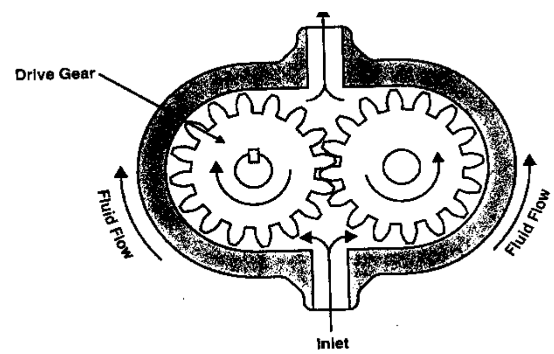
For Any Pump:

$$\boldsymbol{Q_T}\left(\frac{\mathrm{m}^3}{\mathrm{s}}\right) = \frac{\boldsymbol{V_D}\ \left(\frac{\mathrm{m}^3}{\mathrm{rev}}\right) \times \boldsymbol{N}\ (\mathrm{rpm})}{\mathbf{60}}$$

$$\boldsymbol{Q_T}(\mathrm{Gpm}) = \frac{\boldsymbol{V_D}\ \left(\frac{\mathrm{in}^3}{\mathrm{rev}}\right) \times \boldsymbol{N}\ (\mathrm{rpm})}{\mathbf{231}}$$

External Gear Pump:



If $V_{tooth} = V_{space}$

$$\boldsymbol{V_D} = \frac{\boldsymbol{\pi}}{\mathbf{4}}\left(\boldsymbol{D_o^2} - \boldsymbol{D_i^2}\right)\boldsymbol{L}$$

Where:

D_0 = Outside Diameter <u>of</u> Gear Teeth (in, m)

D_i = Inside Diameter of Gear Teeth (in, m)

L = Width of Gear Teeth (in, m)

V_D = Displacement Volume of Pump (in^3/rev,m^3/rev)

N = rpm of Pump

Q_T = Theoretical Pump Flow-Rate (in^3/s, m^3/s)

EXAMPLE 5-1

A gear pump has a 3-in outside diameter, a 2-in inside diameter, and a 1-in width. If the actual pump flow at 1800 rpm and rated pressure is 28 gpm, what is the volumetric efficiency?

Solution Find the displacement volume using Eq. (5-1):

$$V_D = \frac{\pi}{4}[(3)^2 - (2)^2](1) = 3.93 \text{ in}^3$$

Next, use Eq. (5-2) to find the theoretical flow-rate:

$$Q_T = \frac{V_D N}{231} = \frac{(3.93)(1800)}{231} = 30.6 \text{ gpm}$$

The volumetric efficiency is then found using Eg. (5-3):

$$\eta_v = \frac{28}{30.6} = 0.913 = 91.3\%$$

EXAMPLE 5-2

A gear pump has a 75-mm outside diameter, a 50-mm inside diameter, and a 25-mm width. If the volumetric efficiency is 90% at rated pressure, what is the corresponding actual flow-rate? The pump speed is 1000 rpm.

Solution The volume displacement is

$$V_D = \frac{\pi}{4}[(0.075)^2 - (0.050)^2](0.025) = 0.0000614 \text{m}^3/\text{rev}$$

Since 1 L = 0.001 m^3, V_D = 0.0614 L.

Next, combine Eqs. (5-2M) and (5-3) to find the actual flow-rate:

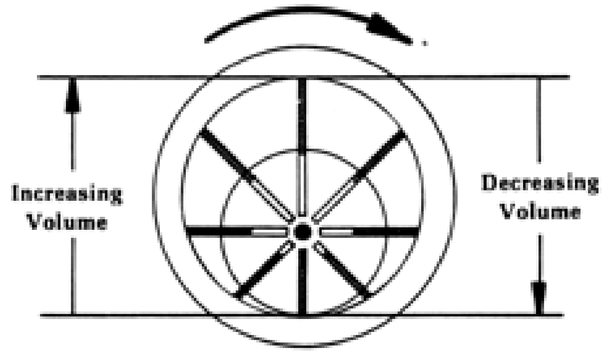
$$Q_A = \eta_v Q_T = \eta_v V_D(\text{m}^3/\text{rev}) \times N(\text{rev/min})$$

$$= 0.90 \times 0.0000614 \times 1000 = 0.0553 \text{ m}^3/\text{min}$$

Since 1 L = 0.001 m^3, we have

$$Q_A = 55.3 \text{ Lpm}$$

Vane Pump:



Note: Ring Does Not Rotate

$$e_{max} = \frac{D_C - D_R}{2}$$

$$V_D = \frac{\pi}{2}(D_C + D_R)eL$$

D_c = Diameter of Cam Ring (in, m)
D_R = Diameter of Rotor (in, m)
L = Width of Rotor (in, m)
V_D = Pump Volumetric Displacement (in^3, m^3)
e = Eccentricity (in, m)
e_{max} = Maximum possible Eccentricity (in, m)
V_{Dmax} = Maximum possible Volumetric Displacement (in^3, m^3)

Example 5-3

A vane pump is to have a volumetric displacement of 5 in^3. It has a rotor diameter of 2 in, a cam ring diameter of 3 in, and a vane width of 2 in. What must be the eccentricity?

Solution Use Eq. (5-4):

$$e = \frac{2V_D}{\pi(D_C + D_R)L} = \frac{(2)(5)}{\pi(2 + 3)(2)} = 0.318 \text{ in}$$

EXAMPLE 5-4

A vane pump has a rotor diameter of 50 mm, a cam ring diameter of 75 mm, and a vane width of 50 mm. If the eccentricity is 8 mm, determine the volumetric displacement.

Solution Substituting values into Eq. (5-4) yields

$$V_D = \frac{\pi}{2}(0.050 + 0.075)(0.008)(0.050) = 0.0000785 \text{ m}^3$$

Since 1 L = 0.001 m^3, $V_D = 0.0785$ L.

EXAMPLE 5-5

A fixed displacement vane pump delivers 1000 psi oil to an extending hydraulic cylinder at 20 gpm. When the cylinder is fully extended, oil leaks past its piston at a rate of 0.7 gpm. The pressure relief valve setting is 1200 psi. If a pressure-compensated vane pump were used it would reduce pump flow from 20 gpm to 0.7 gpm when the cylinder is fully extended to provide the leakage flow at the pressure relief valve setting of 1200 psi. How much hydraulic horsepower would be saved by using the pressure-compensated pump?

Solution The fixed displacement pump produces 20 gpm at 1200 psi when the cylinder is fully extended (0.7 gpm leakage flow through the cylinder and 19.3 gpm through the relief valve). Thus, we have

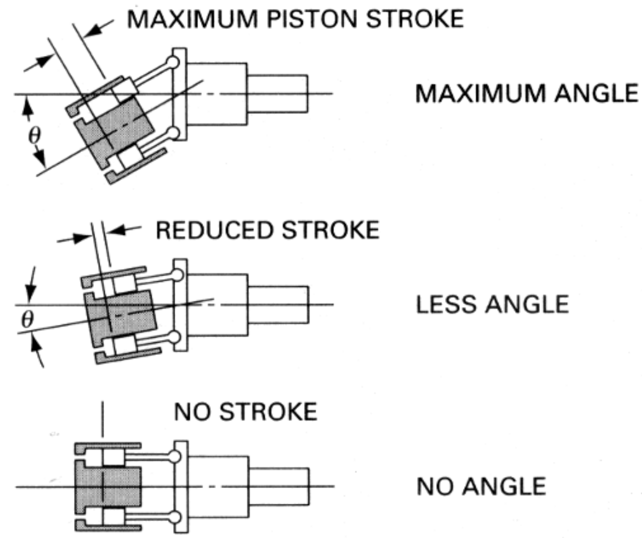
$$\text{hydraulic HP lost} = \frac{pQ}{1714} = \frac{1200 \times 20}{1714} = 14.0 \text{ HP}$$

A pressure-compensated pump would produce only 0.7 gpm at 1200 psi when the cylinder is fully extended. For this case we have

$$\text{hydraulic HP lost} = \frac{pQ}{1714} = \frac{1200 \times 0.7}{1714} = 0.49 \text{ HP}$$

Hence, the hydraulic horsepower saved = 14.0 − 0.49 = 13.51 HP. This horsepower savings occurs only while the cylinder is fully extended because either pump would deliver 1000 psi oil at 20 gpm while the cylinder is extending.

Piston Pump:



$$S = D \tan(\theta)$$

$$V_D = YAS$$

$$V_D = YAD \tan(\theta)$$

Where:
θ = Offset Angle (°)
S = Piston Stroke (in, m)
D = Piston Circle Diameter (in, m)
Y = Number of Pistons
A = Piston Area (in^2, m^2)
N = Pump Speed (rpm)
Q_T = Theoretical Flow-Rate (gpm, m^3/min)

Example 5-6

Find the offset angle for an axial piston pump that delivers 16 gpm at 3000 rpm. The pump has nine $\frac{1}{2}$-in-diameter pistons arranged on a 5-in-diameter piston circle. The volumetric efficiency is 95%.

Solution From Eq. (5-3) we calculate the theoretical flow rate.

$$Q_T = \frac{Q_A}{\eta_v} = \frac{16 \text{ gpm}}{0.95} = 16.8 \text{ gpm}$$

Using Eq. (5-6) yields:

$$\tan(\theta) = \frac{231 Q_T}{DANY} = \frac{231 \times 16.8}{5[\pi/4(1/2)^2] \times 3000 \times 9} = 0.146$$

$$\theta = 8.3°$$

EXAMPLE 5-7

Find the flow rate in units of L/s that an axial piston pump delivers at 1000 rpm. The pump has nine 15-mm-diameter pistons arranged on a 125-mm-diameter piston circle. The offset angle is set at 10° and the volumetric efficiency is 94%.

Solution Substituting directly into Eq. (5-6M) yields

$$Q_T(\text{m}^3/\text{min}) = D(\text{m}) \times A(\text{m}^2) \times N(\text{rev/min}) \times Y \tan(\theta)$$

$$= 0.125\left(\frac{\pi}{4} \times 0.015^2\right) \times 1000 \times 9 \times \tan 10° = 0.0351\ \text{m}^3/\text{min}$$

From Eq. (5-3) we calculate the actual flow-rate.

$$Q_A = Q_T\eta_v = 0.0351\ \text{m}^3/\text{min} \times 0.94 = 0.0330\ \text{m}^3/\text{min}$$

To convert to flow-rate in units of L/s, we perform the following manipulation of units:

$$Q_A(L/s) = Q_A(\text{m}^3/\text{min}) \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{1\ \text{L}}{0.001\ \text{m}^3}$$

$$= 0.0330 \times \frac{1}{60} \times \frac{1}{0.001} = 0.550\ \text{L/s}$$

PUMP PERFORMANCE

1. Volumetric Efficiency (η_v):

$$\eta_v = \frac{\text{actual flow-rate produced by pump}}{\text{theoretical flow-rate pump should produce}} = \frac{Q_A}{Q_T}$$

2. Mechanical Efficiency (η_m):

$$\eta_m = \frac{\text{pump output power assuming no leakage}}{\text{actual power delivered to pump}}$$

$$\eta_m = \frac{pQ_T}{T_A N}$$

Where

P = Pump Discharge Pressure (psi, Pa)
Q_T = Pump Theoretical Flow-rate (gpm, m^3/s)
T_A = Actual Torque delivered to pump (in • Ib, N • m)
N = Pump Speed (rpm, rad/s)

$$\eta_m = \frac{\text{theoretical torque required to operate pump}}{\text{actual torque delivered to pump}} = \frac{T_T}{T_A}$$

$$T_T(\mathrm{N \cdot m}) = \frac{V_D(\mathrm{m}^3) \times p(\mathrm{Pa})}{2\pi}$$

$$T_A = \frac{\text{actual power delivered to pump (W)}}{N \text{ (rad/s)}}$$

3. Overall Efficiency (η_0):

$$\text{overall efficiency} = \frac{\text{actual power delivered by pump}}{\text{actual power delivered to pump}}$$

$$\eta_o = \eta_v \times \eta_m$$

$$\eta_o = \frac{pQ_A}{T_A N} = \frac{\text{actual power delivered by pump}}{\text{actual power delivered to pump}}$$

EXAMPLE 5-9

A pump has a displacement volume of 100 cm^3. It delivers 0.0015 m^3/s at 1000 rpm and 70 bars. If the prime mover input torque is 120 N · m,

a. What is the overall efficiency of the pump?

b. What is the theoretical torque required to operate the pump?

Solution

a. Using Eq. (5-2M), where the volumetric displacement is

$$V_D = 100 \text{ cm}^3/\text{rev} \times \left(\frac{1\text{m}}{100 \text{ cm}}\right)^3 = 0.000100 \text{ m}^3/\text{rev}$$

we have

$$Q_T = V_D N = (0.000100 \text{ m}^3/\text{rev})\left(\frac{1000}{60}\text{ rev/s}\right) = 0.00167 \text{ m}^3/\text{s}$$

Next solve for the volumetric efficiency:

$$\eta_v = \frac{Q_A}{Q_T} = \frac{0.0015}{0.00167} = 0.898 = 89.8\%$$

Then solve for the mechanical efficiency:

$$\eta_m = \frac{pQ_T}{T_A N} = \frac{(70 \times 10^5 \text{ N m}^2)(0.00167 \text{ m}^3/\text{s})}{(120 \text{ N}\cdot\text{m})\left(1000 \times \frac{2\pi}{60}\text{ rad/s}\right)}$$

$$\eta_m = \frac{11{,}690 \text{ N}\cdot\text{m/s}}{12{,}570 \text{ N}\cdot\text{m/s}} = 0.930 = 93.0\%$$

Note that the product T_AN gives power in units of N · m/s (W) where torque (T_A) has units of N · m and shaft speed has units of rad/s. Finally, we solve for the overall efficiency:

$$\eta_o = \eta_v \eta_m = 0.898 \times 0.930 = 0.835 = 83.5\%$$

b. $$T_T = T_A \eta_m = (120)(0.93) = 112 \text{ N}\cdot\text{m}$$

Thus, due to mechanical losses within the pump, 120 N · m of torque are required to drive the pump instead of 112 N · m.

EXAMPLE 5-10

The pump of Example 5-9 is driven by an electric motor having an overall efficiency of 85%. The hydraulic system operates 12 hours per day for 250 days per year. The cost of electricity is \$0.11 per kilowatt hour. Determine

a. The yearly cost of electricity to operate the hydraulic system

b. The amount of the yearly cost of electricity that is due to the inefficiencies of the electric motor and pump

Solution

a. First we calculate the mechanical input power the electric motor delivers to the pump. Per Eq. (3-37) we have

$$\text{Pump input power (kW)} = \frac{T_A(\text{N}\cdot\text{m}) \times N(\text{rpm})}{9550} = \frac{120 \times 1000}{9550} = 12.6\ \text{kW}$$

Next we calculate the electrical input power the electric motor receives.

$$\text{Electric motor input power} = \frac{\text{Electric motor output power}}{\text{Electric motor overall efficiency}}$$

Since the electric motor output power equals the pump input power we have

$$\text{Electric motor input power} = \frac{12.6\ \text{kW}}{0.85} = 14.8\ \text{kW}$$

Finally we determine the yearly cost of electricity

$$\text{yearly cost} = \text{power rate} \times \text{time per year} \times \text{unit cost of electricity}$$

$$= 14.8\ \text{kW} \times 12\frac{\text{hr}}{\text{day}} \times 250\frac{\text{days}}{\text{year}} \times \frac{\$0.11}{\text{kw hr}} = \$4884/\text{yr}$$

b. The total kW loss equals the kW loss due to the electric motor plus the kW loss due to the pump. Thus, we have

$$\text{Total kW loss} = (1 - 0.85) \times 14.8 + (1 - 0.835) \times 12.6$$
$$= 2.22 + 2.08 = 4.30\ \text{kW}$$

$$\text{Yearly cost due to inefficiencies} = \frac{4.3}{14.8} \times \$4884/\text{yr} = \$1419/\text{yr}$$

Since $4.3/14.8 = 0.29$, we conclude that 29% of the total cost of electricity is due to the inefficiencies of the electric motor and pump. This also means that only 71% of the electrical power entering the electric motor is transferred into hydraulic power at the pump outlet port.