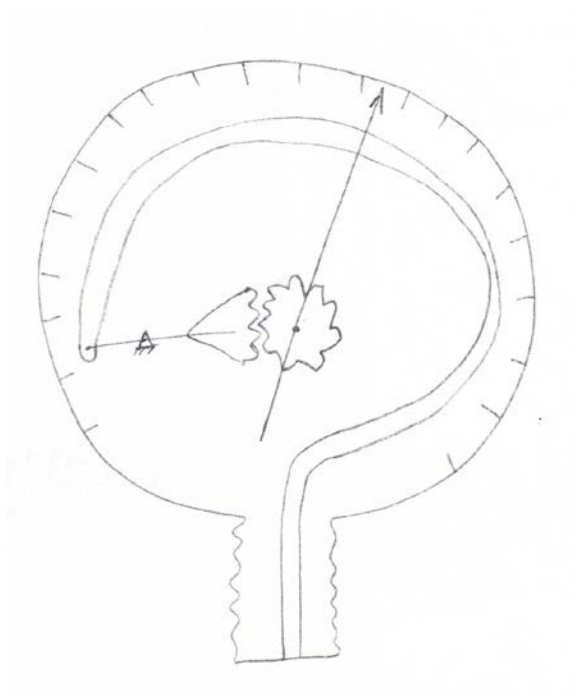
It is used for measuring pressure in almost all ranges except minutely small pressure.

Disadvantages:

1 – Needs calibration on dead weight tester.

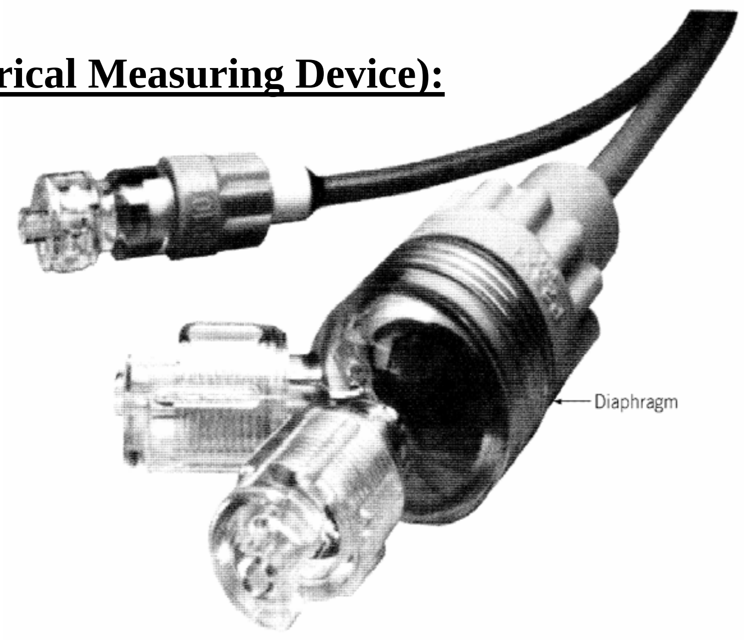
2 – Accuracy is less than liquid Columns.





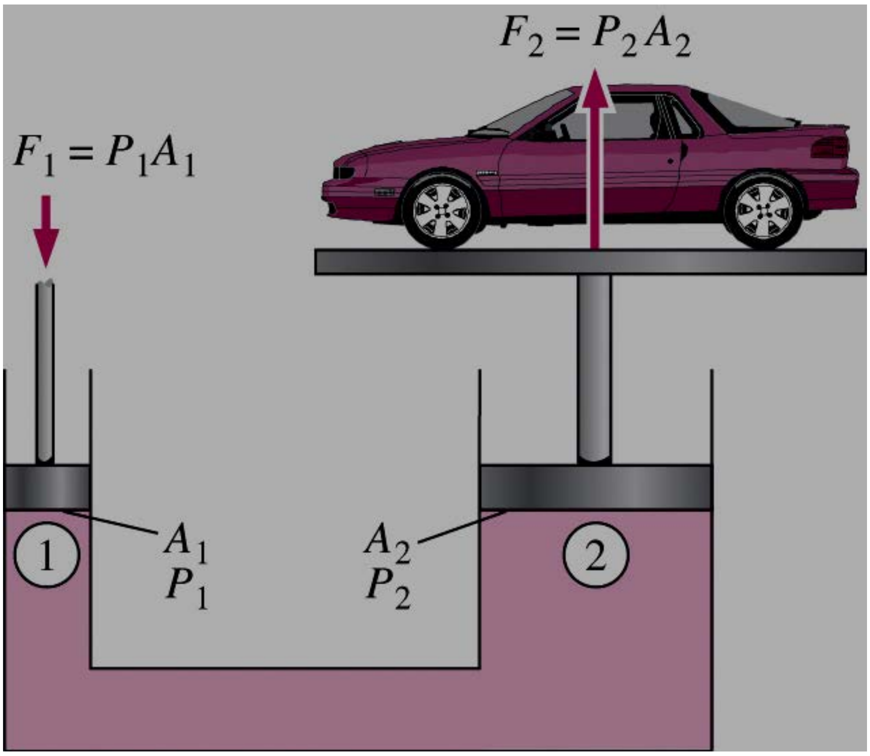
*** Pressure Transducer (Electrical Measuring Device):**

A pressure transducer converts pressure into an electrical output.



Applications

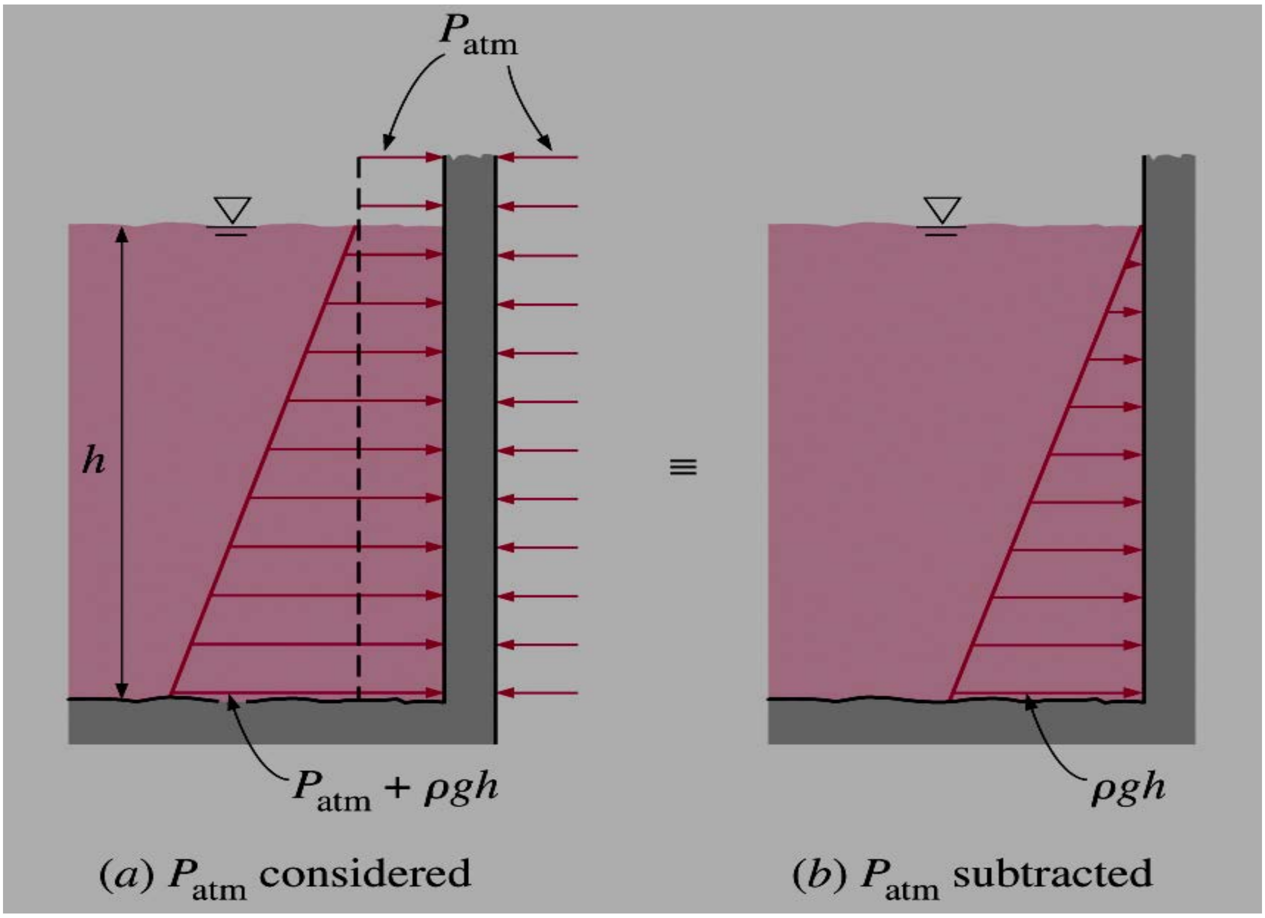
Lifting of a large weight by a small force by the application of Pascal's law.

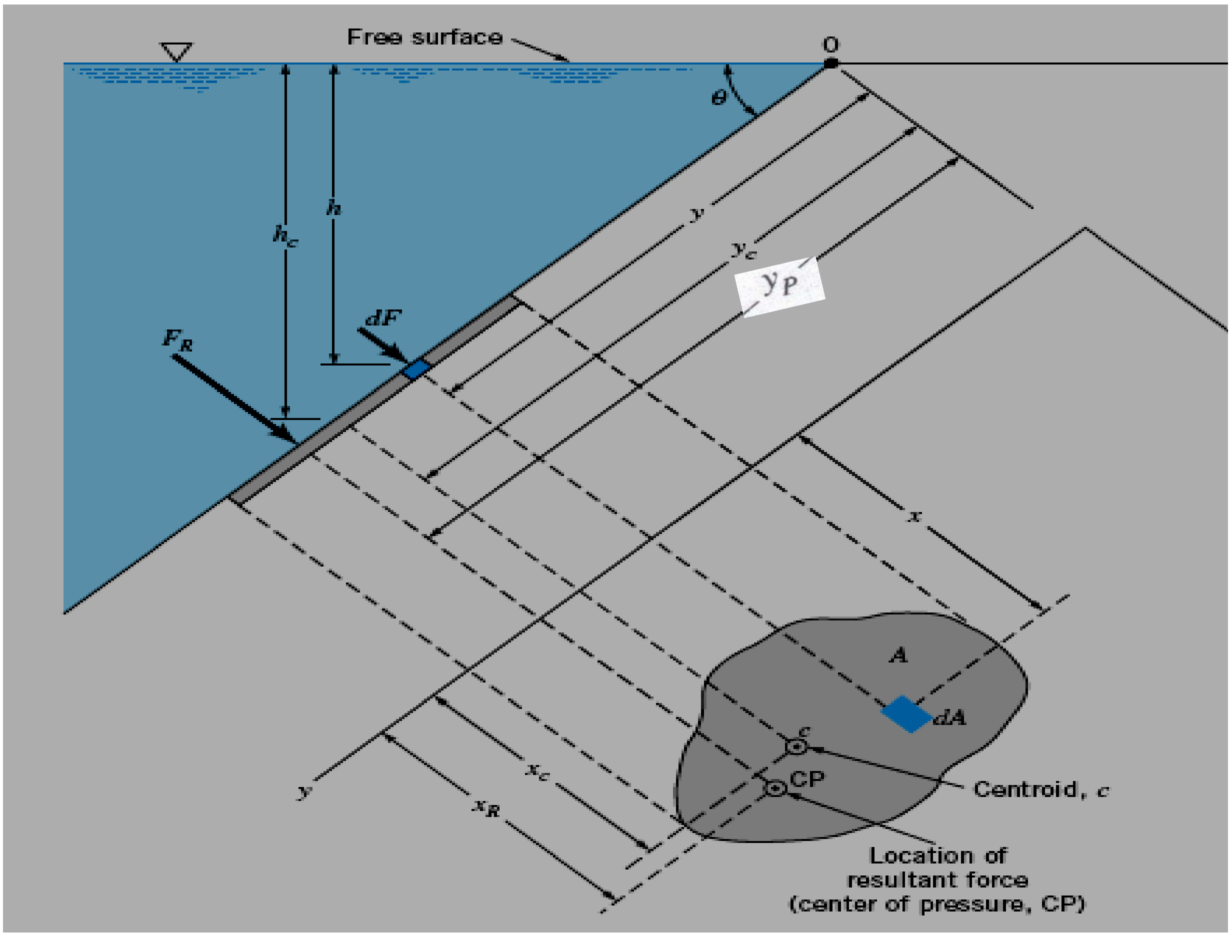


Pascal's law: The pressure applied to a confined fluid increases the pressure throughout by the same amount.

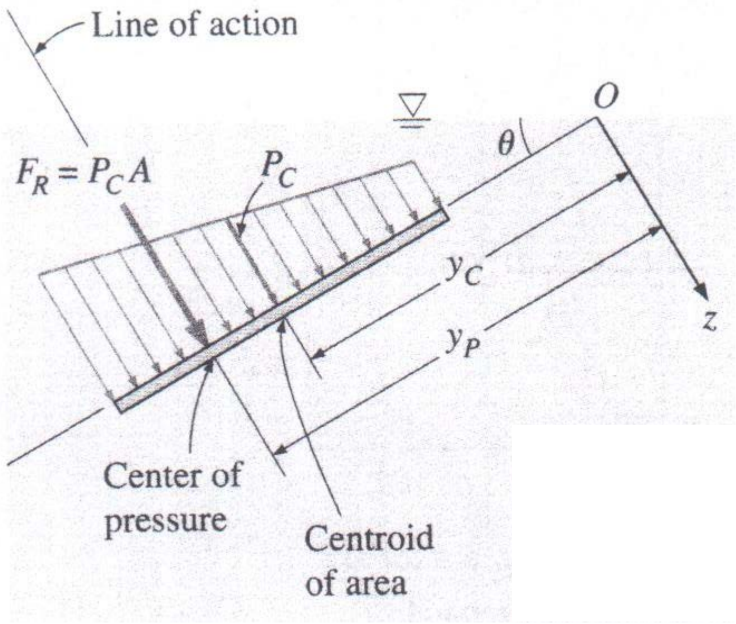
$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

Hydrostatic Forces on Plane Surfaces





$P_{guage} = \rho\ g\ h$
y_c at centroid
y_p at center of pressure



$$P = P_o + \rho\ g\ h$$
$$h = y\ \sin\theta$$
$$P = P_o + \rho\ g\ y\ \sin\theta$$

$$F_R = \int_A P\, dA$$
$$= \int_A (P_o + \rho\ g\ y\ \sin\theta) dA$$
$$= P_o\ A + \rho\ g\ \sin\theta \int_A y\ dA$$

$\int_A y\ dA$: first moment of area

as $y_c = \dfrac{1}{A}\int_A y\ dA$

then $\boxed{F_R = (P_o + \rho\ g\ y_c \sin\theta)\ A}$ $\longrightarrow$ Resultant force (1)

F_R line of action doesn't pass through the centroid of the surface. It lies underneath, where the pressure is higher.
By taking the moment of F_R about x- axis to the moment of the distributed pressure force.

$$y_P F_R = \int_A yP\ dA \quad = \int_A y(P_o + \rho\ g\ y\ \sin\theta) dA \quad = P_o \int_A y\ dA + \rho\ g\ \sin\theta \int_A y^2 dA$$

as $I_{xx,o} = \int_A y^2 dA$

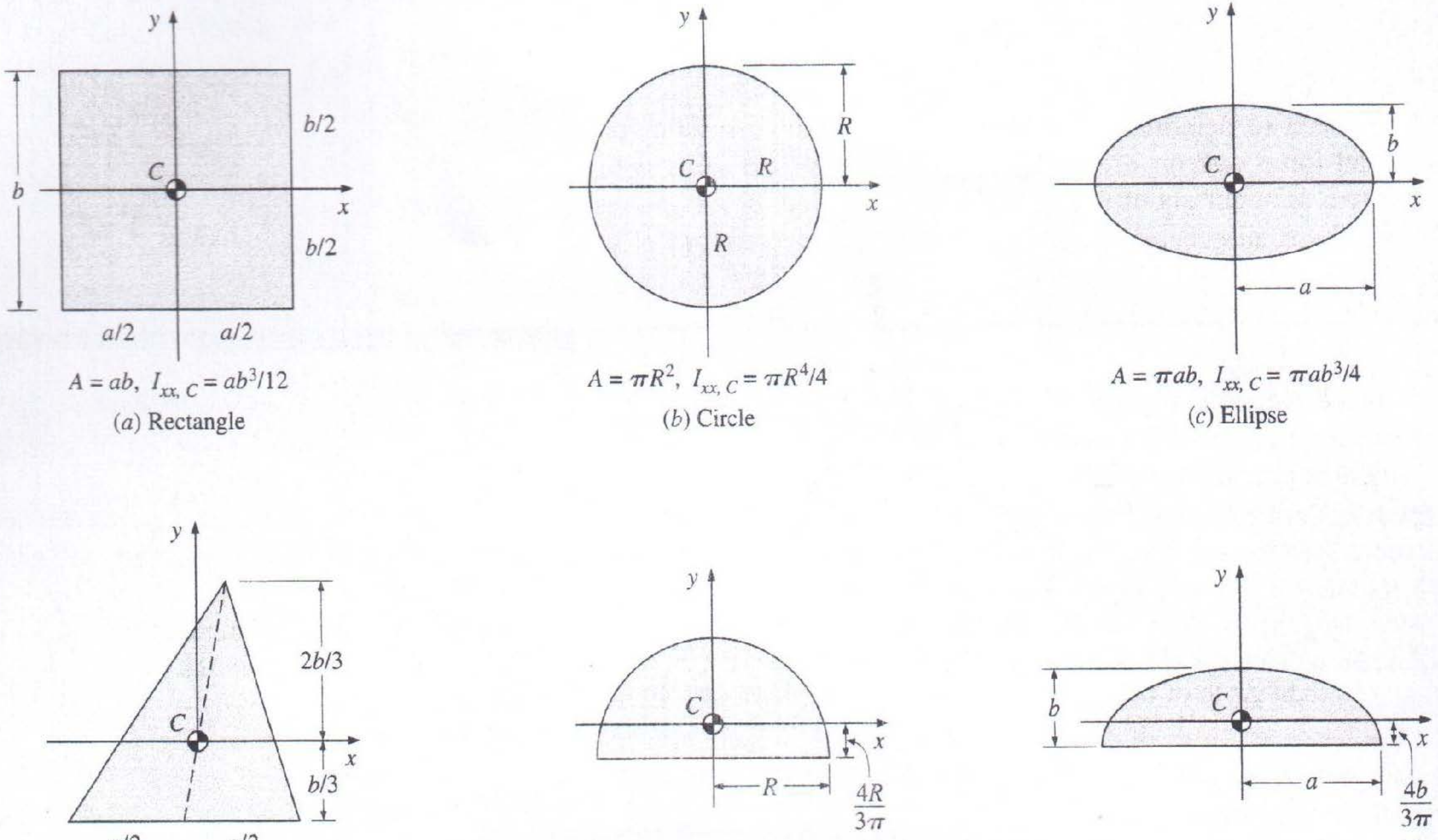
$$y_P F_R = P_o y_c A + \rho\ g\ \sin\theta\ I_{xx,o} \longrightarrow (2)$$

$$I_{xx,o} = I_{xx,c} + y_c^2 A \xrightarrow{\text{Parallel axis theorem}} (3)$$

as $I_{xx,c}$: second moment of area about centroid
From equations (1), (3) in (2)

$$\boxed{y_P = y_c + \frac{I_{xx,c}}{\left[y_c + \dfrac{P_o}{\rho g \sin\theta}\right] A}}$$

at $P_o = 0$ then $\boxed{y_P = y_c + \dfrac{I_{xx,c}}{y_c A}}$



$A = ab,\ I_{xx,\,C} = ab^3/12$
(*a*) Rectangle

$A = \pi R^2,\ I_{xx,\,C} = \pi R^4/4$
(*b*) Circle

$A = \pi ab,\ I_{xx,\,C} = \pi ab^3/4$
(*c*) Ellipse

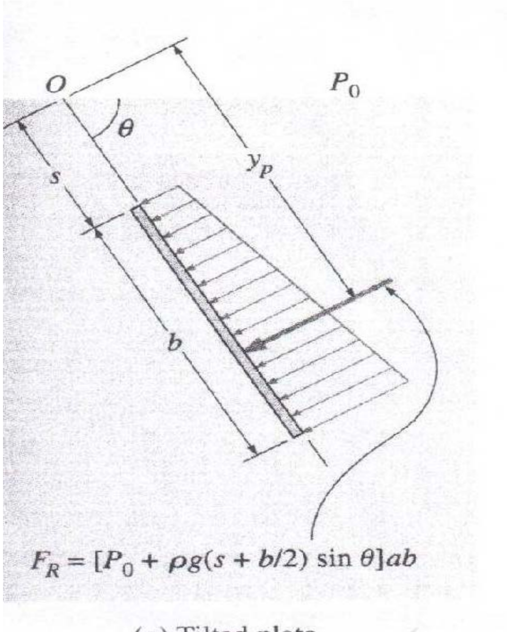
$A = ab/2,\ I_{xx,\,C} = ab^3/36$
(*d*) Triangle

$A = \pi R^2/2,\ I_{xx,\,C} = 0.109757R^4$
(*e*) Semicircle

$A = \pi ab/2,\ I_{xx,\,C} = 0.109757ab^3$
(*f*) Semiellipse

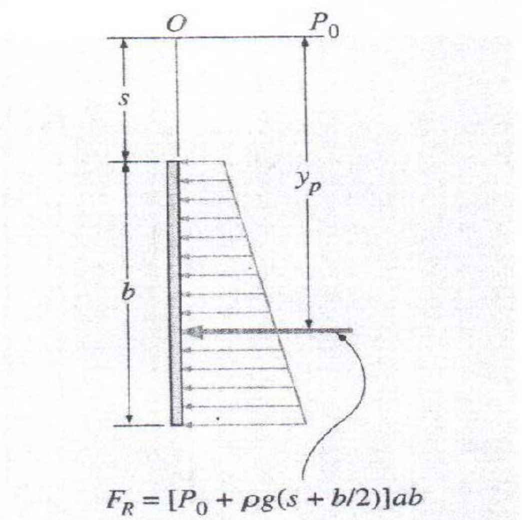
FIGURE 3–28
The centroid and the centroidal moments of inertia for some common geometries.

Hydrostatic force acting on the top surface of a submerged vertical rectangular plate



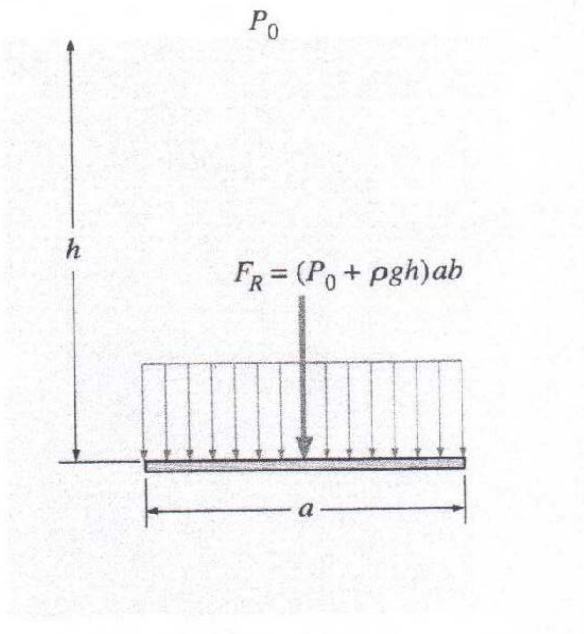
(*a*) Tilted plate

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + \frac{b}{2} + \frac{P_o}{\rho\, g \sin\theta})}$$



(*b*) Vertical plate

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + \frac{b}{2} + \frac{P_o}{\rho\, g})}$$

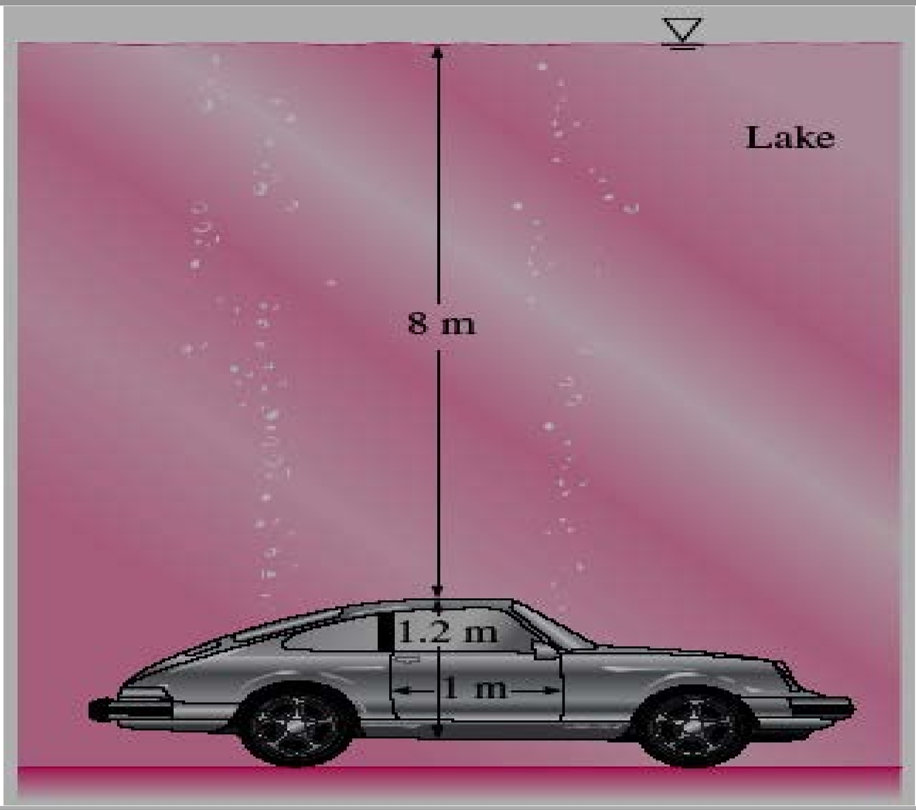


(*c*) Horizontal plate

$$y_P = h$$

EXAMPLE 3–8 Hydrostatic Force Acting on the Door of a Submerged Car

A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels (Fig. 3–33). The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.



Properties We take the density of lake water to be 1000 kg/m^3 throughout.

Analysis The average (gage) pressure on the door is the pressure value at the centroid (midpoint) of the door and is determined to be

$$P_{avg} = P_C = \rho g h_C = \rho g(s + b/2)$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(8 + 1.2/2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{84.4\ kN/m^2}$$

Then the resultant hydrostatic force on the door becomes

$$F_R = P_{avg}A = (84.4\ \text{kN/m}^2)(1\ \text{m} \times 1.2\ \text{m}) = \mathbf{101.3\ kN}$$

The pressure center is directly under the midpoint of the door, and its distance from the surface of the lake is determined from Eq. 3–24 by setting $P_0 = 0$, yielding

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 8 + \frac{1.2}{2} + \frac{1.2^2}{12(8 + 1.2/2)} = \mathbf{8.61\ m}$$

Discussion A strong person can lift 100 kg, whose weight is 981 N or about 1 kN. Also, the person can apply the force at a point farthest from the hinges (1 m farther) for maximum effect and generate a moment of 1 kN · m. The resultant hydrostatic force acts under the midpoint of the door, and thus a distance of 0.5 m from the hinges. This generates a moment of 50.6 kN · m, which is about 50 times the moment the driver can possibly generate. Therefore, it is impossible for the driver to open the door of the car. The driver's best bet is to let some water in (by rolling the window down a little, for example) and to keep his or her head close to the ceiling. The driver should be able to open the door shortly before the car is filled with water since at that point the pressures on both sides of the door are nearly the same and opening the door in water is almost as easy as opening it in air.

By taking moment about hinges

$F_R * 0.5 = F_{man} * 1$

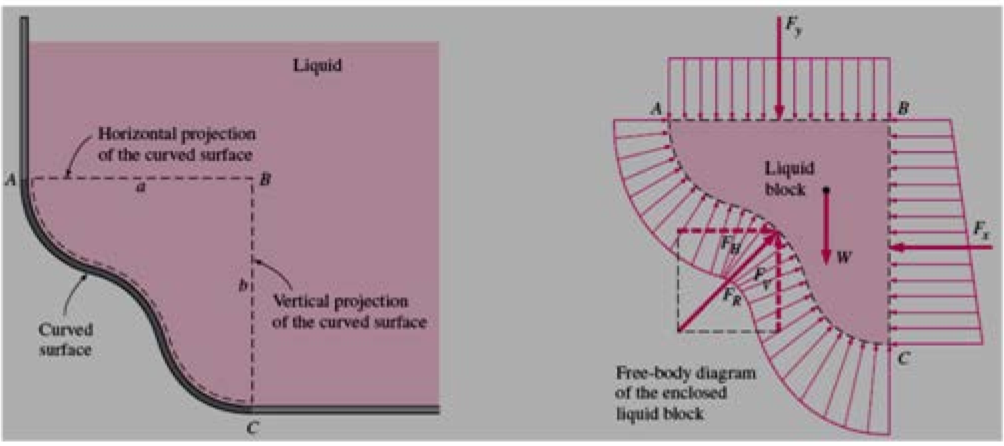
$101.3 * 0.5 = F_{man}$

$F_{man} = 50.6$ KN

A strong man can lift 100 Kg = 981 N ≃ 1KN

about 50 times the effort the driver can possibly generate.

Hydrostatic Forces on Submerged Curved Surfaces



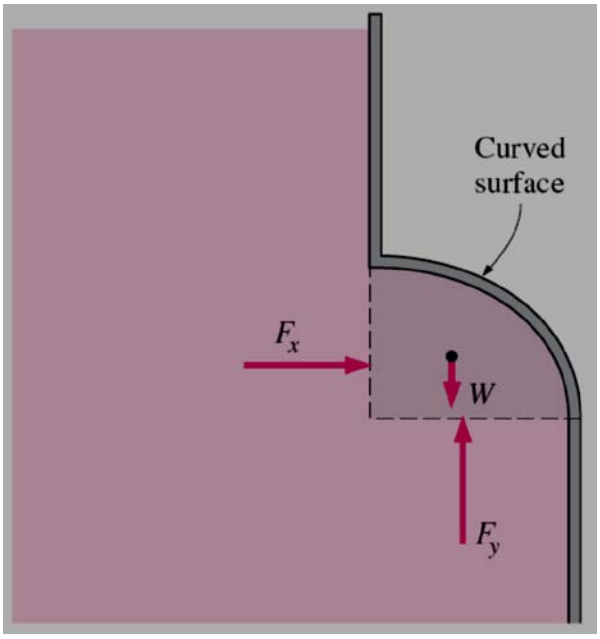
$$F_R = \sqrt{F_H^2 + F_V^2} \qquad \tan\theta = F_V/F_H$$

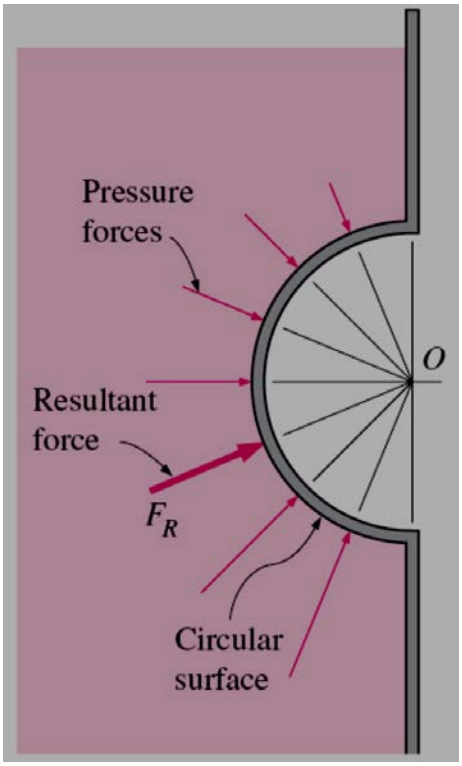
Determination of the hydrostatic force acting on a submerged curved surface.

Horizontal force component on curved surface: $F_H = F_x$

Vertical force component on curved surface: $F_V = F_y + W$

When a curved surface is above the liquid, the weight of the liquid and the vertical component of the hydrostatic force act in the opposite directions.





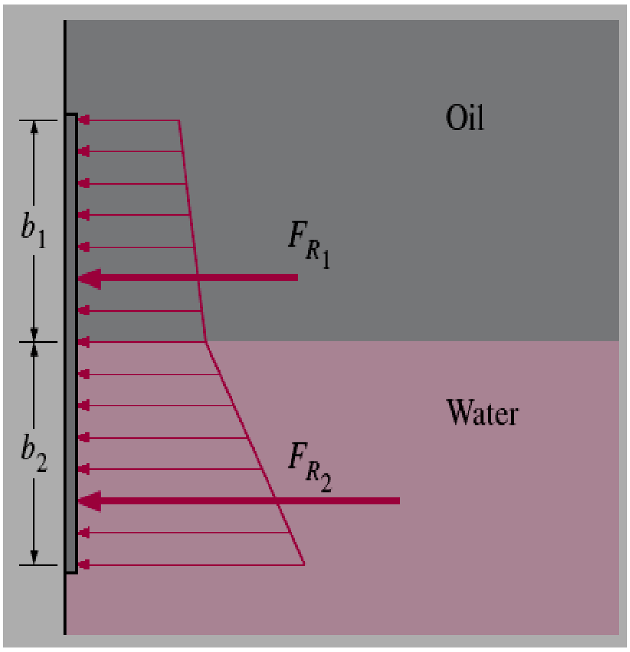
The hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface and they all pass through the center.

In a **multilayered fluid** of different densities can be determined by considering different parts of surfaces in different fluids as different surfaces, finding the force on each part, and then adding them using vector addition. For a plane surface, it can be expressed as

Plane surface in a multilayered fluid: $F_R = \sum F_{R,i} = \sum P_{C,i} A_i$

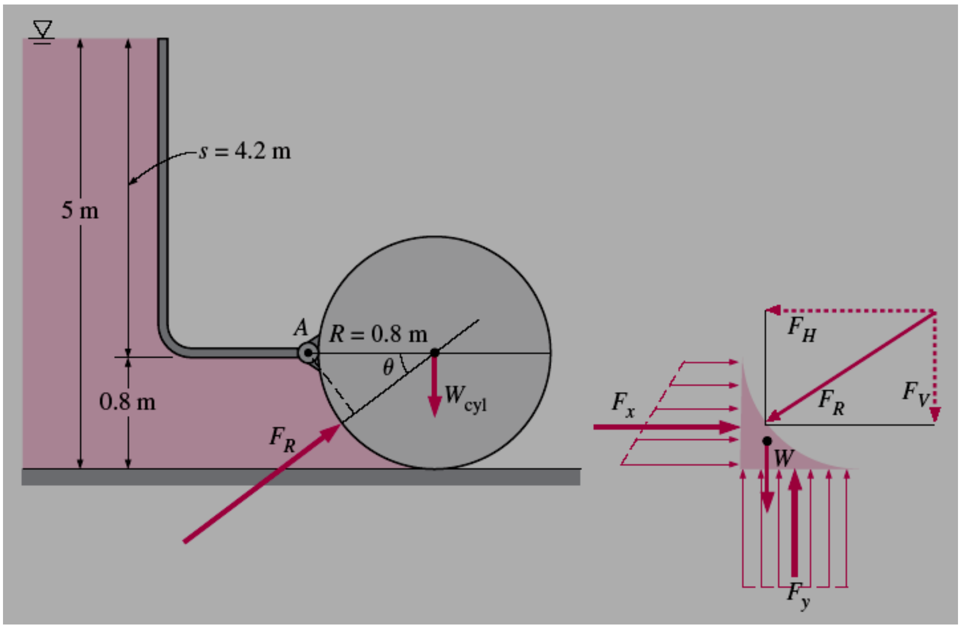
$$P_{C,i} = P_0 + \rho_i g h_{C,i}$$

The hydrostatic force on a surface submerged in a multilayered fluid can be determined by considering parts of the surface in different fluids as different surfaces.



EXAMPLE 3–9 A Gravity-Controlled Cylindrical Gate

A long solid cylinder of radius 0.8 m hinged at point *A* is used as an automatic gate, as shown in Fig. 3–38. When the water level reaches 5 m, the gate opens by turning about the hinge at point *A*. Determine (*a*) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (*b*) the weight of the cylinder per m length of the cylinder.



Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis (*a*) We consider the free-body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as

Horizontal force on vertical surface:

$$F_H = F_x = P_{avg}A = \rho g h_C A = \rho g(s + R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.2 + 0.8/2\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg} \cdot \text{m/s}^2}\right)$$

$$= 36.1\ \text{kN}$$

Vertical force on horizontal surface (upward):

$$F_y = P_{avg}A = \rho g h_C A = \rho g h_{\text{bottom}} A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg} \cdot \text{m/s}^2}\right)$$

$$= 39.2\ \text{kN}$$

Weight of fluid block for one m width into the page (downward):

$$W = mg = \rho g V = \rho g(R^2 - \pi R^2/4)(1\ \text{m})$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.8\ \text{m})^2(1 - \pi/4)(1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg} \cdot \text{m/s}^2}\right)$$

$$= 1.3\ \text{kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.3 = 37.9\ \text{kN}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = \mathbf{52.3\ kN}$$

$$\tan\theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4°$$

Therefore, the magnitude of the hydrostatic force acting on the cylinder is 52.3 kN per m length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.4° with the horizontal.

(*b*) When the water level is 5 m high, the gate is about to open and thus the reaction force at the bottom of the cylinder is zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about point *A* at the location of the hinge and equating it to zero gives

$$F_R R \sin\theta - W_{\text{cyl}} R = 0 \rightarrow W_{\text{cyl}} = F_R \sin\theta = (52.3\ \text{kN}) \sin 46.4° = \mathbf{37.9\ kN}$$

Discussion The weight of the cylinder per m length is determined to be 37.9 kN. It can be shown that this corresponds to a mass of 3863 kg per m length and to a density of 1921 kg/m^3 for the material of the cylinder.