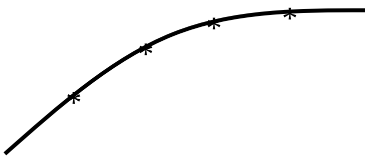
Fluid Kinematics

(Hydrodynamics)

Kinematics describes motion in terms of displacements, velocities, and accelerations regardless to the forces which are associated with these variables.

* Definitions :

*** Streamline :** is a smooth imaginary curve represents one particle in the flow. The tangent of this line gives the direction of velocity at any point.

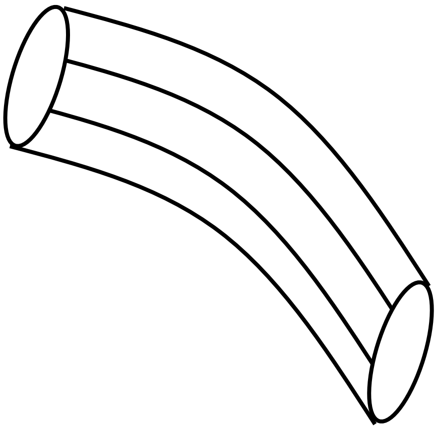


- Streamlines can never intersect.
- They can never have sudden change in direction .

Intersection or sudden change in direction means that there is a point where the velocity vector has two directions in the same time which is impossible .

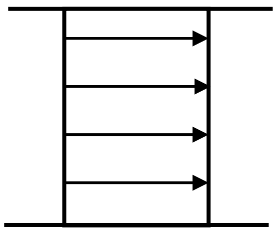
*** Stream tube :** is a tube formed of an infinite number of streamlines which are drawn passing through a closed curve in the flow.

- No flow can go in or out the sides of this tube.

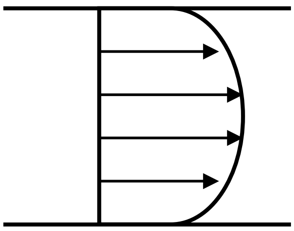


* Types of flow :

1 – Ideal and real flow



Ideal flow

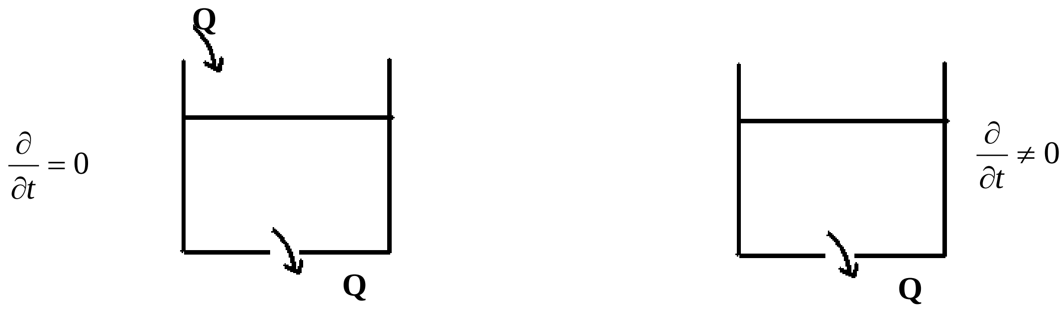


Real flow

*** Ideal flow:** means frictionless flow, no energy is lost, viscosity is considered zero.

*** Real flow:** viscosity can't be neglected, there is friction. Friction causes some of the mechanical energy to be converted into heat energy and can't be restored.

2 – Steady and unsteady flow (with respect to time) (from time to time)



steady flow unsteady flow

*** Steady flow:** pressure, velocity, flow rate (flow parameters) are constant with respect to time.

*** Unsteady flow:** any of the flow parameters change with time.

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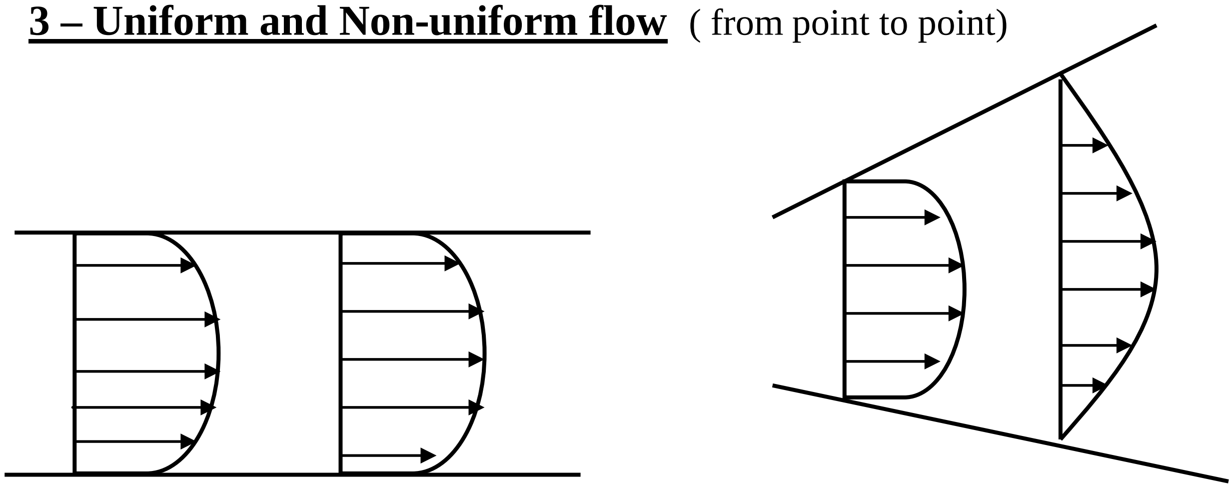
3 – Uniform and Non-uniform flow (from point to point)



Uniform flow Non-uniform flow

*** Uniform flow:** the velocity at a given instant is the same in magnitude and direction at every point in the flow.

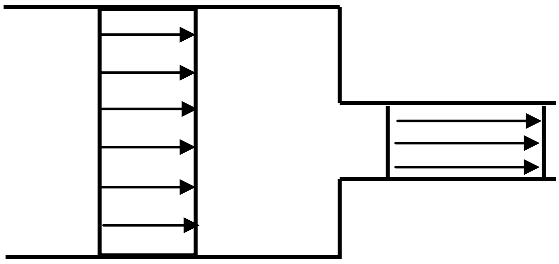
*** Non-uniform flow:** the velocity at a given instant changes from point to point.

4 – one , two and three-dimensional flow :

* One-dimensional flow

Flow variables depend on one dimension only.

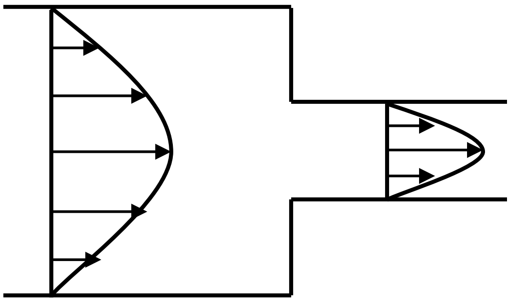
Ex.: Ideal flow in a pipe of stepping cross-section.



* Two-dimensional flow

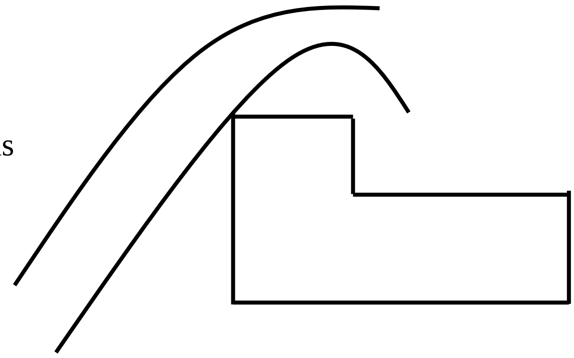
Flow variables depend on two dimensions.

Ex.: real flow in the same pipe.



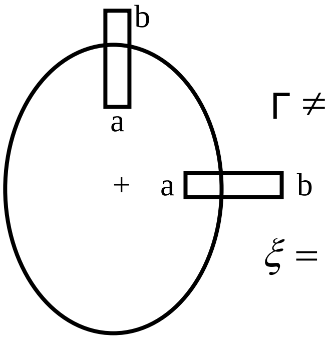
* Three-dimensional flow

Flow variables depends on three dimensions



5 – rotational & irrotational flow :

مثل حركة الماء عند التقليب بواسطة مروحة



$\Gamma \neq 0$

$\xi = 0$

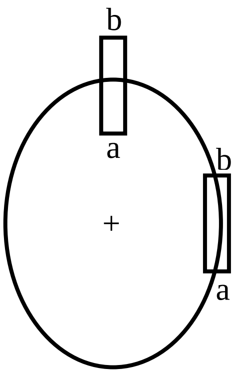
rotational flow

circulation

vorticity

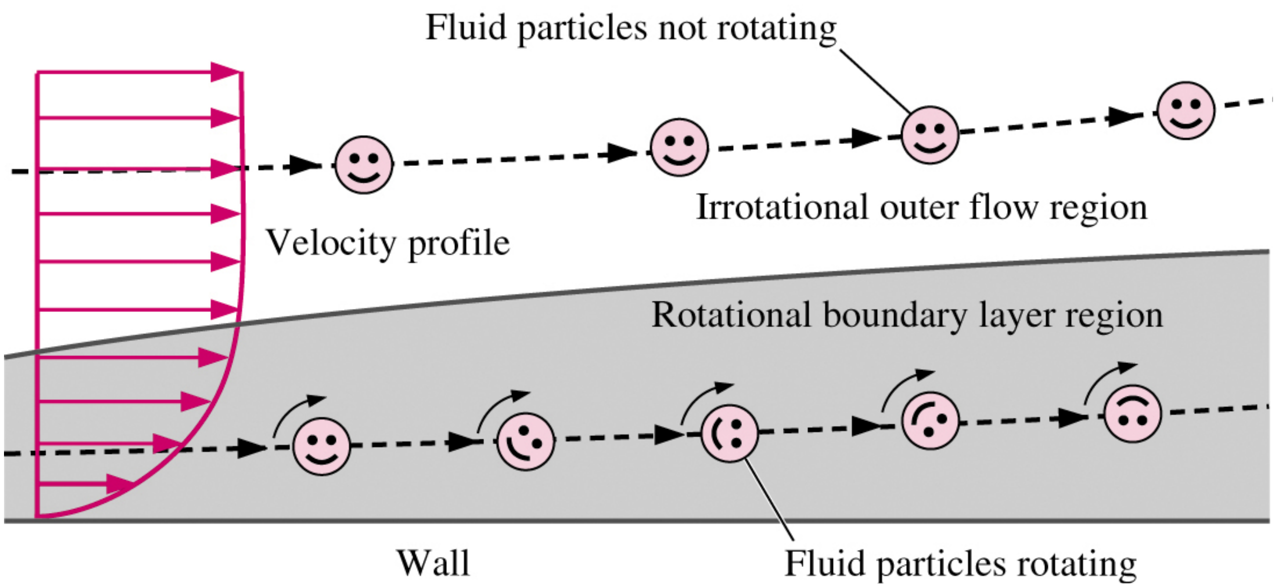
$\Gamma = 0$

$\xi = 0$

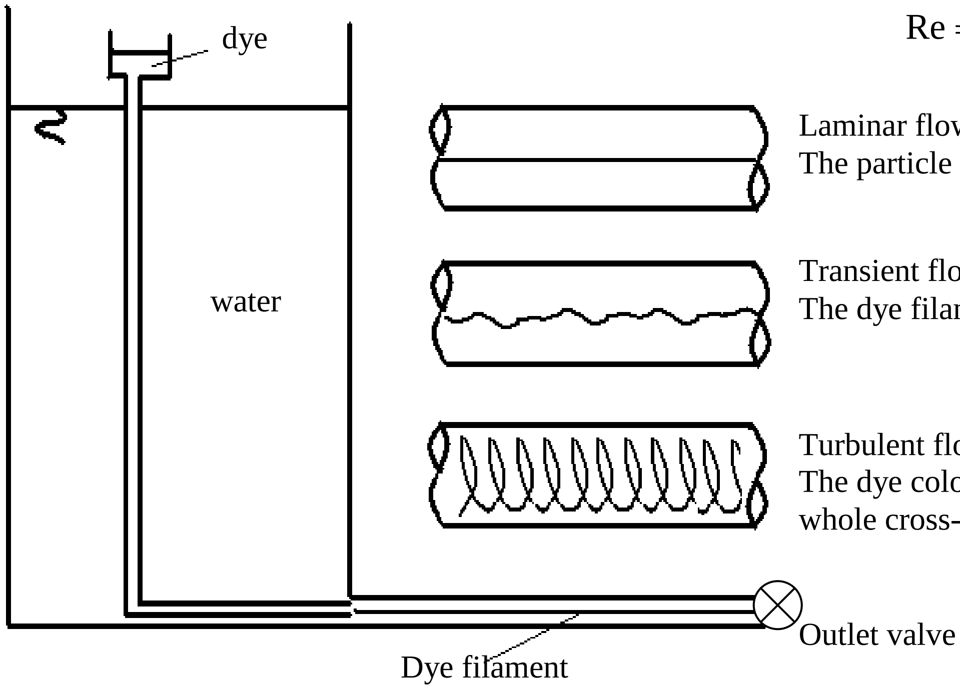


irrotational flow

مثل حركة الماء عند نزع سدادة الحوض



6 - Laminar, Transient and Turbulent flow



$$Re = \frac{\rho v d}{\mu}$$

Laminar flow Re < 2000
The particle moves in parallel layers.

Transient flow 2000 < Re < 4000
The dye filament begins to oscillate.

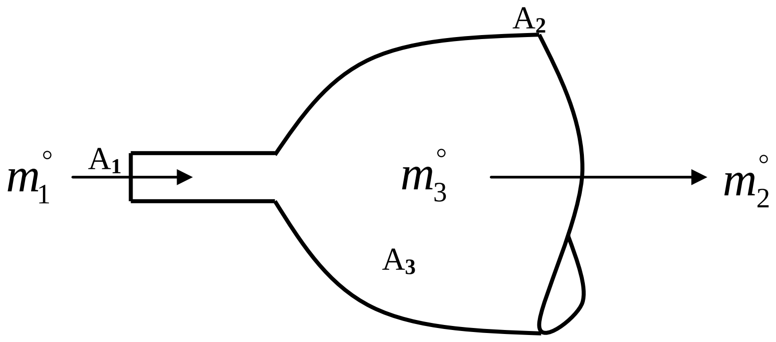
Turbulent flow Re > 4000
The dye color is diffused over the whole cross-section.

Equations of motion :

1 – Continuity equation.

2 – Bernoulli's equation.

1 – Continuity equation:



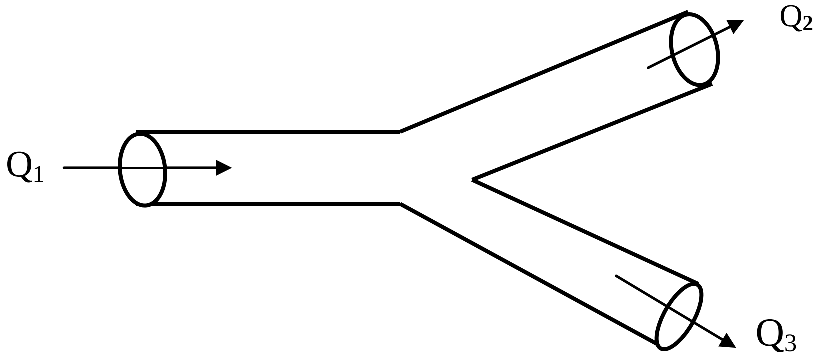
Mass of fluid entering per unit time = Mass of fluid leaving per unit time

$$\overset{\circ}{m}_1 = \overset{\circ}{m}_2 = \overset{\circ}{m}_3 = const$$ mass flow rate

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \rho_3 A_3 V_3$$

* For liquids (incompressible fluid) $\rho_1 = \rho_2 = \rho_3 = const.$

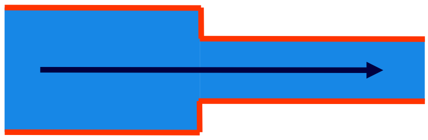
$$\boxed{A_1 V_1 = A_2 V_2 = A_3 V_3 = Q}$$ discharge (volume flow rate)



$$Q_1 = Q_2 + Q_3$$

Ex.:

Assuming the water moving in the pipe is an ideal fluid, relative to its speed in the 1" diameter pipe, how fast is the water going in the 1/2" pipe?



a) 2 v_1 b) 4 v_1 c) 1/2 v_1 c) 1/4 v_1

Solution

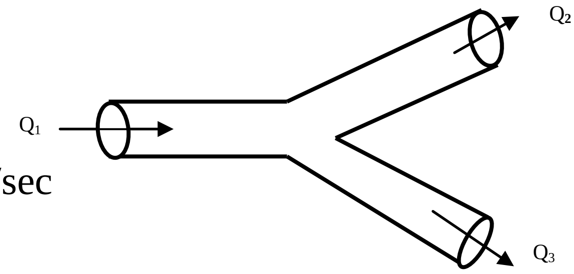
Using the continuity equation

$$A_1 \cdot v_1 = A_2 \cdot v_2$$

$$\rightarrow \quad v_2 = \frac{A_1}{A_2} \cdot v_1 = \left(1 \cdot \frac{2}{1}\right)^2 \cdot v_1 = 4v_1$$

Ex.:

If pipe 1 diameter = 50mm, mean velocity 2m/s, pipe 2 diameter = 40mm takes 30% of total discharge and pipe 3 diameter = 60mm. What are the values of discharge and mean velocity in each pipe?



Solution

$A_1\ v_1 = \frac{\pi}{4}(50x10^{-3})^2 x2 = 3.93x10^{-3}\ m^3/sec$

$Q_2 = 0.3\ x\ Q_1 = 1.178x10^{-3}\ m^3/sec$

$Q_2 = A_2\ v_2$

$1.178x10^{-3} = \frac{\pi}{4}(40x10^{-3})^2 x\ v_2$

$v_2 = 0.9375\ m/s$

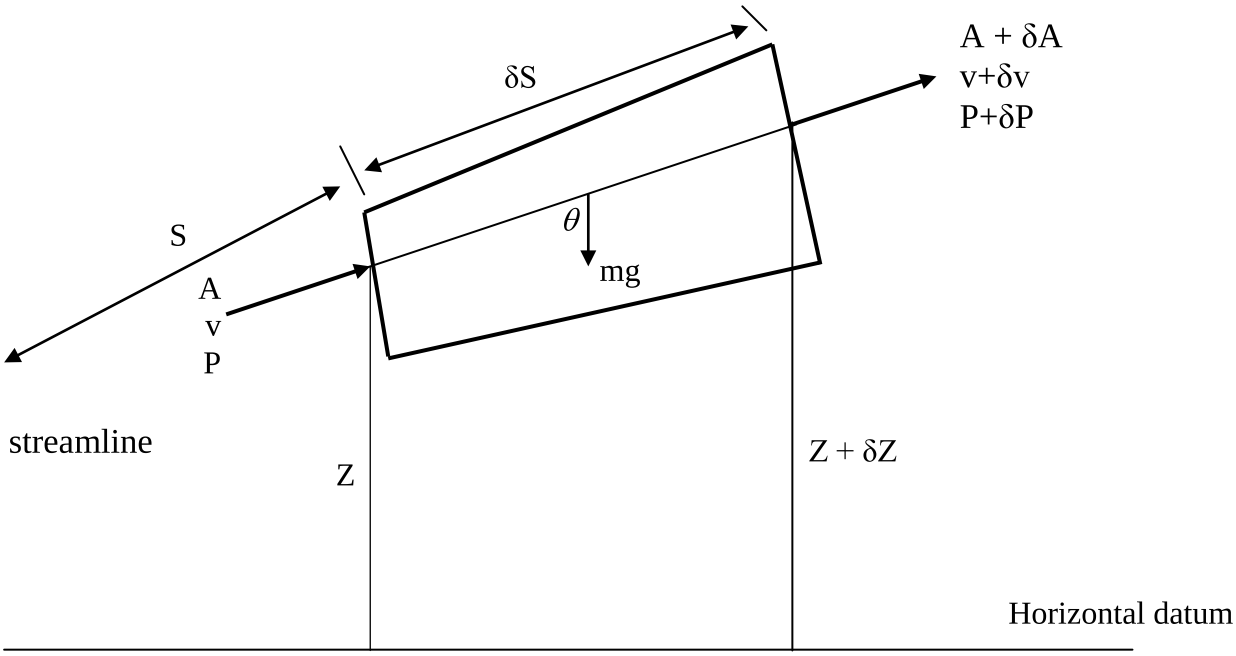
$Q_1 = Q_2 + Q_3$

$3.93x10^{-3} = 1.178x10^{-3} + Q_3 \qquad \therefore Q_3 = 2.752x10^{-3}\ m^3/sec$

$Q_3 = A_3\ v_3$

$2.752x10^{-3} = \frac{\pi}{4}(60x10^{-3})^2 x\ v_3 \qquad \therefore v_3 = 0.9733\ m/sec$

2 – Bernoulli's equation (energy equation)



$$E = Z + \frac{P}{\rho g} + \frac{V^2}{2g} = const$$

E : total energy per unit weight (m)

Z : potential energy per unit weight (m)

$\frac{P}{\rho g}$: pressure energy per unit weight (m)

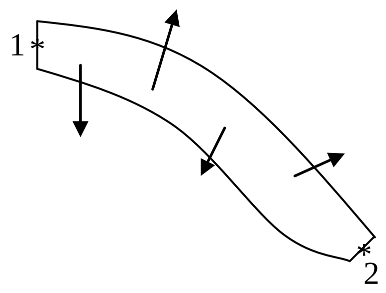
$\frac{V^2}{2g}$: kinetic energy (velocity energy) per unit weight (m)

*** For ideal flow :**

$$E_1 = E_2$$

$$Z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

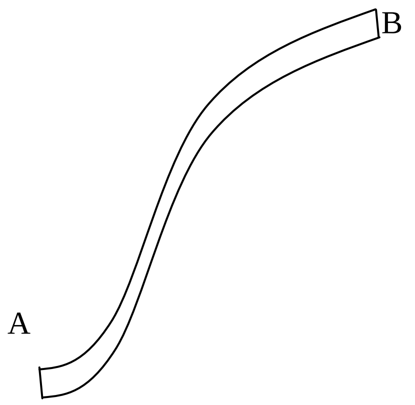
* For Real flow



$$E_{\mathbf{1}} - h_{\textbf{loss 1 to 2}} = E_{\mathbf{2}}$$

$$E_{\mathbf{1}} = E_{\mathbf{2}} + h_{\textbf{loss 1 to 2}}$$

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} + h_{loss1to2}$$



$$\frac{P_A}{\rho g} + Z_A + \cancel{\frac{V_A^2}{2g}} = \underbrace{\frac{P_B}{\rho g}}_{=0\ \text{atm}} + Z_B + \cancel{\frac{V_B^2}{2g}} + h_{loss1to2}$$

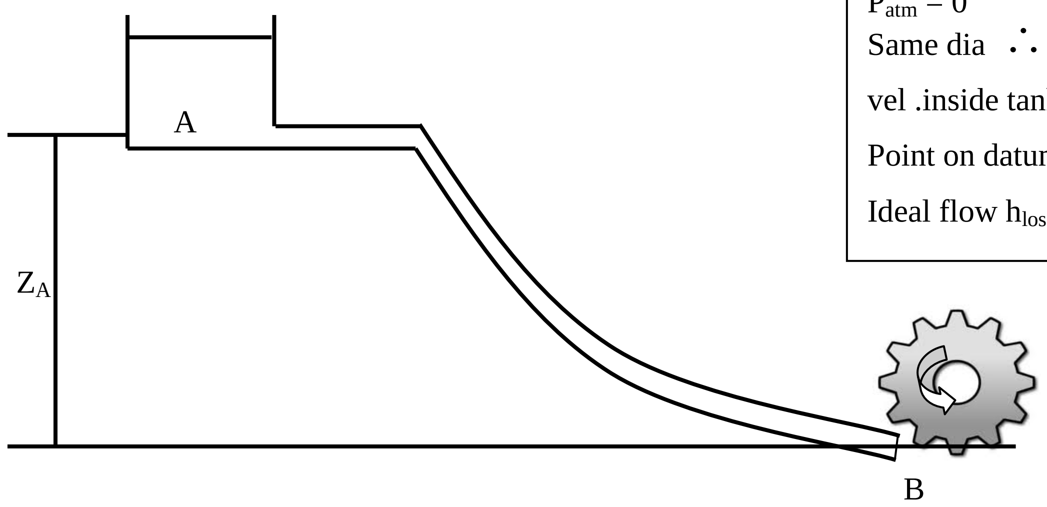
$A_{\mathbf{A}} = A_{\mathbf{B}}$

$\therefore V_{\mathbf{A}} = V_{\mathbf{B}}$

$$\frac{P_A}{\rho g} = \left(Z_B - Z_A\right) + h_{loss1to2}$$

$$\boxed{E_{\mathbf{1}} + h_p = E_{\mathbf{2}} + h_T + h_{\text{loss 1 to 2}}}$$

$P_{atm} = 0$
Same dia $\therefore$ same vel.
vel .inside tank = 0
Point on datum Z = 0
Ideal flow $h_{loss} = 0$



$$\frac{P_A}{\rho g} + Z_A + \underbrace{\cancel{\frac{V_A^2}{2g}}}_{=0} = \underbrace{\cancel{\frac{P_B}{\rho g}}}_{=0} + \underbrace{\cancel{Z_B}}_{=0} + \frac{V_B^2}{2g} + h_{lossAtoB}$$

vel. Inside Tank atm. dotum

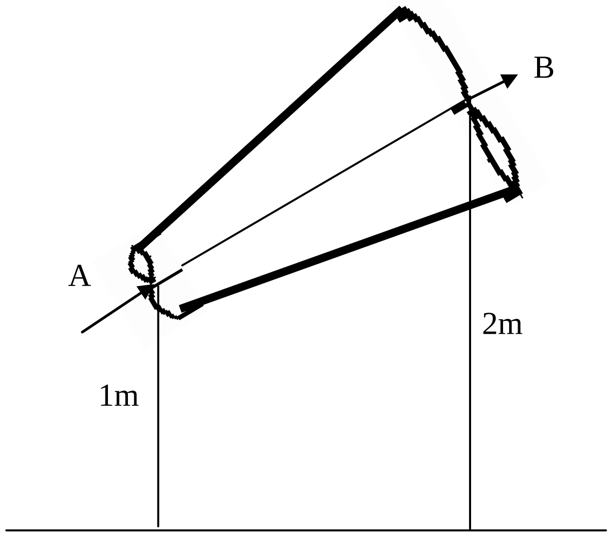
$$\frac{P_A}{\rho g} + Z_A = \frac{V_B^2}{2g} + h_{lossAtoB}$$

Ex.:

d_A = 1 cm d_B = 5 cm

h_A = 2m h_B = 5m

$$\frac{V_A^2}{2g} = 5m$$



Determine:

1 – The velocity at B

2 –Direction of flow

Solution take g = 10 m/s^2

$$\frac{V_A^2}{2g} = 5m \qquad V_A = \sqrt{2*10*5} = 10m/s$$

C.E from A to B $A_A V_A = A_B V_B$

$$\frac{\pi}{4}(1)^2 * 10 = \frac{\pi}{4}(5)^2 * V_B$$

$$\therefore V_B = \frac{10}{25} = 0.4m/\sec$$

$$E_A = h_A + Z_A + \frac{V_A^2}{2g} = 2 + 1 + 5 = 8m$$

$$E_B = h_B + Z_B + \frac{V_B^2}{2g} = 5 + 2 + \frac{(0.4)^2}{2*10} = 7.008m$$

$$E_A > E_B$$

$\therefore$ The flow from A to B

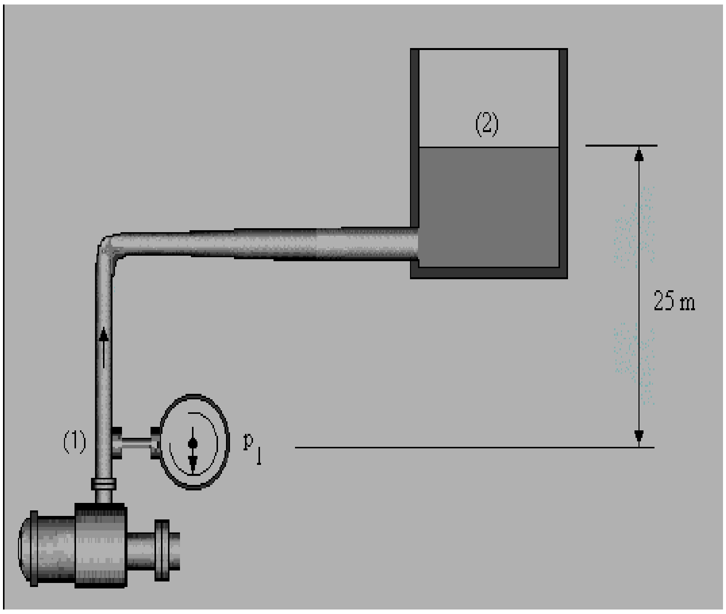
$$h_{loss} = E_A - E_B$$

$= 8 - 7.008$

$= 0.992 \cong 1m$

Ex.:

The diagram shows a pump delivering water through as pipe 30 mm bore to a tank. Find the pressure at point (1) when the flow rate is 0.0014 m^3/s. The density of water is 1000 kg/m^3. The loss of pressure due to friction is 50 kPa ($h_{loss} = 5$m of water).



Solution

d = 0.03 m $\quad Q = 0.0014\ m^3/s \quad h_{loss} = 5$m of water

$P_1 =$?? $\quad \rho = 1000\ kg/m^3$

$$Q = A_1 V_1$$

$$0.0014 = \frac{\pi}{4}(0.03)^2 V_1$$

$$V_1 = 1.98 \text{ m/s}$$

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} + h_{loss1to2}$$

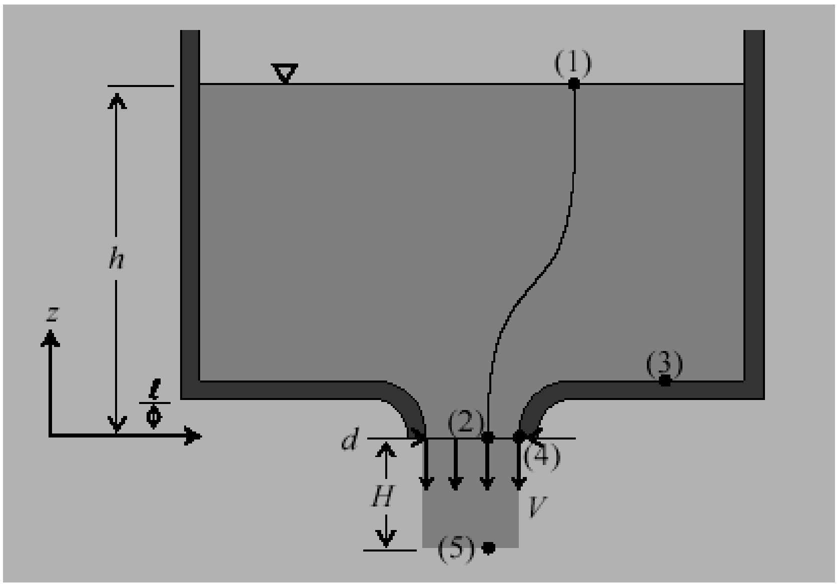
$$\frac{P_1}{\rho g} + 0 + \frac{(1.98)^2}{2g} = \frac{0}{\rho g} + 25 + \frac{0}{2g} + 5$$

$$\frac{P_1}{9800} = 25 + 5 - \frac{(1.98)^2}{2*10}$$

$$P_1 = 9800 * \left[30 - \frac{(1.98)^2}{2*10}\right]$$

$$P_1 = 293.29 kPa$$

Ex.:



At point (1), the pressure is atmospheric ($p_1 = p_0$), or the gage pressure is zero, and the fluid is almost at rest ($V_1 = 0$). At point (2), the exit pressure is also atmospheric ($p_2 = p_0$), and the fluid moves at a velocity V. By using point (2) as the datum where $z_2 = 0$ and the elevation of point (1) is h, the above relation can be reduced to

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

$$\frac{0}{\rho g} + h + \frac{0}{2g} = \frac{0}{\rho g} + 0 + \frac{V_2^2}{2g}$$

$$2gh = V_2^2$$

Hence we can formulate the velocity V to be $V_2 = \sqrt{2gh}$

Notice that we can also obtain the similar relation by using the relation between point (3) and point (4). The pressure and the velocity for point (4) is similar to point (2). However, the pressure for point (3) is the hydrostatic pressure, i.e. $p_3 = p_0 + \rho g(h - \ell)$ and the velocity is also zero due to an assumption of a large tank. Hence, the relation becomes

$$\frac{P_3}{\rho g} + Z_3 + \frac{V_3^2}{2g} = \frac{P_4}{\rho g} + Z_4 + \frac{V_4^2}{2g}$$

$$h - l + l + \frac{0}{2g} = \frac{0}{\rho g} + 0 + \frac{V_4^2}{2g}$$

$$2\mathrm{gh} = \mathrm{V}_4^2$$

Hence we can formulate the velocity *V* to be $V_4 = \sqrt{2gh}$

If we applied Bernoulli's equation between points (1) & (5)

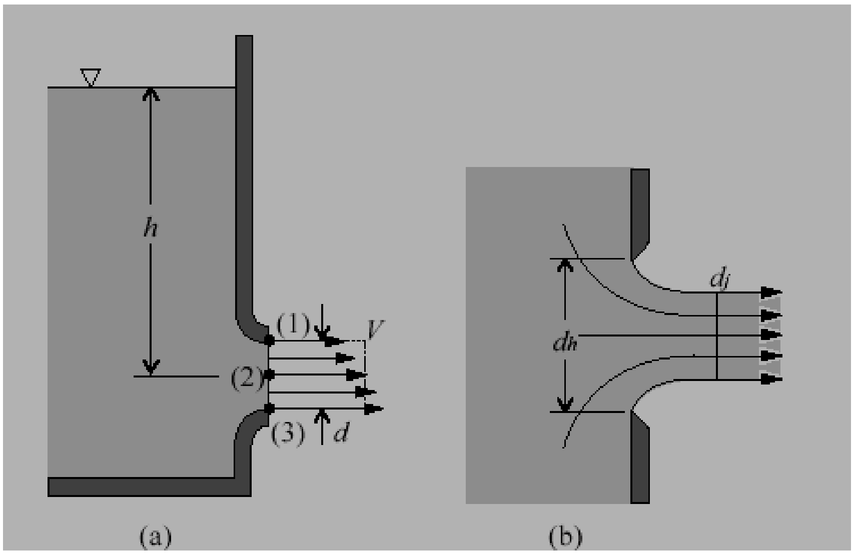
$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_5}{\rho g} + Z_5 + \frac{V_5^2}{2g}$$

$$\frac{0}{\rho g} + h + \frac{0}{2g} = \frac{0}{\rho g} - H + \frac{V_5^2}{2g}$$

$$V_5 = \sqrt{2g(h + H)}$$

where (*h* + *H*) is the vertical distance from point (1) to point (5).

<u>Ex.:</u>



The velocity *V* is only dependent on the depth of the centre of the nozzle from the free surface *h*. If the edge of the nozzle is sharp, as illustrated in Figure, flow contraction will be occurred to the flow. This phenomenon is known as *vena contracta*, which is a result of the inability for the fluid to turn at the sharp corner 90°. This effect causes losses to the flow.

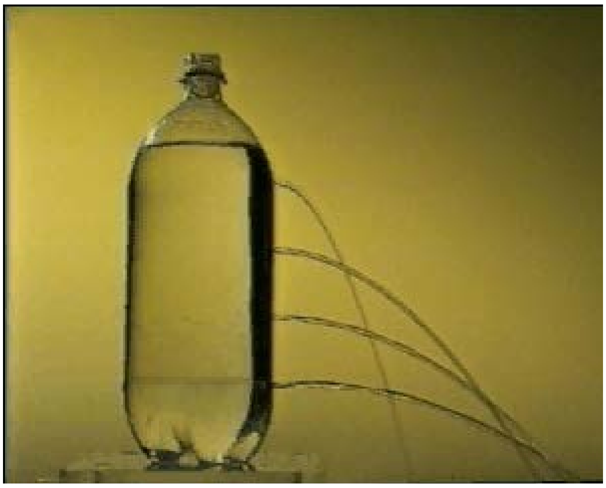
For a nozzle located at the side wall of the tank as in Figure (b), we can also form a similar relation for the Bernoulli equation, i.e.

$$V_1 = \sqrt{2g(h-d/2)} \quad V_2 = \sqrt{2gh} \quad V_3 = \sqrt{2g(h+d/2)}$$

For a nozzle having a small diameter (d ∨ h), then we can conclude that

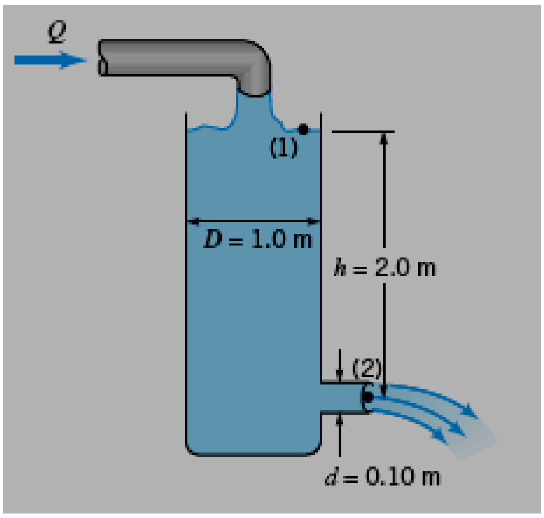
$V_1 \cong V_2 \cong V_3 = V \approx \sqrt{2gh}$

According to the Bernoulli equation, the velocity of a fluid flowing through a hole in the side of an open tank or reservoir is proportional to the square root of the depth of fluid above the hole. The velocity of a jet of water from an open pop bottle containing four holes is clearly related to the depth of water above the hole. The greater the depth, the higher the velocity. Similar behavior can be seen as water flows at a very high velocity from the reservoir behind Glenn Canyon dam in Colorado.



Example:

A stream of water of diameter $d = 0.1$ m flows steadily from a tank of diameter $D = 1.0$ m as shown in Fig. E3.7*a*. Determine the flowrate, Q, needed from the inflow pipe if the water depth remains constant, $h = 2.0$ m.



$$\frac{P_1}{\rho g}+Z_1+\frac{V_1^2}{2g}=\frac{P_2}{\rho g}+Z_2+\frac{V_2^2}{2g}$$

$$\frac{0}{\rho g}+h+\frac{V_1^2}{2g}=\frac{0}{\rho g}+0+\frac{V_2^2}{2g}$$

$$V_1^2-V_2^2=2gh$$

Although the water level remains constant (h = constant), there is an average velocity, V_1, across section (1) because of the flow from the tank. From Eq. 3.19 for steady incompressible flow, conservation of mass requires $Q_1 = Q_2$, where $Q = AV$. Thus, $A_1V_1 = A_2V_2$, or

$$\frac{\pi}{4}D^2V_1=\frac{\pi}{4}d^2V_2$$

Hence,

$$V_1=\left(\frac{d}{D}\right)^2V_2 \qquad \textbf{(3)}$$

Equations 1 and 3 can be combined to give

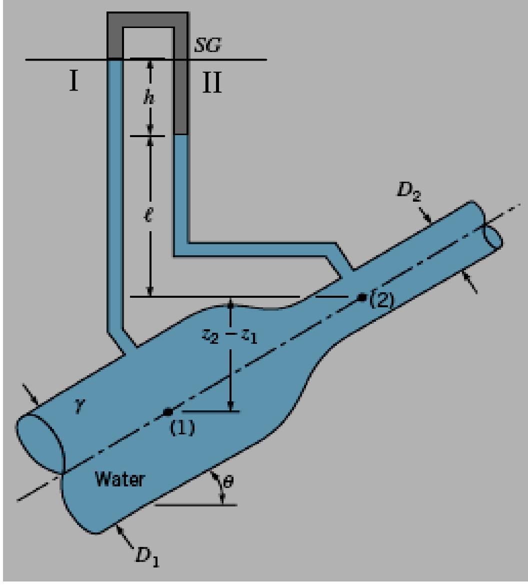
$$V_2=\sqrt{\frac{2gh}{1-(d/D)^4}}=\sqrt{\frac{2(9.81\text{ m/s}^2)(2.0\text{ m})}{1-(0.1\text{m}/1\text{m})^4}}=6.26\text{ m/s}$$

Thus,

$$Q=A_1V_1=A_2V_2=\frac{\pi}{4}(0.1\text{ m})^2(6.26\text{ m/s})=0.0492\text{ m}^3/\text{s} \qquad \textbf{(Ans)}$$

Example:

Water flows through a pipe reducer as is shown in Fig. E3.9. The static pressures at (1) and (2) are measured by the inverted U-tube manometer containing oil of specific gravity, SG, less than one. Determine the manometer reading, h.



■ FIGURE E3.9

$$P_I = P_{II}$$

$$P_1 - \rho_w g(z_2 - z_1) - \rho_w gl - \rho_w gh = P_2 - \rho_w gl - \rho_o gh$$

$$P_1 - P_2 = \rho_w g(z_2 - z_1) + \rho_w gh + \rho_w gl - \rho_w gl - \rho_o gh$$

$$P_1 - P_2 = \rho_w g(z_2 - z_1) + \rho_w gh - \rho_o gh$$

$$P_1 - P_2 = \rho_w g(z_2 - z_1) + \rho_w gh\left(1 - \frac{\rho_o}{\rho_w}\right)$$

$$\frac{P_1 - P_2}{\rho_w g} = (z_2 - z_1) + h\left(1 - SG_o\right) \quad \text{-------------} \ (1)$$

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2}{A_1} V_2 = \frac{\frac{\pi}{4} D_2^2}{\frac{\pi}{4} D_1^2} V_2$$

$$V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 \quad \text{------------------------} \ (2)$$

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho_w g} = z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g} \quad \text{-------------} \ (3)$$

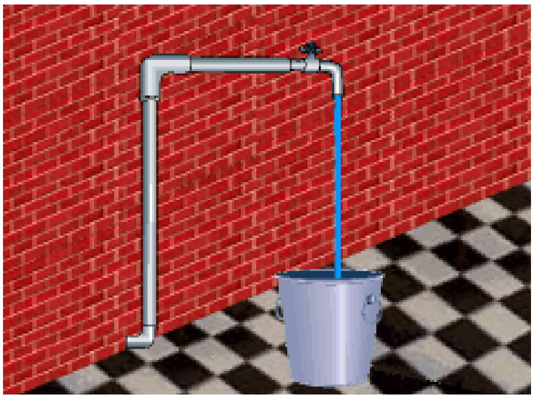
From (1) & (2) in (3)

$$(z_2 - z_1) + h\left(1 - SG_o\right) = z_2 - z_1 + \frac{V_2^2 - \left(\frac{D_2}{D_1}\right)^4 V_2^2}{2g}$$

$$\boxed{h = \frac{V_2^2\left[1 - \left(\frac{D_2}{D_1}\right)^4\right]}{2g\left(1 - SG_o\right)}}$$

Application of Bernoulli`s equation:

Simple Flowrate Measurement



The simplest technique to determine the steady flow rate of a liquid is to measure the amount of liquid collected over a period of time. For example, one could collect the liquid in a container of known size. If the time needed to fill the container is recorded, then the flow rate can be easily determined from the equation $\boxed{Q = \text{Volume} / \text{Time}}$

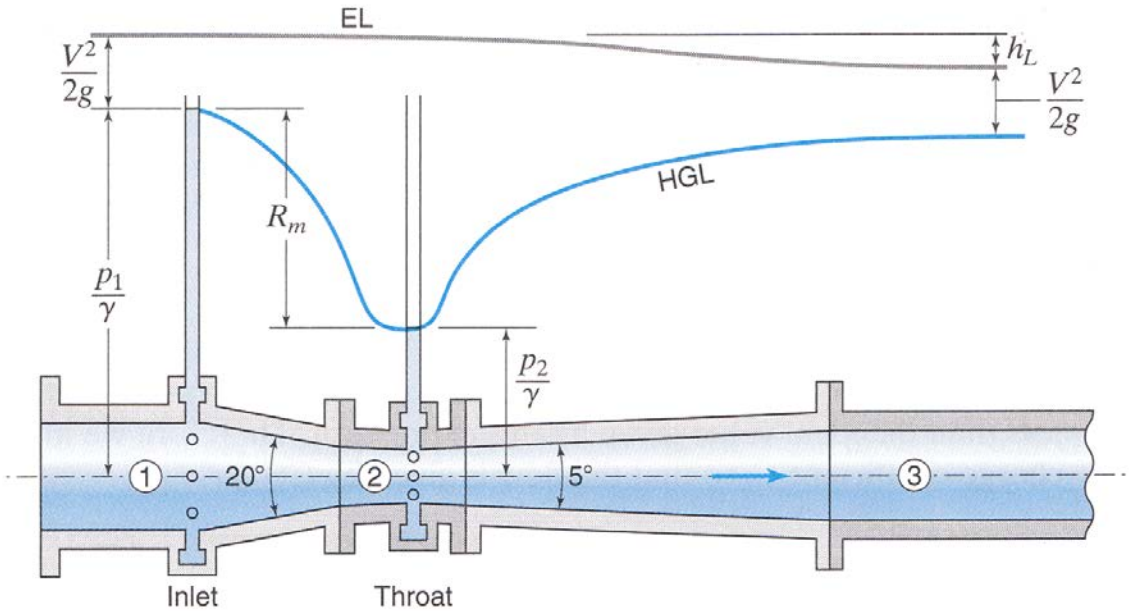
Measurements of flow rate:

A change in the cross-section area of a stream tube has been seen to produce an accelerated flow (change in flow velocity) and fall of pressure**.** By an excellent meter, flow may be calculated from pressure measurements, such as:

1. Venturi meter.
2. Orifice meter.

1. Venturi meter:

The **converging tube** is an efficient device for converting pressure head to velocity head, while the **diverging tube** converts velocity head to pressure head. Both of them may be combined to form a **Venturi tube**. As shown in Fig., it consists of a tube with a **constricted throat** (*smooth entrance cone of angle about 20^{0} as converging part*), which produces an increase in velocity accompanied by a reduction in pressure, followed by a gradually diverging portion of 5^{0} to 7^{0} cone angle in which the velocity is transformed back into pressure with slight friction loss.



As there is a definite relation between *the pressure differential and the rate of flow*, the tube may be made to serve as a metering device known as a **"Venturi Meter".** The pressure difference between the inlet section (1) and the throat section (2) is usually measured using **differential manometer.** Writing the Bernoulli equation between sections (1) (inlet) and (2) (throat), we have:

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_{loss_{1\to 2}}$$

$$V_2^2 - V_1^2 = 2g\left[\left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + (z_1 - z_2) - h_{loss_{1\to 2}} \right]$$

Continuity Eq.:

$$A_1 V_1 = A_2 V_2 = Q \quad \text{then,} \quad V_1 = \frac{Q}{A_1} \quad \& \quad V_2 = \frac{Q}{A_2}$$

Then, Bernoulli equation between sections (1) (inlet) and (2) (throat), can be rewritten as:

$$Q^2\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) = 2g\left[\frac{P_1 - P_2}{\rho g} + (z_1 - z_2) - h_{loss_{1\to 2}}\right]$$

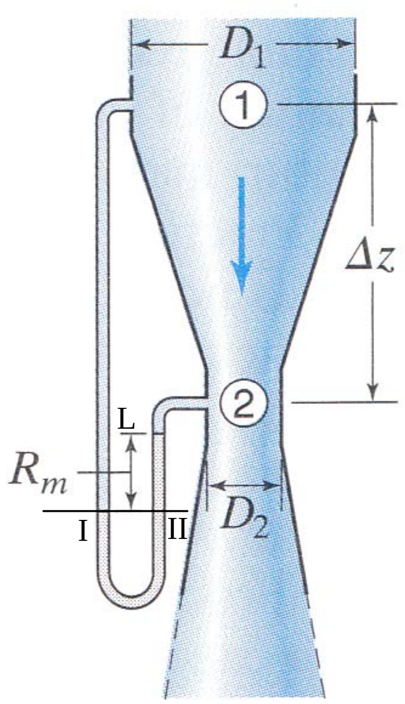
$$Q^2 = \left(\frac{A_1^2 A_2^2}{A_1^2 - A_2^2}\right) 2g\left[\frac{P_1 - P_2}{\rho g} + (z_1 - z_2) - h_{loss_{1\to 2}}\right]$$

$$Q = \sqrt{\left(\frac{A_1^2 A_2^2}{A_1^2 - A_2^2}\right) 2g\left[\frac{P_1 - P_2}{\rho g} + (z_1 - z_2) - h_{loss_{1\to 2}}\right]}$$

$$Q=\frac{A_1\,A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2g(H-h_{loss_{1\to2}})}\quad \text{or}\quad Q=C_d\,\frac{A_1\,A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gH}$$

Where: $H=\left(\frac{P_1-P_2}{\rho g}+z_1-z_2\right)$ and $\mathbf{C_d}$ is a **discharge coefficient**

Example:



Find the discharge rate of water through the venturi tube of discharge coefficient $\mathbf{C_d}$ = 0.988 that shown in Figure, if $\mathbf{D_1}$ = 800 mm, $\mathbf{D_2}$ = 400 mm, $\mathbf{\Delta z}$ = 2 m, and ***Rm*** =150 mmHg.. Use [ρ_w = *1000kg/m*^{3} , ρ_{Hg} = *13600kg/m*^{3} , and g = *9.807m/sec*^{2}]

$$Q=C_d\,\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gH}$$

$$A_1=\frac{\pi}{4}D_1^2=0.5m^2\quad \&\quad A_2=\frac{\pi}{4}D_2^2=0.125m^2$$

$$H=\frac{P_1-P_2}{\rho g}+\Delta z$$

For U_tube manometer $P_I=P_{II}$

$$P_1+\rho_w g(R_m+L+\Delta z)=P_2+\rho_w gL+\rho_m gR_m$$

$$P_1-P_2=\rho_w gL+\rho_m gR_m-\rho_w gR_m-\rho_w gL-\rho_w g\Delta z$$

$$P_1-P_2=\rho_m gR_m-\rho_w gR_m-\rho_w g\Delta z$$

$$P_1-P_2+\rho_w g\Delta z=\rho_w gR_m\left(\frac{\rho_m}{\rho_w}-1\right)$$

$$\frac{P_1-P_2}{\rho_w g}+\Delta z=R_m\left(\frac{\rho_m}{\rho_w}-1\right)=H$$

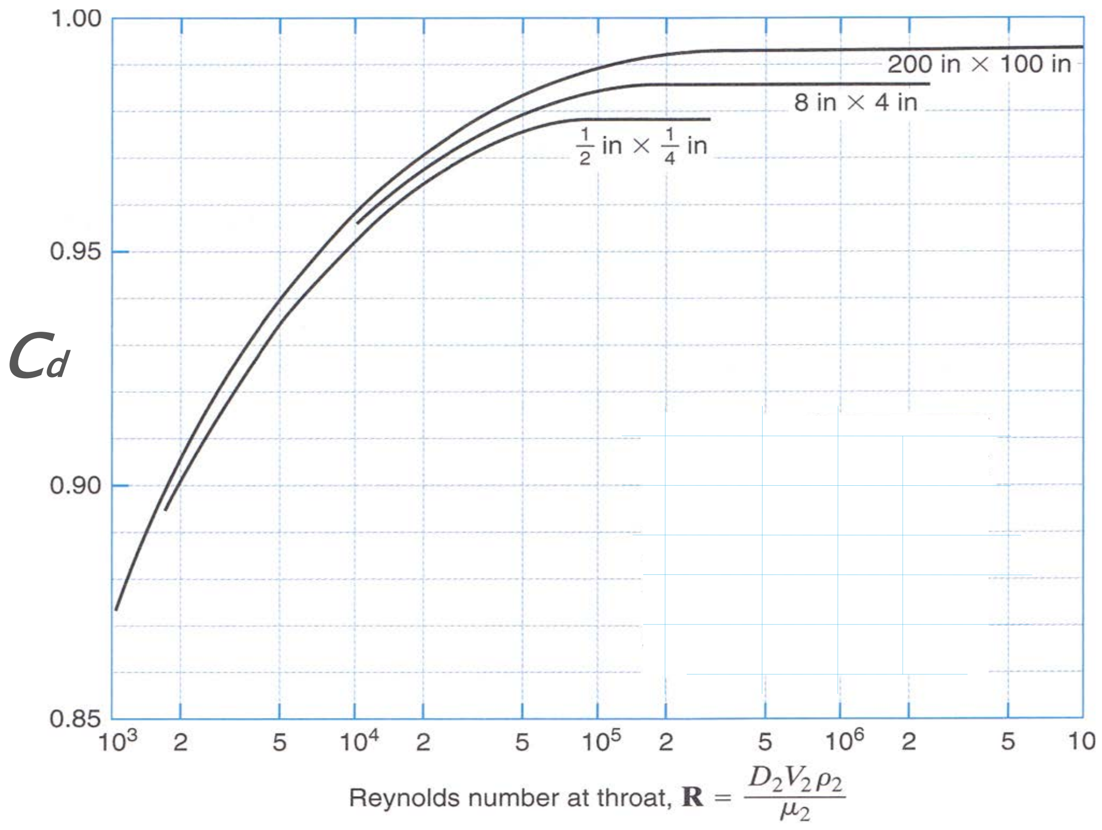
Then $H=0.15\left(\frac{13600g}{1000g}-1\right)=1.89m$

Now; $Q=C_d\,\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gH}$

$$Q=0.988\times\frac{0.5\times0.125}{\sqrt{(0.5)^2-(0.125)^2}}\sqrt{2\times9.807\times1.89}=0.777\,m^3/\text{sec}$$

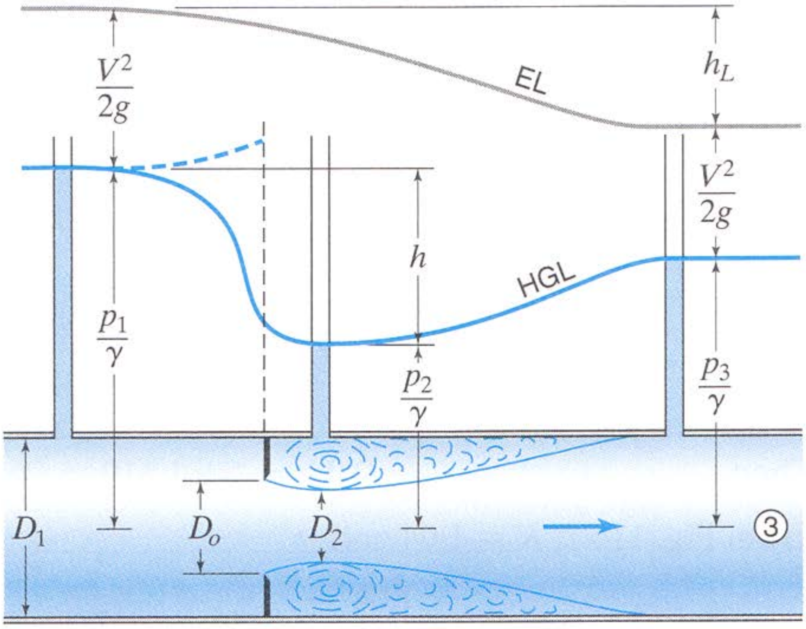
Unless specific information is available for a given venturi tube, we can assume the value of **Cd** is about 0.99 for **large tubes** and about 0.97 or 0.98 for **small ones**, provided the flow is such as to give **Reynolds numbers** greater than about 10^5.

So, the dimensional analysis of a venturi tube indicates that the **coefficient of discharge** C_d should be a function of **Reynolds number** and of the **geometric parameters** D_1 and D_2. Values of venturi tube coefficients are shown in figure.



2. Orifice meter:

We can use an orifice in a pipeline as a meter in the same manner as the venturi tube. The orifice meter consists of a concentric sharp edged circular hole in a thin plate which is clamped between two flanges in a pipe line.



Thin-plate orifice in a pipe. (Scale distorted: the region of eddying turbulence will usually extend $4D_1$ to $8D_1$ downstream, depending on the Reynolds number.)

The flow characteristics of the orifice is similar to the flow in a venturi-meter **except** that the minimum section of the stream tube A_2 occurs down stream from the orifice (section 2) owing to the formation of **vena contraction**. The ratio between the minimum area A_2 and the area of orifice A_o is known as **coefficient of contraction *Cc***, i.e., $C_c = \frac{A_2}{A_o}$

Writing the Bernoulli equation between sections (1) and (2) (downstream), we have: $\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_{loss_{1\to 2}}$

For the same horizontal level $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_{loss_{1\to 2}}$

Continuity Eq.: $A_1V_1 = A_2V_2 = Q$ i.e., $V_1 = V_2\frac{A_2}{A_1} = \frac{C_cA_o}{A_1}V_2$

$$\therefore V_2 = \sqrt{2g\left(\frac{P_1 - P_2}{\rho g} + \frac{V_1^2}{2g} - h_{loss_{1\to 2}}\right)}$$

$$V_2 = C_v\sqrt{2g\left(\frac{P_1 - P_2}{\rho g} + \frac{V_1^2}{2g}\right)} = C_v\sqrt{2gH}$$

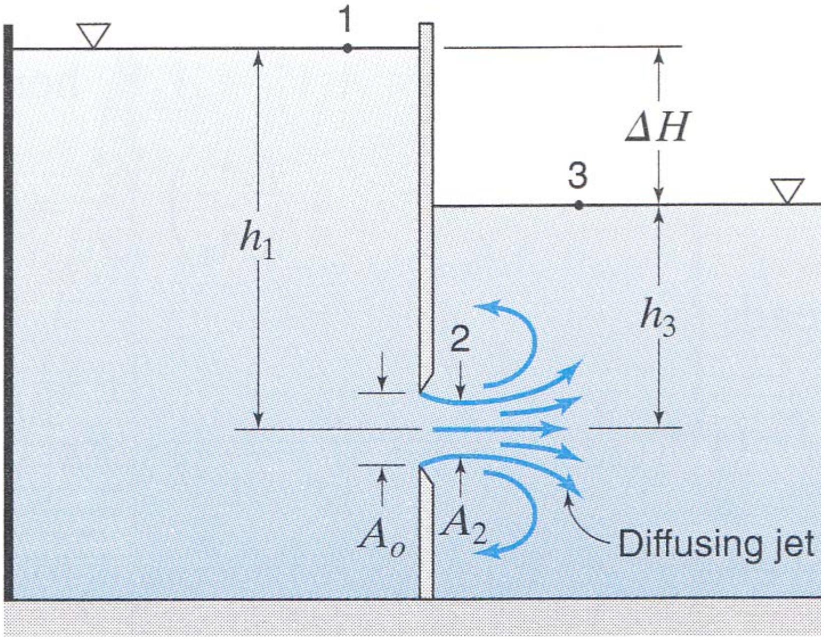
Where $H = \left(\frac{P_1 - P_2}{\rho g} + \frac{V_1^2}{2g} \right)$ and C_v is a **velocity coefficient**.

Then, $Q = A_2 V_2 = C_c A_o C_v \sqrt{2gH} = C_d A_o \sqrt{2gH}$

Where, $C_d = C_c \times C_v$ -------------- discharge coefficient.

Orifice in a Tank

for an orifice in a tank, the ideal energy equation is written between 1 and 2, Thus:



$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

$$\frac{0}{\rho g} + h_1 + \frac{0}{2g} = h_3 + 0 + \frac{V_2^2}{2g}$$

$$h_1 = h_3 + \frac{V_2^2}{2g}$$

$$\therefore V_2 = \sqrt{2g(h_1 - h_3)} = \sqrt{2g\Delta H}$$

Where, V_2 is the ideal velocity at the vena contract

& ***ΔH*** is the net head differential.

Hence **Q** will be: $Q = C_d A_o \sqrt{2g\Delta H}$

Example:

Water flows through a 60-mm-diameter sharp-edged orifice which connects two adjacent tanks. The head on one side of the orifice is 2.5 m and 0.5 m on the other. Given C_c = 0.62 and C_v = 0.95, calculate the flow rate. $A_o = \frac{\pi}{4} D_o^2 = \frac{\pi}{4}(0.06)^2 = 2.827 \times 10^{-3} m^2$

$$Q = C_d A_o \sqrt{2g\Delta H} \quad \& \quad C_d = C_c \times C_v = 0.62 \times 0.95 = 0.589$$

Then $Q = 0.589 \times 2.827 \times 10^{-3} \times \sqrt{2 \times 9.807 \times (2.5 - 0.5)}$

$$= 0.0104289 m^3 / s = 10.43 L / s$$