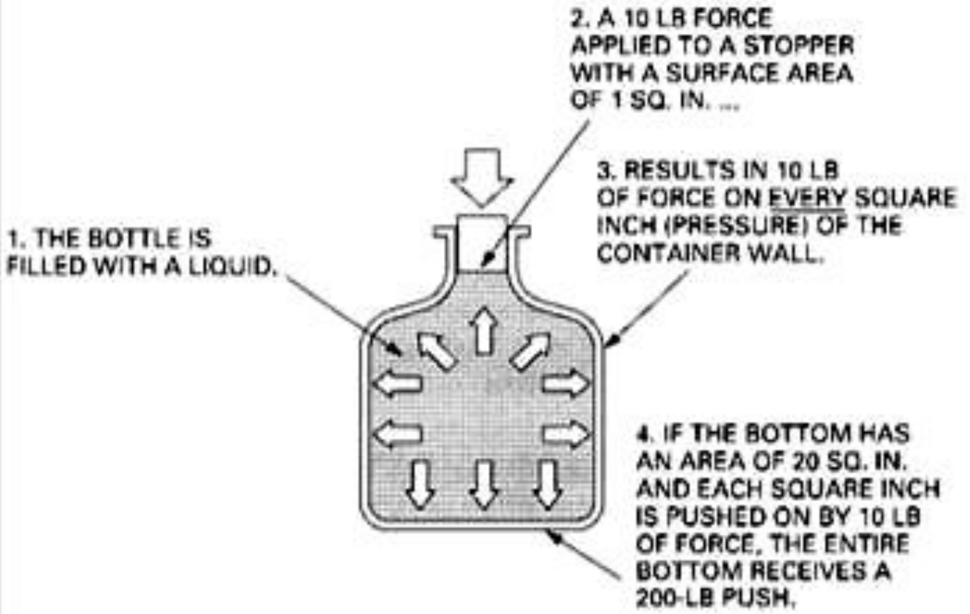
# MULTIPLICATION OF FORCE (PASCAL'S LAW)

**"Pressure** applied to a **confined** **Fluid** is **Transmitted Undiminished** in **All Directions** throughout the **Fluid** and **acts perpendicular** to the **surfaces** in contact with the **Fluid".**

The **Liquid** **Transmits** the **Pressure,** created by the **Force** of the **Stopper**, throughout the **Container**

**Fig. 3-4**.

**Figure 3.4 Demonstration of Pascal's Law.**



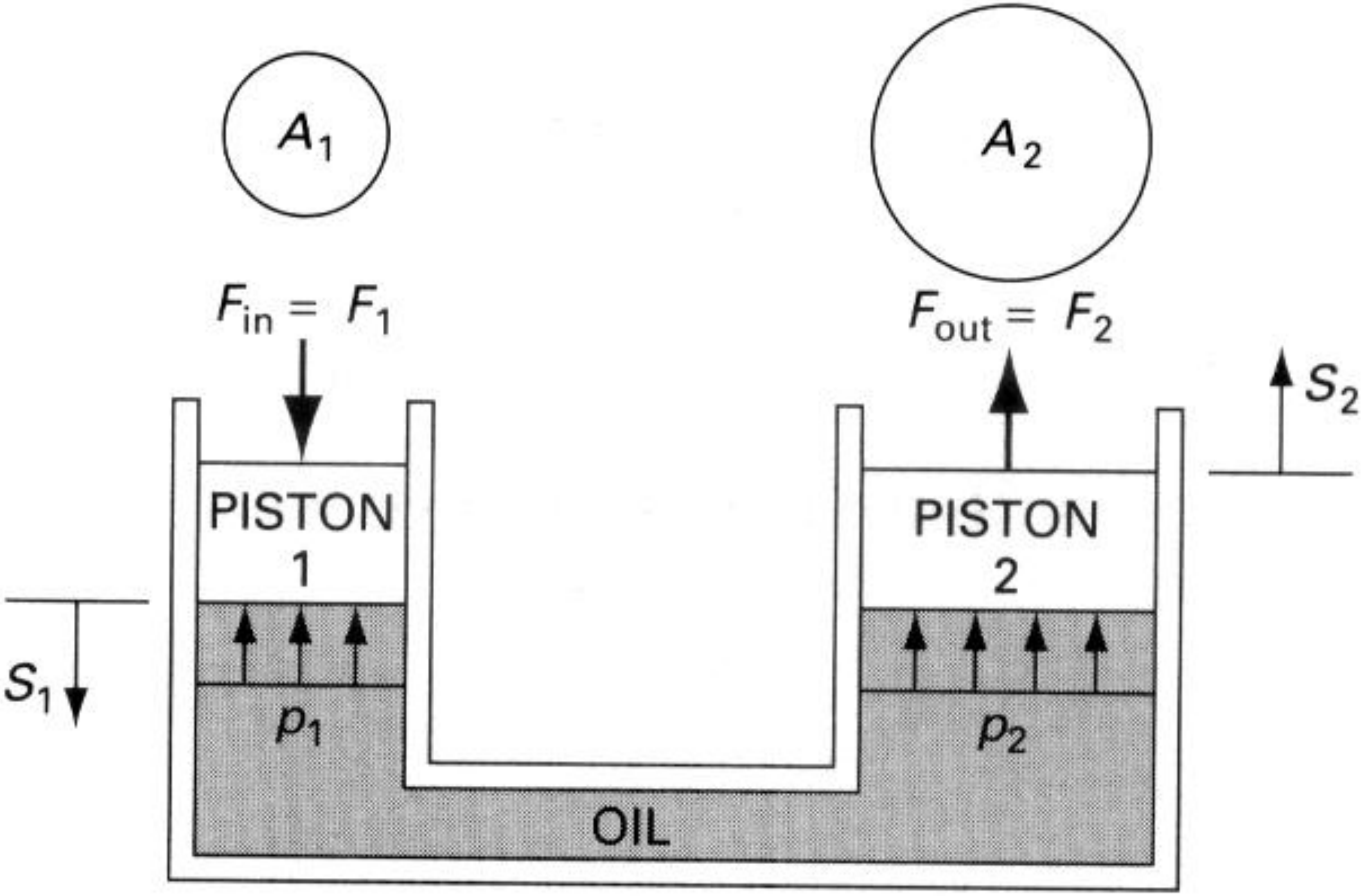



**Figure3-6**

**Operation of Simple Hydraulic Jack**

$$(F_1/A_1) = (F_2/A_2)$$
$$(F_2/F_1) = (A_2/A_1)$$

$$V_1 = V_2$$

$$A_1S_1 = A_2S_2$$

$$(S_1/S_2) = (A_2/A_1)$$

$$(F_1/F_2) = (S_2/S_1)$$

$$F_1S_1 = F_2S_2$$

$$Work_1 = Work_2$$

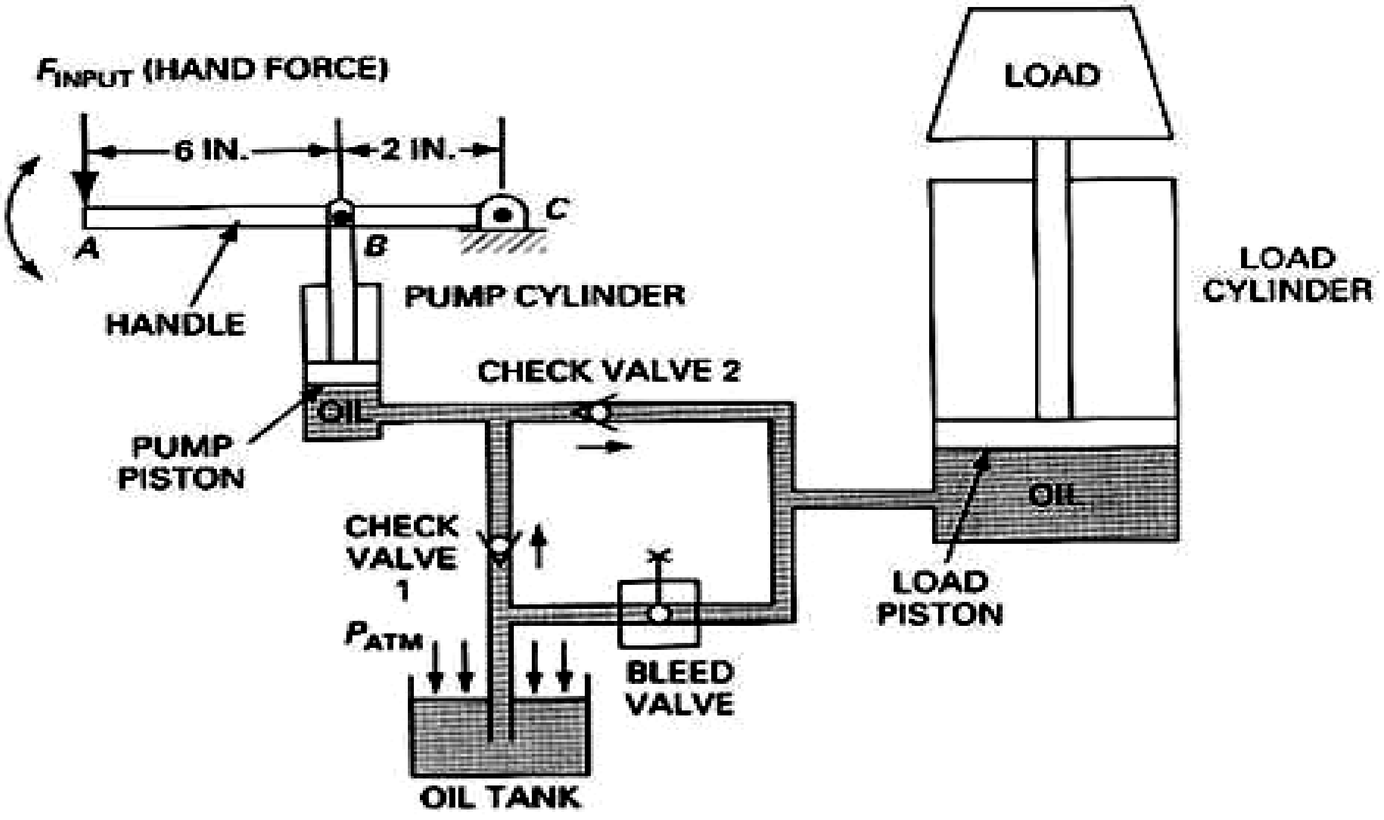



**Figure 3-7**

**Hand-Operated Hydraulic Jack**

**EXAMPLE 3-5**

**An operator makes one complete cycle per second interval using the hydraulic jack of Figure 3-7. Each complete cycle consists of two pump cylinder strokes (intake and power). The pump cylinder has a 1-in-diameter piston and the load cylinder has a 3.25-in-diameter piston. If the average hand force is 25 lb during the power stroke,**

**a.** How much load can be lifted?

**b.** How many cycles are required to lift the load 10 in assuming no oil leakage? The pump piston has a 2-in stroke.

**c.** What is the output HP assuming 100% efficiency?

**d.** What is the output HP assuming 80% efficiency?

***Solution***

**a.** First determine the force acting on the rod of the pump cylinder due to the mechanical advantage of the input handle:

$$F_{\text{rod}} = \frac{8}{2} \times F_{\text{input}} = \frac{8}{2}(25) = 100 \text{ lb}$$

Next, calculate the pump cylinder discharge pressure $p$:

$$p = \frac{\text{rod force}}{\text{piston area}} = \frac{F_{\text{rod}}}{A_{\text{pump piston}}} = \frac{100 \text{ lb}}{(\pi/4)(1)^2 \text{in}^2} = 127 \text{ psi}$$

Per Pascal's law this is also the same pressure acting on the load piston. We can now calculate the load-carrying capacity:

$$F_{\text{load}} = pA_{\text{load piston}} = (127)\text{lb/in}^2\left[\frac{\pi}{4}(3.25)^2\right]\text{in}^2 = 1055 \text{ lb}$$

**b.** To find the load displacement, assume the oil to be incompressible. Therefore, the total volume of oil ejected from the pump cylinder equals the volume of oil displacing the load cylinder:

$$(A \times S)_{\text{pump piston}} \times (\text{no. of cycles}) = (A \times S)_{\text{load piston}}$$

Substituting, we have

$$\frac{\pi}{4}(1)^2\,\text{in}^2 \times 2\text{ in} \times (\text{no. of cycles}) = \frac{\pi}{4}(3.25)^2\,\text{in}^2 \times 10\text{ in}$$

$$1.57\text{ in}^3 \times (\text{no. of cycles}) = 82.7\text{ in}^3$$

$$\text{no. of cycles} = 52.7$$

**c.**

$$\text{Power} = \frac{FS}{t} = \frac{(1055\text{ lb})\left(\frac{10}{12}\text{ ft}\right)}{52.7\text{ s}} = 16.7\text{ ft}\cdot\text{lb/s}$$

**EXAMPLE 3-6**

Figure 3-12 shows a pressure booster used to drive a load $F$ via a hydraulic cylinder. The following data are given:

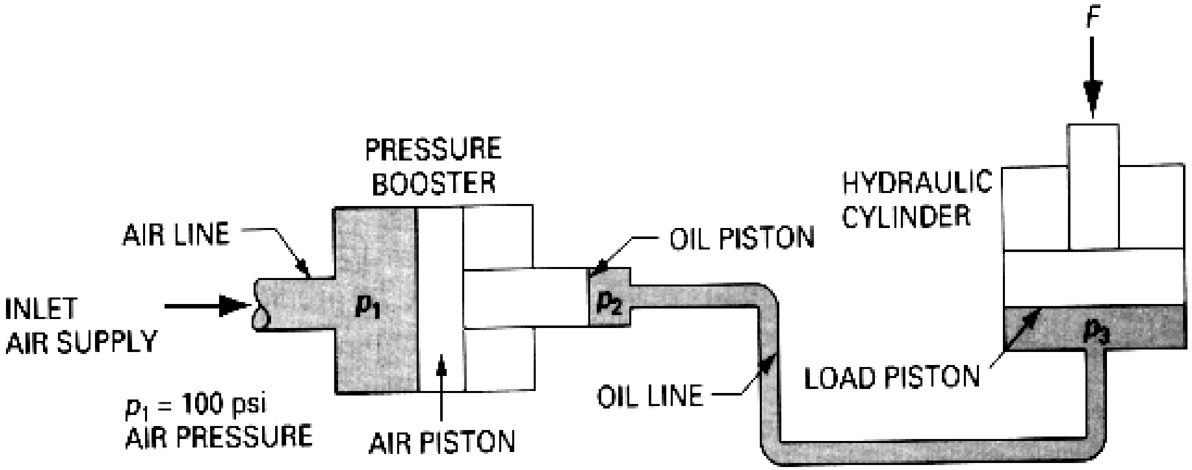
$$\text{inlet air pressure } (p_1) = 100 \text{ psi}$$

$$\text{air piston area } (A_1) = 20 \text{ in}^2$$

$$\text{oil piston area } (A_2) = 1 \text{ in}^2$$

$$\text{load piston area } (A_3) = 25 \text{ in}^2 \text{ (diameter} = 5.64 \text{ in)}$$

Find the load-carrying capacity $F$ of the system.



***Solution*** First, find the booster discharge pressure $p_2$:

$$\text{booster input force} = \text{booster output force}$$

$$p_1A_1 = p_2A_2$$

$$p_2 = \frac{p_1A_1}{A_2} = (100)\left(\frac{20}{1}\right) = 2000\ psi$$

Per Pascal's law, $p_3 = p_2 = 2000$ psi:

$$F = p_3A_3 = (2000)(25) = 50{,}000 \text{ lb}$$

To produce this force without the booster would require a 500-in$^2$-area load piston (diameter = 25.2 in), assuming 100-psi air pressure.

**EXAMPLE 3-8**

A hydraulic cylinder is to compress a car body down to bale size in 10 s. The operation requires a 10-ft stroke and a 8000-lb force. If a 1000-psi pump has been selected, and assuming the cylinder is 100% efficient, find

**a.** The required piston area
**b.** The necessary pump flow rate
**c.** The hydraulic horsepower delivered to the cylinder
**d.** The output horsepower delivered by the cylinder to the load

***Solution***

**a.** $$A = \frac{F_{\text{load}}}{p} = \frac{8000 \text{ lb}}{1000 \text{ lb/in}^2} = 8 \text{ in}^2$$

**b.** $$Q(\text{ft}^3/\text{s}) = \frac{A(\text{ft}^2) \times S(\text{ft})}{t(s)} = \frac{\left(\frac{8}{144}\right)(10)}{10} = 0.0556 \text{ ft}^3/\text{s}$$

Per Appendix E, 1 $\text{ft}^3/\text{s} = 449$ gpm. Thus,

$$Q(\text{gpm}) = 449Q(\text{ft}^3/\text{s}) = (449)(0.0556) = 24.9 \text{ gpm}$$

**c.** $$\text{HHP} = \frac{(1000)(24.9)}{1714} = 14.5 \text{ HP}$$

**d.** $$\text{OHP} = \text{HHP} \times \eta = 14.5 \times 1.0 = 14.5 \text{ HP}$$

Thus, assuming a 100% efficient cylinder (losses are negligibly small), the hydraulic horsepower equals the output horsepower.

**EXAMPLE 3-9**

Solve the problem of Example 3-8 assuming a frictional force of 100 lb and a leakage of 0.2 gpm.

### *Solution*

**a.** $$A = \frac{F_{\text{load}} + F_{\text{friction}}}{p} = \frac{8000 \text{ lb} + 100 \text{ lb}}{1000 \text{ lb/in}^2} = 8.10 \text{ in}^2$$

**b.** $$Q_{\text{theoretical}} = \frac{A(\text{ft}^2) \times S(\text{ft})}{t(\text{s})} = \frac{\left(\frac{8.10}{144}\right)(10)}{10} = 0.0563 \text{ ft}^3\text{/s} = 25.2 \text{ gpm}$$

$$Q_{\text{actual}} = Q_{\text{theoretical}} + Q_{\text{leakage}} = 25.2 + 0.2 = 25.4 \text{ gpm}$$

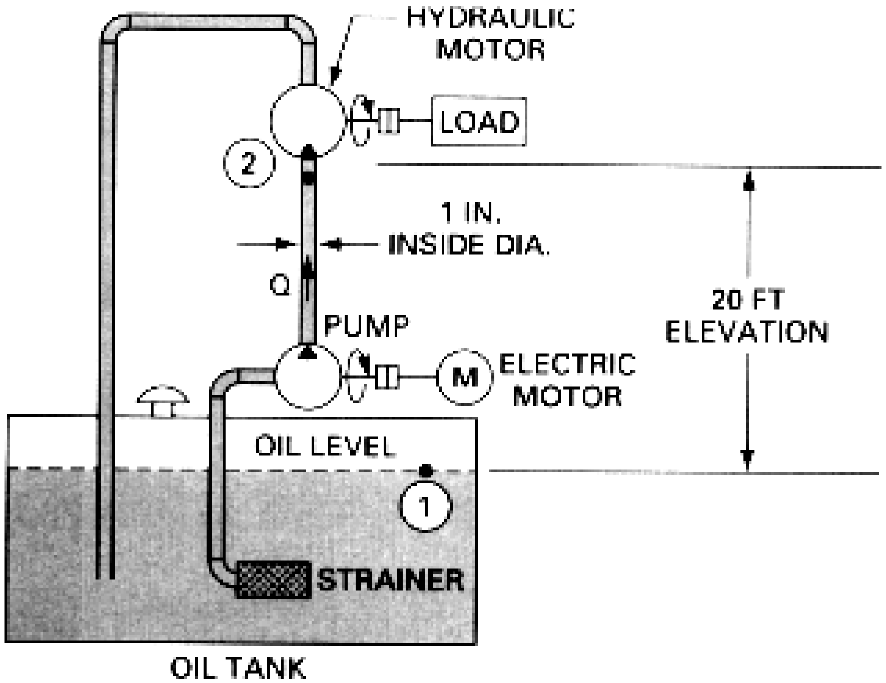
**c.** $$\text{HHP} = \frac{1000 \times 25.4}{1714} = 14.8 \text{ HP}$$

**d.** $$\text{OHP} = \frac{F(\text{lb}) \times v(\text{ft/s})}{550} = \frac{8000 \times 1}{550} = 14.5 \text{ HP}$$

**EXAMPLE 3-17**

For the hydraulic system of Figure 3-20, the following SI metric data are given:

- **a.** The pump is adding 3.73 kW (pump hydraulic power = 3.73 kW) to the fluid.
- **b.** Pump flow is 0.001896 $m^3/s$.
- **c.** The pipe has a 0.0254-m inside diameter. Note that this size can also be represented in units of centimeters or millimeters as 2.54 cm or 25.4 mm, respectively.
- **d.** The specific gravity of the oil is 0.9.
- **e.** The elevation difference between stations 1 and 2 is 6.096 m.



Find the pressure available at the inlet to the hydraulic motor (station 2). The pressure at station 1 in the hydraulic tank is atmospheric (0 Pa or 0 $N/m^2$ gage). The head loss $H_L$ due to friction between stations 1 and 2 is 9.144 m of oil.

***Solution*** Writing the energy equation between stations 1 and 2, we have

$$Z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + H_p - H_m - H_L = Z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

Since there is no hydraulic motor between stations 1 and 2, $H_m = 0$. Also, $v_1 = 0$ because the cross section of an oil tank is large. Also, $H_L = 9.144$ m and $p_1 = 0$ per the given input data.

Substituting known values, we have

$$Z_1 + 0 + 0 + H_p - 0 - 9.144 = Z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

Solving for $p_2/\gamma$, we have

$$\frac{p_2}{\gamma} = (Z_1 - Z_2) + H_p - \frac{v_2^2}{2g} - 9.144$$

Since $Z_2 - Z_1 = 6.096$ m, we have

$$\frac{p_2}{\gamma} = H_p - \frac{v_2^2}{2g} - 15.24$$

From Eq. (3-36) we solve for the pump head:

$$H_p(\mathrm{m}) = \frac{\text{pump hydraulic power(W)}}{\gamma(\mathrm{N/m^3}) \times Q(\mathrm{m^3/s})}$$

where

$$\gamma_{\text{oil}} = (\mathrm{SG})\,\gamma_{\text{water}} = 0.9 \times 9797\ \mathrm{N/m^3} = 8817\ \mathrm{N/m^3},$$

$$H_p(\mathrm{m}) = \frac{3730}{8817 \times 0.001896} = 223.1\ \mathrm{m}.$$

Next we solve for $v_2$ and $v_2^2/2g$:

$$v_2(\text{m/s}) = \frac{Q(\text{m}^3/\text{s})}{A(\text{m}^2)} = \frac{0.001896}{(\pi/4)(0.0254\ \text{m})^2} = 3.74\ \text{m/s}$$

$$\frac{v_2^2}{2g} = \frac{(3.74\ \text{m/s})^2}{(2)(9.80\ \text{m/s}^2)} = 0.714\ \text{m}$$

On final substitution, we have

$$\frac{p_2}{\gamma} = 223.1 - 0.714 - 15.24 = 207.1\ \text{m}$$

Solving for $p_2$ yields

$$p_2(\text{N/m}^2) = (207.1\ \text{m})\ \gamma\ (\text{N/m}^3)$$

$$p_2 = (207.1)(8817) = 1{,}826{,}000\ \text{Pa} = 1826\ \text{kPa gage}$$