Extension Stroke:

$$v_{ext}(\mathrm{m/s}) = \frac{Q_{in}(\mathrm{m^3/s})}{A_p(\mathrm{m^2})}$$, $F_{ext} = P * A_P$

Retraction Stroke:

$$v_{ret}(\mathrm{m/s}) = \frac{Q_{in}(\mathrm{m^3/s})}{(A_p - A_r)\mathrm{m^2}}$$, $F_{ret} = P * (A_P - A_r)$

$V_{ret} > V_{ext}$ & $F_{ret} < F_{ext}$

EXAMPLE 6-1

A pump supplies oil at 20 gpm to a 2-in-diameter double-acting hydraulic cylinder. If the load is 1000 lb (extending and retracting) and the rod diameter is 1 in, find

a. The hydraulic pressure during the extending stroke
b. The piston velocity during the extending stroke
c. The cylinder horsepower during the extending stroke
d. The hydraulic pressure during the retraction stroke
e. The piston velocity during the retraction stroke
f. The cylinder horsepower during the retraction stroke

Solution

a.

$$p_{ext} = \frac{F_{ext}(\text{lb})}{A_p(\text{in}^2)} = \frac{1000}{(\pi/4)(2)^2} = \frac{1000}{3.14} = 318 \text{ psi}$$

b.

$$v_{ext} = \frac{Q_{in}(\text{ft}^3/\text{s})}{A_p(\text{ft}^2)} = \frac{20/449}{3.14/144} = \frac{0.0446}{0.0218} = 2.05 \text{ ft/s}$$

c.

$$\text{HP}_{ext} = \frac{v_{ext}(\text{ft/s}) \times F_{ext}(\text{lb})}{550} = \frac{2.05 \times 1000}{550} = 3.72 \text{ HP}$$

or

$$\text{HP}_{ext} = \frac{Q_{in}(\text{gpm}) \times p_{ext}(\text{psi})}{1714} = \frac{20 \times 318}{1714} = 3.72 \text{ HP}$$

d.

$$p_{ret} = \frac{F_{ret}(\text{lb})}{(A_p - A_r)\text{ in}^2} = \frac{1000}{3.14 - (\pi/4)(1)^2} = \frac{1000}{2.355} = 425 \text{ psi}$$

Therefore, as expected, more pressure is required to retract than to extend the same load due to the effect of the rod.

e.

$$v_{ret} = \frac{Q_{in}(\text{ft}^3/\text{s})}{(A_p - A_r)\text{ ft}^2} = \frac{0.0446}{2.355/144} = 2.73 \text{ ft/s}$$

Hence, as expected (for the same pump flow), the piston retraction velocity is greater than that for extension due to the effect of the rod.

f.

$$\text{HP}_{ret} = \frac{v_{ret}(\text{ft/s}) \times F_{ret}(\text{lb})}{550} = \frac{2.73 \times 1000}{550} = 4.96 \text{ HP}$$

or

$$\text{HP}_{ret} = \frac{Q_{in}(\text{gpm}) \times p_{ret}(\text{psi})}{1714} = \frac{20 \times 425}{1714} = 4.96 \text{ HP}$$

Thus, more horsepower is supplied by the cylinder during the retraction stroke because the piston velocity is greater during retraction and the load force remained the same during both strokes. This, of course, was accomplished by the greater pressure level during the retraction stroke. Recall that the pump output flow-rate is constant, with a value of 20 gpm.

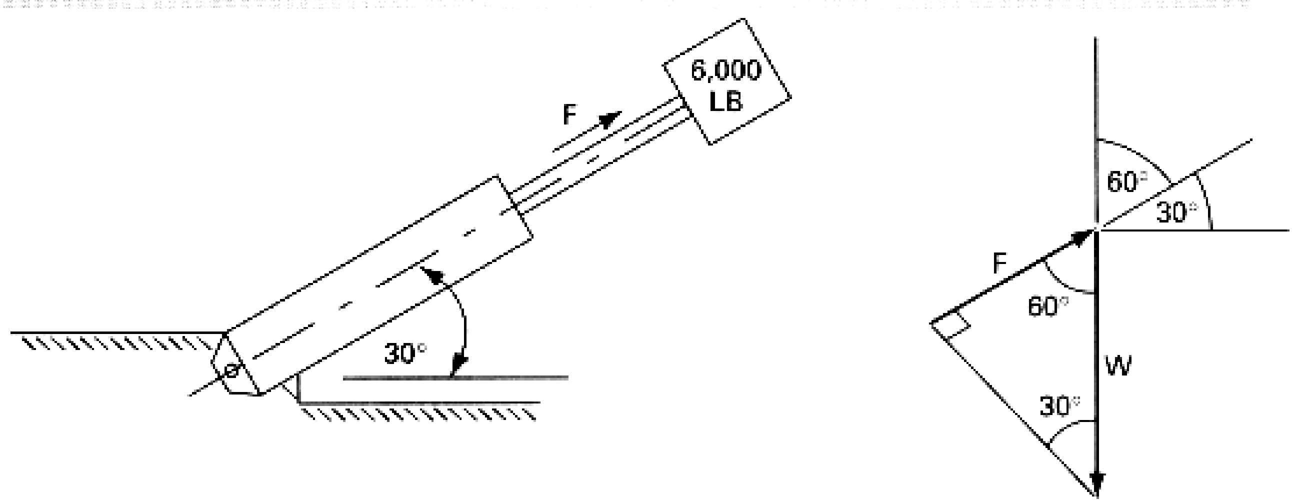
EXAMPLE 6-2

Find the cylinder force F required to move a 6000-lb weight W along a horizontal surface at a constant velocity. The coefficient of friction (CF) between the weight and horizontal support surface equals 0.14.

Solution

The frictional force f between the weight and its horizontal supporting surface equals CF times W. Thus, we have

$$F = f = (CF) \times W = 0.14 \times 6000 \text{ lb} = 840 \text{ lb}$$



(a) (b)

EXAMPLE 6-3

Find the cylinder force F required to lift the 6000-lb weight W of Example 6-2 along a direction which is 30° from the horizontal, as shown in Figure 6-8(a). The weight is moved at constant velocity.

Solution

As shown in Figure 6-8(b), the cylinder force F must equal the component of the weight acting along the centerline of the cylinder. Thus, we have a right triangle with the hypotenuse W and the side F forming a 60° angle. From trigonometry we have $\sin 30° = F/W$. Solving for F yields

$$F = W \sin 30° = 6000 \text{ lb} \times \sin 30° = 3000 \text{ lb}$$

EXAMPLE 6-4

The 6000-lb weight of Example 6-2 is to be lifted upward in a vertical direction. Find the cylinder force required to

a. Move the weight at a constant velocity of 8 ft/s
b. Accelerate the weight from zero velocity to a velocity of 8 ft/s in 0.50 s.

Solution

a. For a constant velocity the cylinder force simply equals the weight of 6000 lb.
b. From Newton's law of motion, the force required to accelerate a mass m equals the product of the mass m and its acceleration a. Noting that mass equals weight divided by the acceleration of gravity g, we have

$$a = \frac{8 \text{ ft/s} - 0 \text{ ft/s}}{0.50 \text{ s}} = 16 \text{ ft/s}^2$$

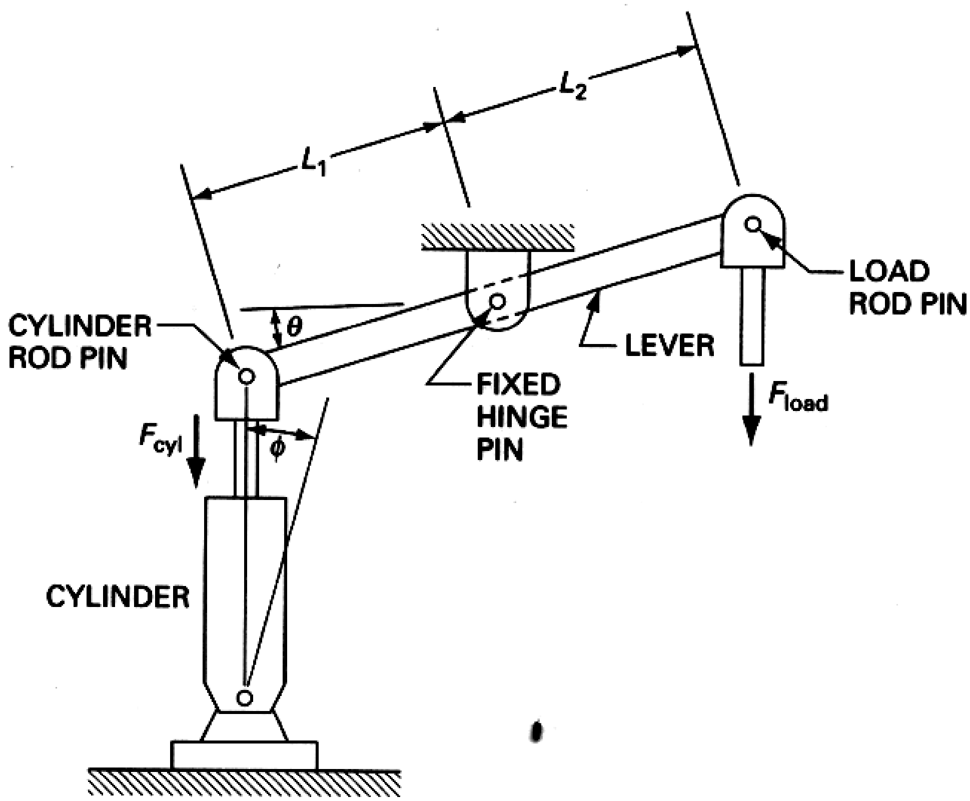
Thus, the force required to accelerate the weight is

$$F_{\text{accel}} = \frac{6000 \text{ lb}}{32.2 \text{ ft/s}^2} \times 16 \text{ ft/s}^2 = 2980 \text{ lb}$$

The cylinder force F_{cyl} required equals the sum of the weight and the acceleration force.

$$F_{\text{cyl}} = 6000 \text{ lb} + 2980 \text{ lb} = 8980 \text{ lb}$$

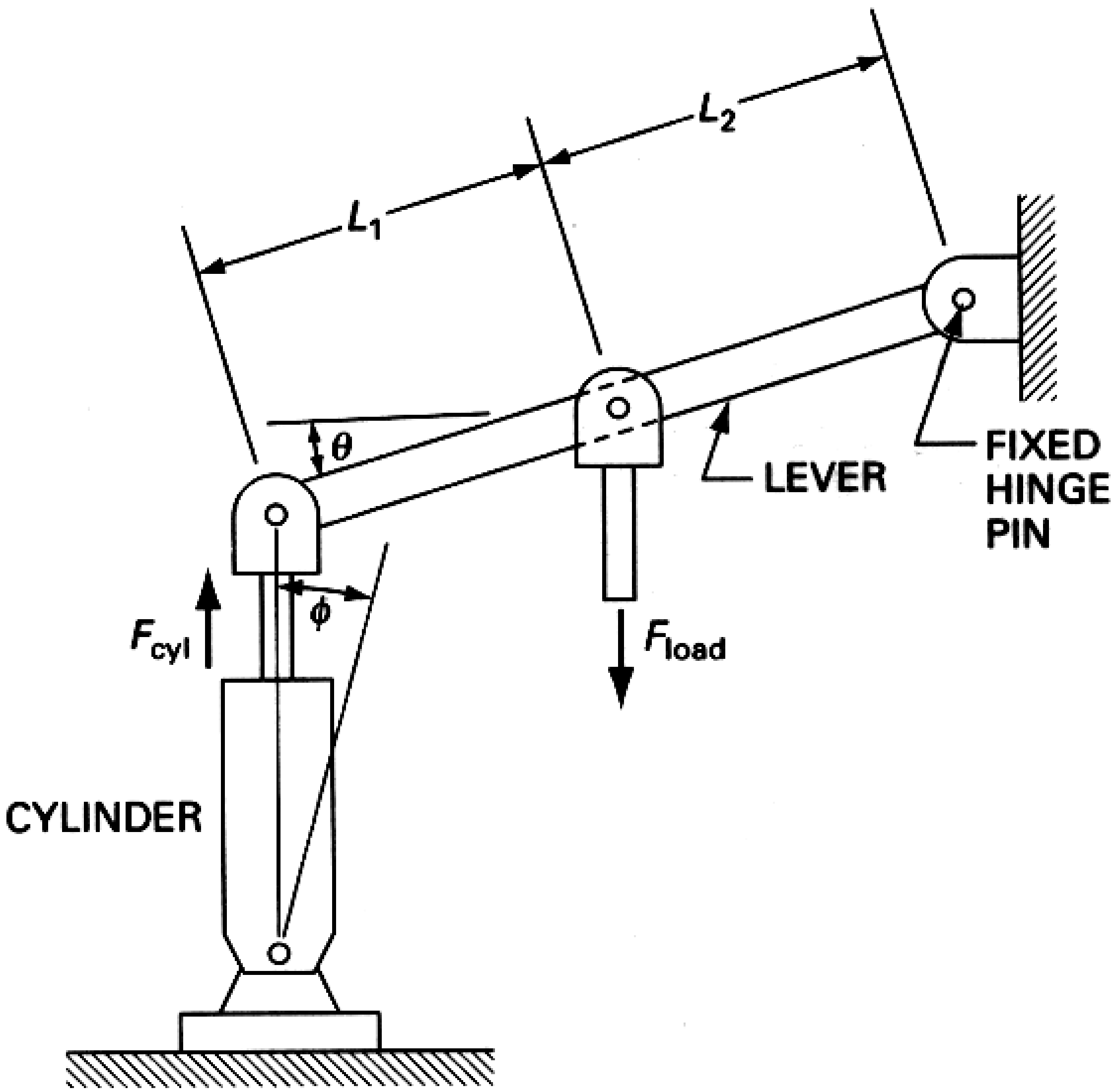
First-Class Lever System



$$F_{\text{cyl}}(L_1 \cos\theta \times cos\ \phi) = F_{\text{load}}(L_2 \cos\theta)$$

$$F_{\text{cyl}} = \frac{L_2}{L_1 \cos\phi} F_{\text{load}}$$

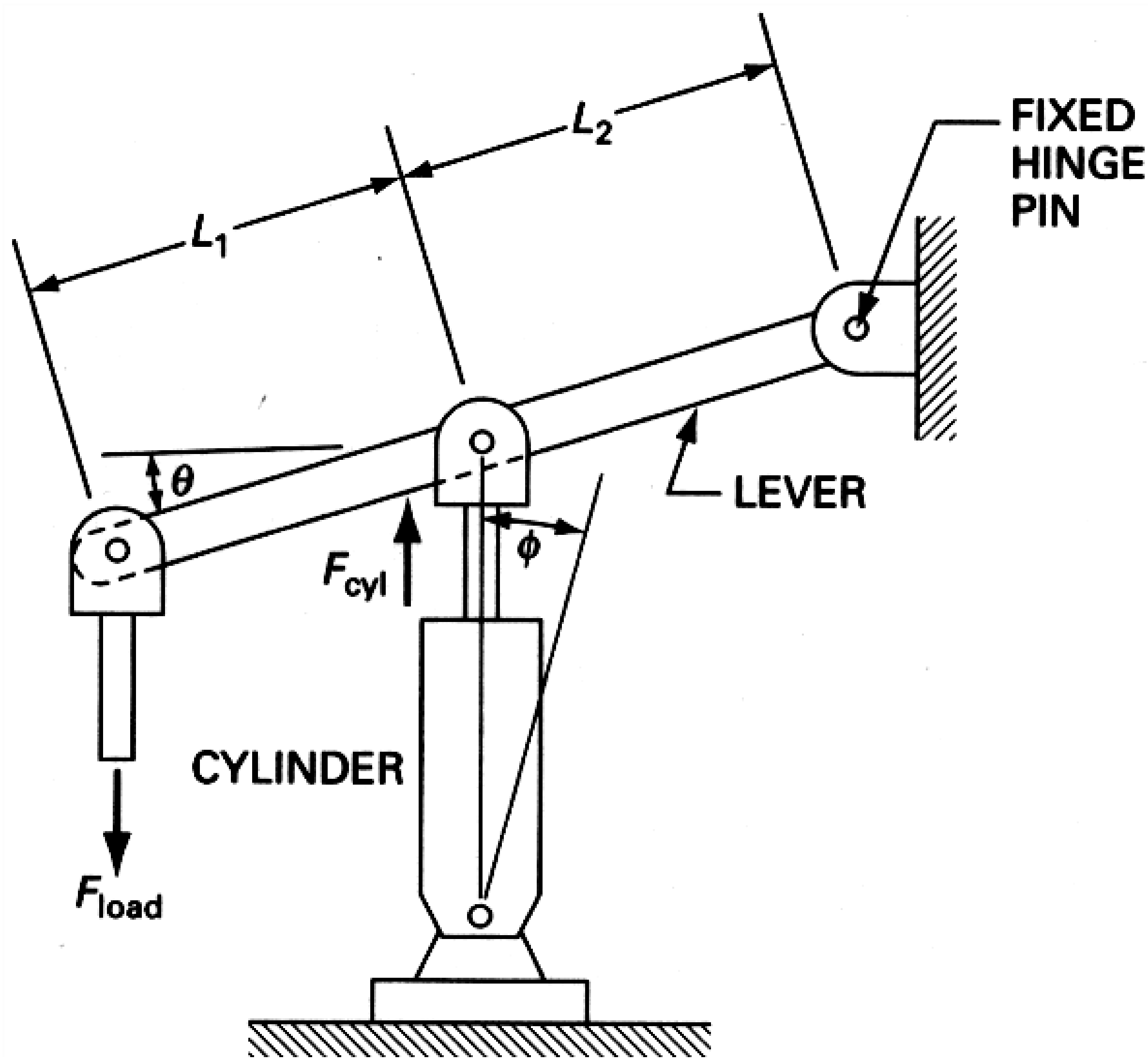
Second-Class Lever System



$$F_{\text{cyl}} \cos \phi \; (L_1 + L_2)\cos \theta = F_{\text{load}} (L_2 \cos \theta)$$

$$F_{\text{cyl}} = \frac{L_2}{(L_1 + L_2)\cos \phi} F_{\text{load}}$$

Third-Class Lever System



$$F_{\mathrm{cyl}} \cos \phi \,(L_2 \cos \theta) = F_{\mathrm{load}}(L_1 + L_2) \cos \theta$$

$$F_{\mathrm{cyl}} = \frac{L_1 + L_2}{L_2 \cos \phi} F_{\mathrm{load}}$$

EXAMPLE 6-5

For the first-, second-, and third-class lever systems of Figures 6-12, 6-13, and 6-14 the following data are given:

$$L_1 = L_2 = 10 \text{ in}$$

$$\phi = 0°$$

$$F_{\text{load}} = 1000 \text{ lb}$$

Find the cylinder force required to overcome the load force for the

a. First-class lever
b. Second-class lever
c. Third-class lever

Solution

a. Per Eq. (6-7) we have

$$F_{\text{cyl}} = \frac{L_2}{L_1 \cos \phi} F_{\text{load}} = \frac{10}{10 \times 1}(1000) = 1000 \text{ lb}$$

b. Using Eq. (6-8) yields

$$F_{\text{cyl}} = \frac{L_2}{(L_1 + L_2)\cos \phi} F_{\text{load}} = \frac{10}{(10 + 10) \times 1}(1000) = 500 \text{ lb}$$

c. Substituting into Eq. (6-9), we have

$$F_{\text{cyl}} = \frac{L_1 + L_2}{L_2 \cos \phi} F_{\text{load}} = \frac{(10 + 10)}{10 \times 1}(1000) = 2000 \text{ lb}$$

Thus, as expected, the second-class lever requires the smallest cylinder force, whereas the third-class lever requires the largest cylinder force.