College of Engineering & Technology

Department: Mechanical Engineering — Marks: 15
Lecturer: Dr. Rola Afify — Time: 12:30 – 2:00
Course Code: ME416 — Date: 15/12/2015

Name: **Model Answer** R.N.:

Answer the following questions:

Question one (5 marks)

A steady two-dimensional flow field described by $|\vec{V}| = \sqrt{5y^2 + x^2 + 4xy}$ m/s with $xy + y^2 = k$ (streamlines), determine:

a) The velocity components.

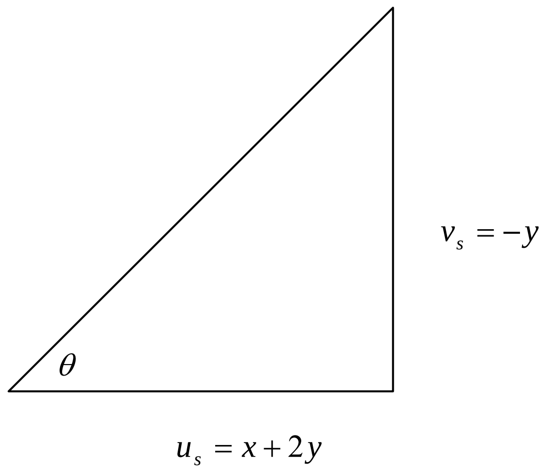
$xy + y^2 = k$ (equation of a streamline)

Differentiate w.r.t. x

$$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x + 2y) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x+2y} = \frac{v_s}{u_s}$$



From the triangle $|\vec{V}_s| = \sqrt{5y^2 + x^2 + 4xy}$

$$u = |V|\cos\theta = \sqrt{5y^2 + x^2 + 4xy}\,\frac{(x+2y)}{\sqrt{5y^2 + x^2 + 4xy}}$$

then $\boxed{u = x + 2y}$

$$v = |V|\sin\theta = \sqrt{5y^2 + x^2 + 4xy}\,\frac{(-y)}{\sqrt{5y^2 + x^2 + 4xy}}$$

and $\boxed{v = -y}$

b) The location of any stagnation point.

Stagnation point occurs at $u = 0$ & $v = 0$

$u = x + 2y = 0$ then $x = -2y$

$v = -y = 0$ then $y = 0$

Stagnation point exists at $x = 0$ & $y = 0$

c) The acceleration vector.

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = (x + 2y)(1) - y(2) = x$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial x} = (x + 2y)(0) - y(-1) = y$$

$\vec{a} = a_x\vec{i} + a_y\vec{j}$ then $\vec{a} = x\,\vec{i} + y\,\vec{j}$

d) The resultant acceleration, if it passes by point (1,2).

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{5} = 2.3261 \ m/s^2$$

$$\tan\alpha = \frac{a_y}{a_x} = \frac{y}{x} = \frac{2}{1} = 2 \qquad \& \qquad \alpha = 1.1071 \ rad = 63.4^o$$

<u>Question two (7 marks)</u>

The three components of velocity are given by:

$u = x^2 + y^2 + z^2$, $v = xy + yz + z^2$, and $w = -3xz - \frac{z^2}{2} + 4$, determine:

1. The volumetric dilatation rate.

$$\nabla.V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$u = x^2 + y^2 + z^2 \quad \text{then} \quad \frac{\partial u}{\partial x} = 2x$$

$$v = xy + yz + z^2 \quad \text{then} \quad \frac{\partial v}{\partial y} = x + z$$

$$w = -3xz - \frac{z^2}{2} + 4 \quad \text{then} \quad \frac{\partial w}{\partial z} = -3x - z$$

$$\nabla.V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = (2x) + (x + z) + (-3x - z) = 0$$

2. Is this incompressible fluid?

This flow is incompressible flow because volumetric dilatation equals to zero.

3. Is it satisfied the conservation of mass (continuity equation)?

conservation of mass (continuity equation) $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$

For incompressible flow $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

This fluid flow satisfies the conservation of mass (continuity equation)

4. Is it a physically possible flow field?

This is a physically possible flow field, because it satisfy the Continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

5. The rotation vectors.

$$\boldsymbol{\omega} = \tfrac{1}{2}\,\text{curl}\,\mathbf{V} = \tfrac{1}{2}\,\nabla \times \mathbf{V}$$

since by definition of the vector operator $\nabla \times \mathbf{V}$

$$\frac{1}{2}\nabla \times \mathbf{V} = \frac{1}{2}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\omega=\frac{1}{2}\nabla\times V=\frac{1}{2}\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\vec{i}+\frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\vec{j}+\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\vec{k}$$

$$\omega=\frac{1}{2}(0-(y+2z))\vec{i}+\frac{1}{2}((2z)-(-3z))\vec{j}+\frac{1}{2}((y)-(2y))\vec{k}$$

$$\omega=\left(-\frac{y}{2}-z\right)\vec{i}+\left(\frac{5z}{2}\right)\vec{j}+\left(\frac{-y}{2}\right)\vec{k}$$

6. The vorticity.

The *vorticity*, ζ, is defined as a vector that is twice the rotation vector; that is,

$$\zeta = 2\,\boldsymbol{\omega} = \nabla \times \mathbf{V}$$

$$\varsigma=2\omega=(-y-2z)\vec{i}+(5z)\vec{j}+(-y)\vec{k}$$

7. Is this an irrotational flow field?

This flow isn't irrotational flow because the vorticity doesn't equal to zero.

Question three (3 marks)

The two components of velocity are given by:
$u=4y$ & $v=4x$, determine:

1. The rate of angular deformation (rate of shearing strain).

$$\dot{\gamma}=\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}=4+4=8$$

2. The stream function.

$$u=\frac{\partial\psi}{\partial y}\quad \text{then } \psi=\int u\,dy=\int 4y\,dy=2y^2+f_1(x)$$

$$v=-\frac{\partial\psi}{\partial x}\quad \text{then } \psi=\int -v\,dx=\int -4x\,dx=-2x^2+f_2(y)$$

$$\psi=-2x^2+2y^2+C\quad \text{as}\quad f_1(x)=-2x^2\quad \&\quad f_2(y)=2y^2$$

3. The velocity potential.

$$u=\frac{\partial\phi}{\partial x}\quad \text{then } \phi=\int u\,dx=\int 4y\,dx=4xy+f_3(y)$$

$$v=\frac{\partial\phi}{\partial y}\quad \text{then } \phi=\int v\,dy=\int 4x\,dy=4xy+f_4(x)$$

$$\phi=4xy+C\quad \text{as}\quad f_3(x)=0\quad \&\quad f_4(y)=0$$