COLLEGE OF ENGINEERING & TECHNOLOGY

Department: Mechanical Engineering

Lecturer : Prof. Dr. Kamal Abdelaziz Ibrahim & Prof.Dr. Ashraf S. Ismail

Date: 12/1/2015

Course Title: Fluid Mechanics — Marks: 40

Course Code: ME461 — Time: 2:00-4:00

Final Exam

FINAL EXAMINATION PAPER

Answer all the Following Questions:

1-a: Starting with Navier-stokes equations, derive an equation for estimating the volume flow rate through a circular pipe of radius (R) . Assuming the flow is viscous incompressible and steady.

(4Marks)

1-b: Differentiate clearly between the following items:

a- Uniform and Non-uniform flow.
b- Free vortex and Forced vortex flow.
c- Steady and unsteady flow.
d- Laminar and turbulent flow.

(4Marks)

2- Find an expression for the acceleration vector of a fluid particle as it moves through the point (1,1) in steady two dimensional flow described by ; $|V| = 2xy\sqrt{(x^2+y^2)}$ with $xy = c$ (Streamline Equation) , where; C is constant.

Find also the velocity potential if it exists. (8Marks)

3- The velocity component (v) in the y direction for a certain steady incompressible two- dimensional ideal flow field is given by

$$v = x^2 - y^2$$

a- Describe this flow field.
b- Determine the value of pressure gradient in the x- direction ($\partial p/\partial x$) at a point (1, 2). Neglect body forces .
C- locate any stagnation point in the flow field.

(8Marks)

Members of Course Examination Committee:	Signature:	Date:
Lecturer: Prof.Dr Kamal Abdelaziz Prof.Dr. Ashraf Ismail		
Course Coordinator: Prof. Kamal Abdelaziz		
Head of Department: Prof. Dr. Elsaied Saber		

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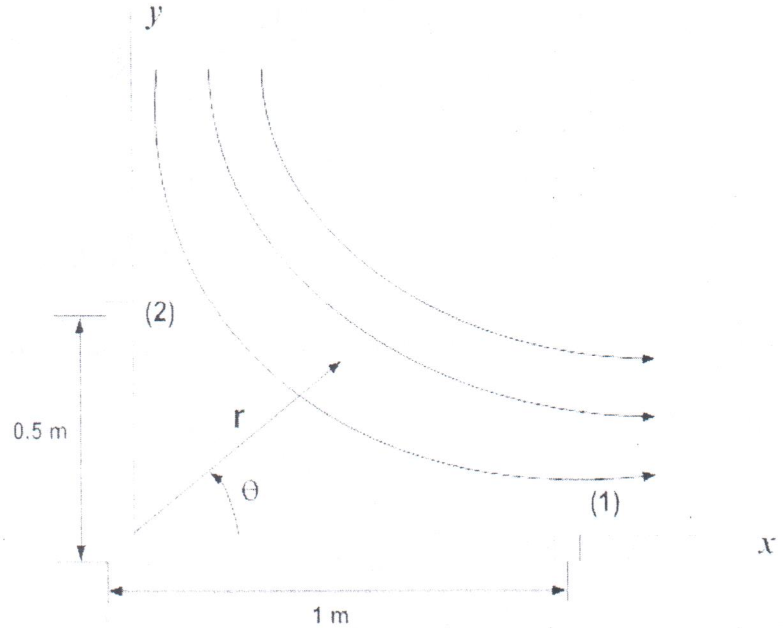
4- The two-dimensional flow of an inviscid, incompressible fluid flow in the vicinity of the 90° corner, as shown in the figure, is described by the stream function, ($\Psi = 2r^2 \sin 2\theta$) where Ψ has units of m^2/s, and r is in m.

Assume the fluid density is 1000 kg/m^3 and the x-y plane is horizontal (that is, there is no difference in elevation between points 1 and 2).

a- Determine, if possible the corresponding velocity potential.

b- What is the pressure at point (2) if the pressure at point (1) is 30 kPa ?

(8 Marks)



5-A two dimensional flow field is formed by adding a c.w free vortex at the origin of the coordinate system to the velocity potential $\phi = r^2 \cos 2\Theta$. Find

i. The pressure variation along (r)

ii. Locate any stagnation points in the flow field for plane ($0 \le \Theta \le \pi$)

(8Marks)

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List of formulae:

$\vec{V} = u\,\vec{i} + v\,\vec{j} + w\,\vec{k}$, $\dfrac{dx}{u} = \dfrac{dy}{v} = \dfrac{dz}{w}$ (Equations for streamlines)

$\dfrac{dy}{dx} = \tan\theta = \dfrac{v}{u}$ (For two - dimensional flow)

$\vec{a} = a_x\,\vec{i} + a_y\,\vec{j} + a_z\,\vec{k}$

$a_x = \dfrac{Du}{Dt} = \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z}$, $a_y = \dfrac{Dv}{Dt} = \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial v}{\partial z}$

$a_z = \dfrac{Dw}{Dt} = \dfrac{\partial w}{\partial t} + u\dfrac{\partial w}{\partial x} + v\dfrac{\partial w}{\partial y} + w\dfrac{\partial w}{\partial z}$

$\vec{\nabla}\cdot\vec{V} = \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}$, $\dot{\gamma} = \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}$ (For two - dimensional flow)

$\vec{\nabla}\times\vec{V} = \vec{\zeta} = 2\vec{\omega} = \zeta_x\,\vec{i} + \zeta_y\,\vec{j} + \zeta_z\,\vec{k} = 2(\omega_x\,\vec{i} + \omega_y\,\vec{j} + \omega_z\,\vec{k})$

$\omega_x = \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y} - \dfrac{\partial v}{\partial z}\right)$, $\omega_y = \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z} - \dfrac{\partial w}{\partial x}\right)$, $\omega_z = \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right)$

If $\vec{\nabla}\times\vec{V} = 0$, $\vec{V} = \vec{\nabla}\phi$, $\vec{V} = \dfrac{\partial\phi}{\partial x}\vec{i} + \dfrac{\partial\phi}{\partial y}\vec{j} + \dfrac{\partial\phi}{\partial z}\vec{k}$

$u = \dfrac{\partial\phi}{\partial x}$, $v = \dfrac{\partial\phi}{\partial y}$, $w = \dfrac{\partial\phi}{\partial z}$, $v_r = \dfrac{\partial\phi}{\partial r}$, $v_\theta = \dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta}$, $v_z = \dfrac{\partial\phi}{\partial z}$

$\dfrac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{V}) = 0$, $\dfrac{\partial\rho}{\partial t} + \dfrac{\partial(\rho u)}{\partial x} + \dfrac{\partial(\rho v)}{\partial y} + \dfrac{\partial(\rho w)}{\partial z} = 0$

$\dfrac{\partial\rho}{\partial t} + \dfrac{1}{r}\dfrac{\partial(\rho r v_r)}{\partial r} + \dfrac{1}{r}\dfrac{\partial(\rho v_\theta)}{\partial\theta} + \dfrac{\partial(\rho v_z)}{\partial z} = 0$, $\vec{\nabla}\cdot\vec{V} = 0$ (Incompressible flow)

$\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0$, $\dfrac{1}{r}\dfrac{\partial(r v_r)}{\partial r} + \dfrac{1}{r}\dfrac{\partial v_\theta}{\partial\theta} + \dfrac{\partial v_z}{\partial z} = 0$

$\xi_z = \dfrac{1}{r}\left(\dfrac{\partial(r v_\theta)}{\partial r} - \dfrac{\partial v_r}{\partial\theta}\right).$

$u = \dfrac{\partial\psi}{\partial y}$, $v = -\dfrac{\partial\psi}{\partial x}$, $v_r = \dfrac{1}{r}\dfrac{\partial\psi}{\partial\theta}$, $v_\theta = -\dfrac{\partial\psi}{\partial r}$

$\rho\vec{g} - \vec{\nabla}p = \rho\left(\dfrac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{V}\right)$, $\rho g_x - \dfrac{\partial p}{\partial x} = \rho\left(\dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z}\right)$

$\rho g_y - \dfrac{\partial p}{\partial y} = \rho\left(\dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial v}{\partial z}\right)$, $\rho g_z - \dfrac{\partial p}{\partial z} = \rho\left(\dfrac{\partial w}{\partial t} + u\dfrac{\partial w}{\partial x} + v\dfrac{\partial w}{\partial y} + w\dfrac{\partial w}{\partial z}\right)$

$\rho g_r - \dfrac{\partial p}{\partial r} = \rho\left(\dfrac{\partial v_r}{\partial t} + v_r\dfrac{\partial v_r}{\partial r} + \dfrac{v_\theta}{r}\dfrac{\partial v_r}{\partial\theta} + v_z\dfrac{\partial v_r}{\partial z} - \dfrac{v_\theta^2}{r}\right)$

$\rho g_\theta - \dfrac{1}{r}\dfrac{\partial p}{\partial\theta} = \rho\left(\dfrac{\partial v_\theta}{\partial t} + v_r\dfrac{\partial v_\theta}{\partial r} + \dfrac{v_\theta}{r}\dfrac{\partial v_\theta}{\partial\theta} + v_z\dfrac{\partial v_\theta}{\partial z} + \dfrac{v_r v_\theta}{r}\right)$

$\rho g_z - \dfrac{\partial p}{\partial z} = \rho\left(\dfrac{\partial v_z}{\partial t} + v_r\dfrac{\partial v_z}{\partial r} + \dfrac{v_\theta}{r}\dfrac{\partial v_z}{\partial\theta} + v_z\dfrac{\partial v_z}{\partial z}\right)$

$\dfrac{p}{\rho} + \dfrac{V^2}{2} + gZ = \text{constant}$, $V^2 = u^2 + v^2$, $V^2 = v_r^{\,2} + v_\theta^{\,2}$

Uniform flow: $\psi = U(y\cos\alpha - x\sin\alpha)$, $\phi = U(x\cos\alpha + y\sin\alpha)$

Source: $\psi = \dfrac{m}{2\pi}\theta$, $\phi = \dfrac{m}{2\pi}\ln r$

Sink: $\psi = -\dfrac{m}{2\pi}\theta$, $\phi = -\dfrac{m}{2\pi}\ln r$

Vortex: $\psi = -\dfrac{\Gamma}{2\pi}\ln r$, $\phi = \dfrac{\Gamma}{2\pi}\theta$

Doublet: $\psi = -\dfrac{k}{r}\sin\theta$, $\phi = \dfrac{k}{r}\cos\theta$

Navier–Stokes Equations

(x direction)

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

(y direction)

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

(z direction)

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

(r direction)

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial r} + \rho g_r + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right) - \frac{v_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

(θ direction)

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right)$$

$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) - \frac{v_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right]$$

(z direction)

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial z} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$