

③ Normal due to bending

N.A هو محور القادل *

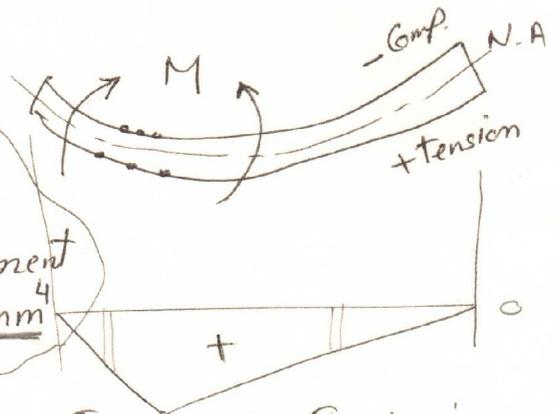
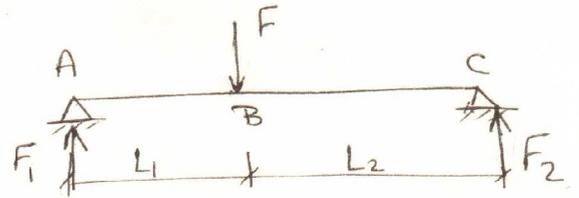
* السطح العلوي للكمرة تعرض للضغط

* السطح السفلي للكمرة تعرض للتension

* لايجاد قيم ردود الافعال يتم استخدام

معادلات الاتزان $\sum F = 0, \sum M = 0$

* يتم حساب قيم العزم عند النقط المطلوبه من ناحيه الجانب الايسر



النقط المائنه من N.A الى النقط $\sigma = \frac{My}{I}$

العزم عند النقط $I \leftarrow$ second moment of Area mm^4

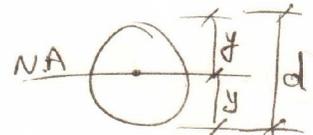
* في حالة الكمره الواحده تكون قيم I, y ثابتة وتتغير M بتغير

* يتم رسم العزم عكس اتجاه السهم (مع ذيل السهم)

Beam type
A) circular

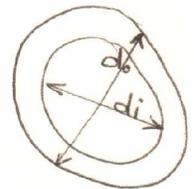
$$y = \frac{d}{2}$$

$$I = \frac{\pi}{64} d^4$$



B) hollow cylinder $y = \frac{d_o}{2}$

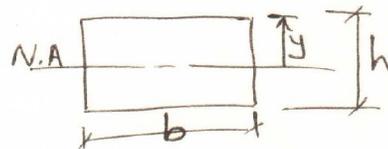
$$I = \frac{\pi}{64} (d_o^4 - d_i^4)$$



C) rectangular section

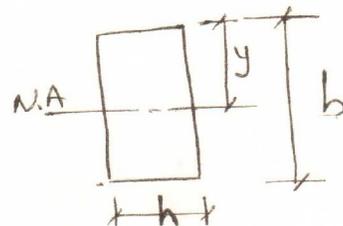
$$y = \frac{h}{2}$$

$$I = \frac{bh^3}{12}$$



$$y = \frac{b}{2}$$

$$I = \frac{hb^3}{12}$$

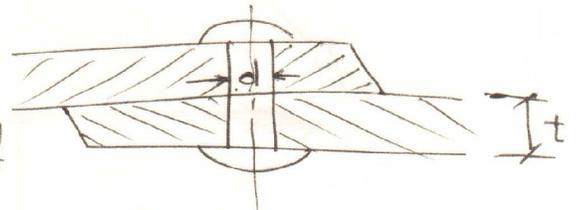


* Bearing stress (Crushing stress)

$$\sigma_c = \sigma_b = \frac{P}{d \cdot t \cdot n}$$

projected area

no. of rivets



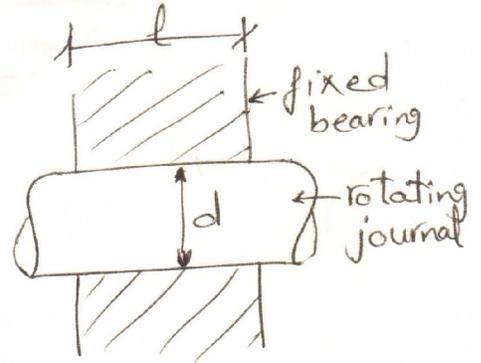
average bearing pressure

$$P_b = \frac{P}{l \cdot d}$$

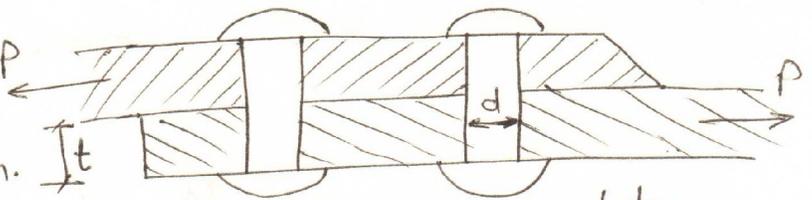
radial load

diameter of journal

length of journal in contact



ex. two plate 16 mm thickness t are joined by double riveted lap joint as shown.



Find the crushing stress induced between the plates and the rivet, if the max. tensile load on the joint is 48 kN and the rivet diameter is 25 mm

$$\sigma_c = \frac{P}{d \cdot t \cdot n} = \frac{48 \times 1000}{25 \times 16 \times 2} = 60 \text{ N/mm}^2$$

ex. A journal 25 mm in diameter is supported in sliding bearings has a max. end reaction of 2500 N . Assuming an allowable bearing pressure of 5 N/mm^2 , find the length of the sliding bearing.

$$P_b = \frac{P}{l \cdot d}$$

$$5 = \frac{2500}{l \cdot 25} \Rightarrow l = 20\text{ mm}$$

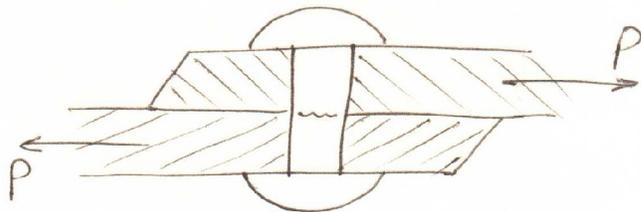
* shear stress

① Direct shear

Ⓐ single shear

$$\tau = \frac{\text{Force}}{\text{area of failure}}$$

$$\tau = \frac{P}{\frac{\pi}{4} d^2}$$



Ⓑ double shear

shear stress

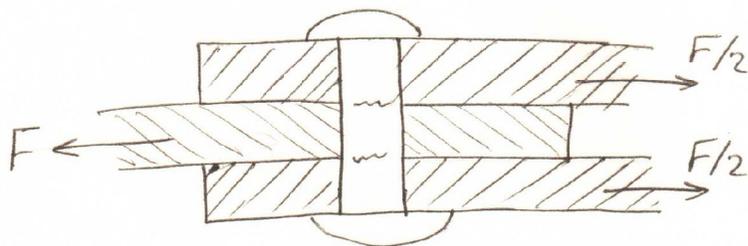
$$\tau = \frac{\text{Force}}{n * A}$$

Thickness

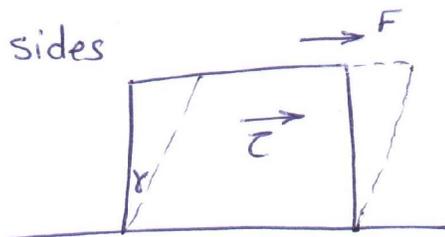
n=1 single shear

n=2 double shear

Area
 $A = \frac{\pi}{4} d^2$



γ shear strain = change in \perp sides of the block
 in radians



$\tau \propto \gamma$ in elastic zone

$\tau = G\gamma$ as G = modulus of rigidity (Pa)

$$E = 2G(1 + D)$$

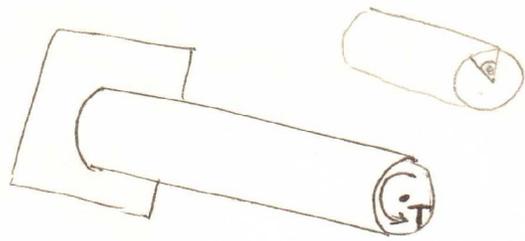
D = Poisson's ratio = 0.3 for most materials

$$\therefore E = 2.6G$$

② shear due to torsion

$$\tau = \frac{T * r}{J}$$

Torque (N-mm) \swarrow
 radius (mm) \swarrow
 polar moment of Area (mm⁴) \swarrow
 shear stress (MPa) \swarrow



Beam type

Ⓐ circular

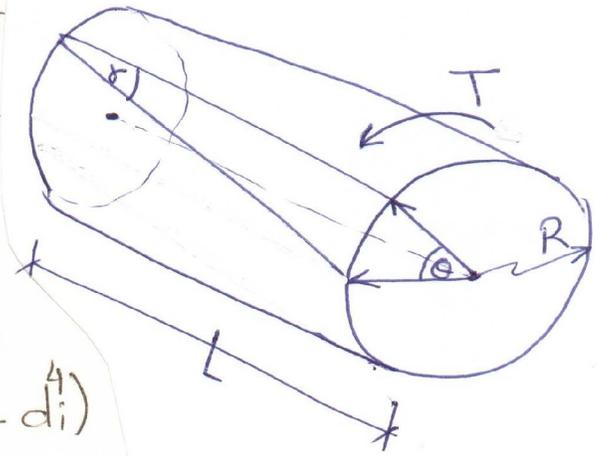
$$r = \frac{d}{2}$$

$$J = \frac{\pi}{32} d^4$$

Ⓑ hollow cylinder

$$r = \frac{d_o}{2}$$

$$J = \frac{\pi}{32} (d_o^4 - d_i^4)$$



θ twisting angle

$$r\theta = \delta L$$

δ shear strain angle

L total length

R max radius

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$$

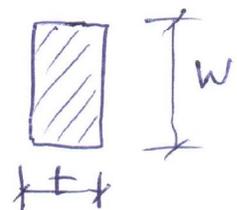
τ shear stress

r radius

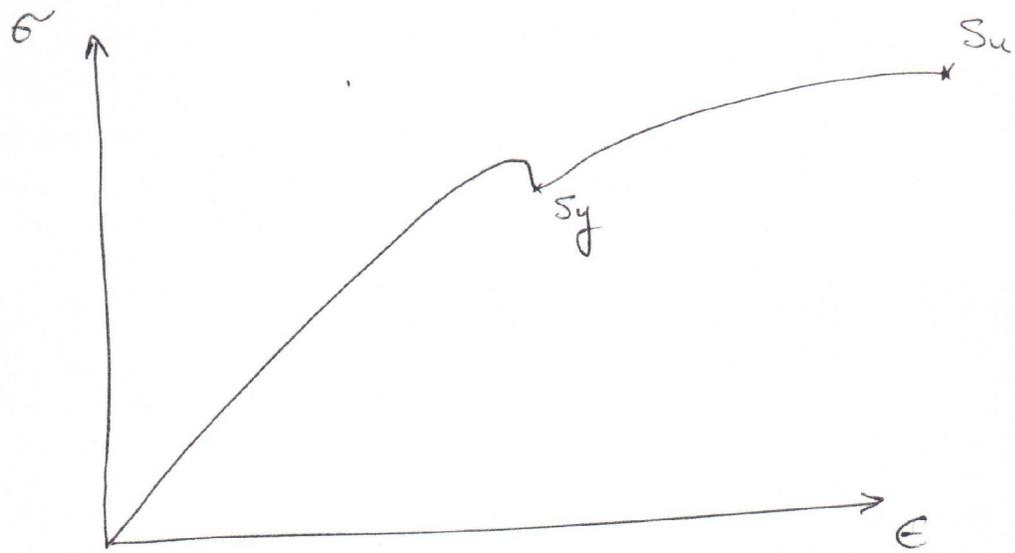
T twisting moment

Ⓒ rectangular

$$\tau_{max} = \frac{T}{wt^2} \left(3 + 1.8 \frac{t}{w} \right)$$



yield strength and ultimate strength



S_y : yield strength (MPa) (اجهاد الخضوع) (اجهاد التناهي)

S_u : ultimate strength (MPa) (اجهاد الكسر) (أقصى اجهد يتقبله الجسم)

f.s. : factor of safety (معامل الأمان) (2 ~ 3)

σ_{all} : maximum allowable stress

$$\sigma_{all} = \frac{S_y}{f.s.}$$

τ_{all} : maximum allowable shear stress

$$\tau_{all} = \frac{0.5 S_y}{f.s.}$$

Combined stresses

from the same kind

tension
or
Comp. + bending

from different kind

Bending + torque
(σ) (τ)

From the same kind

$$\sigma_{\text{tension}} = \frac{F}{A}, \quad A = \frac{\pi}{4}d^2$$

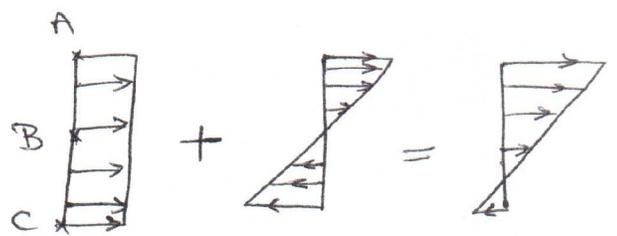
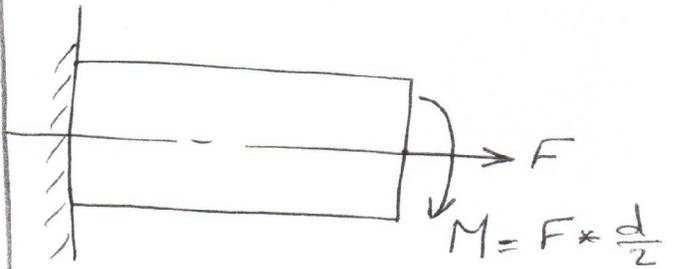
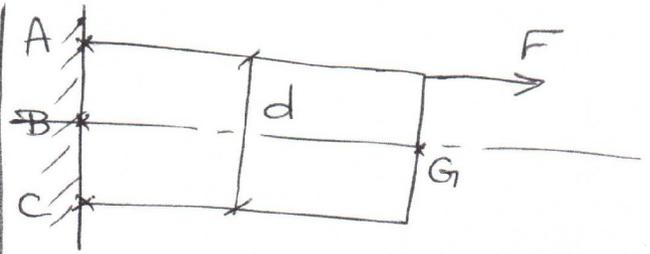
$$\sigma_{\text{bending}} = \frac{My}{I}, \quad y = \frac{d}{2}, \quad I = \frac{\pi}{64}d^4$$

At point A (top surface)

$$\sigma_{\text{max}} = \sigma_{\text{tension}} + \sigma_{\text{bending}}$$

At point C (bottom surface)

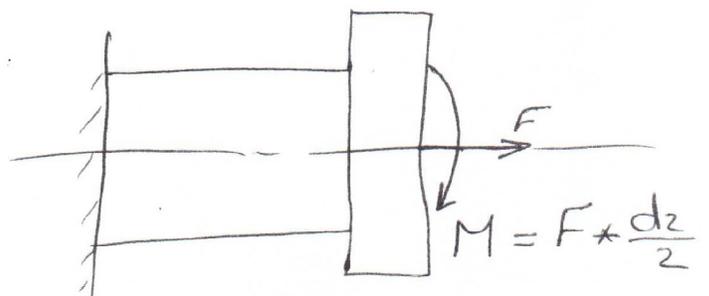
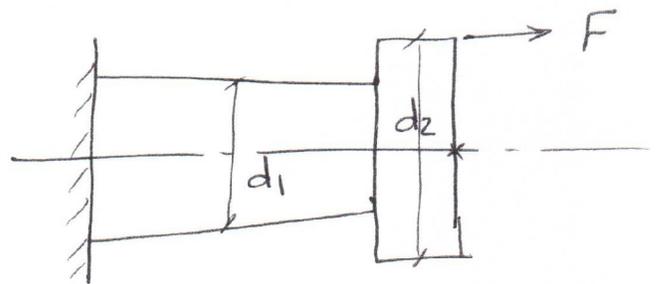
$$\sigma_{\text{min}} = \sigma_{\text{tension}} - \sigma_{\text{bending}}$$



نتيجة القوى نتيجة العزم

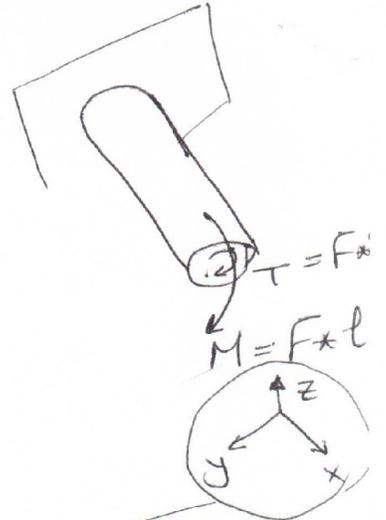
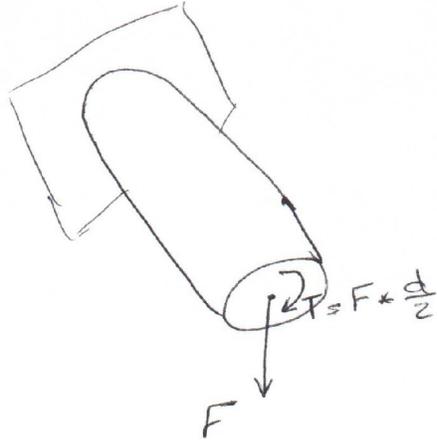
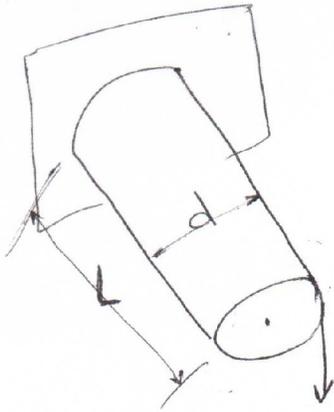
$$\sigma_{\text{tension}} = \frac{F}{\frac{\pi}{4}d_1^2}$$

$$\sigma_{\text{bending}} = \frac{M * \frac{d_1}{2}}{\frac{\pi}{64}d_1^4}$$



في حالة وجود أكثر من قطر لـ shaft يتم استخدام أقل قطر لأن الأكثر عرضة للكسر

From different kind (σ with τ)



$$\sigma_{\text{bending}} = \frac{M * y}{I}$$

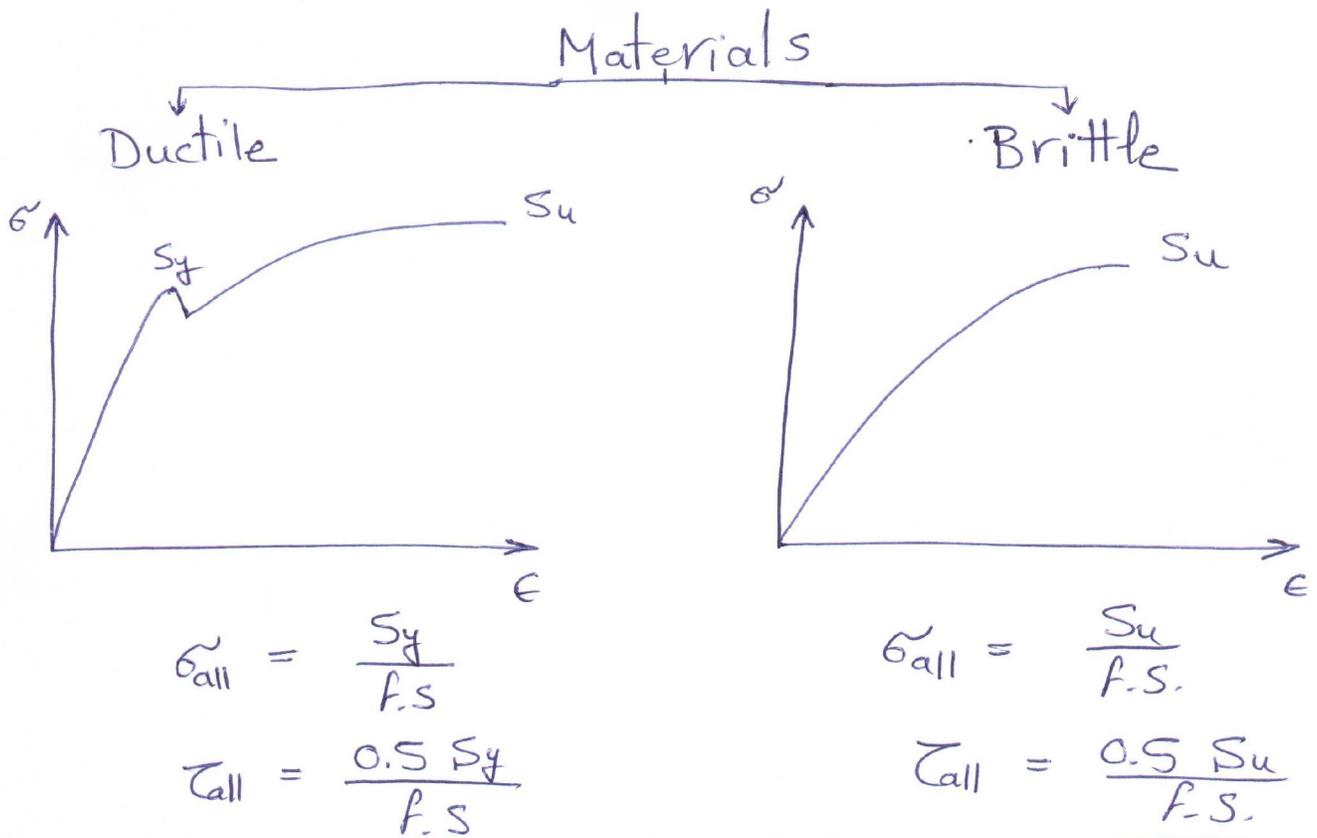
$$\tau = \frac{T * r}{J}$$

max. & min. Normal stress

$$\sigma_{\text{max}} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \leq \frac{S_y}{F.S.}$$

$$\tau_{\text{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \leq \frac{0.5 S_y}{F.S.}$$

هنا M قبل انثناء
محاور x و y و z
 $\sigma_x = \frac{M y}{I}$
& $\sigma_y = 0$



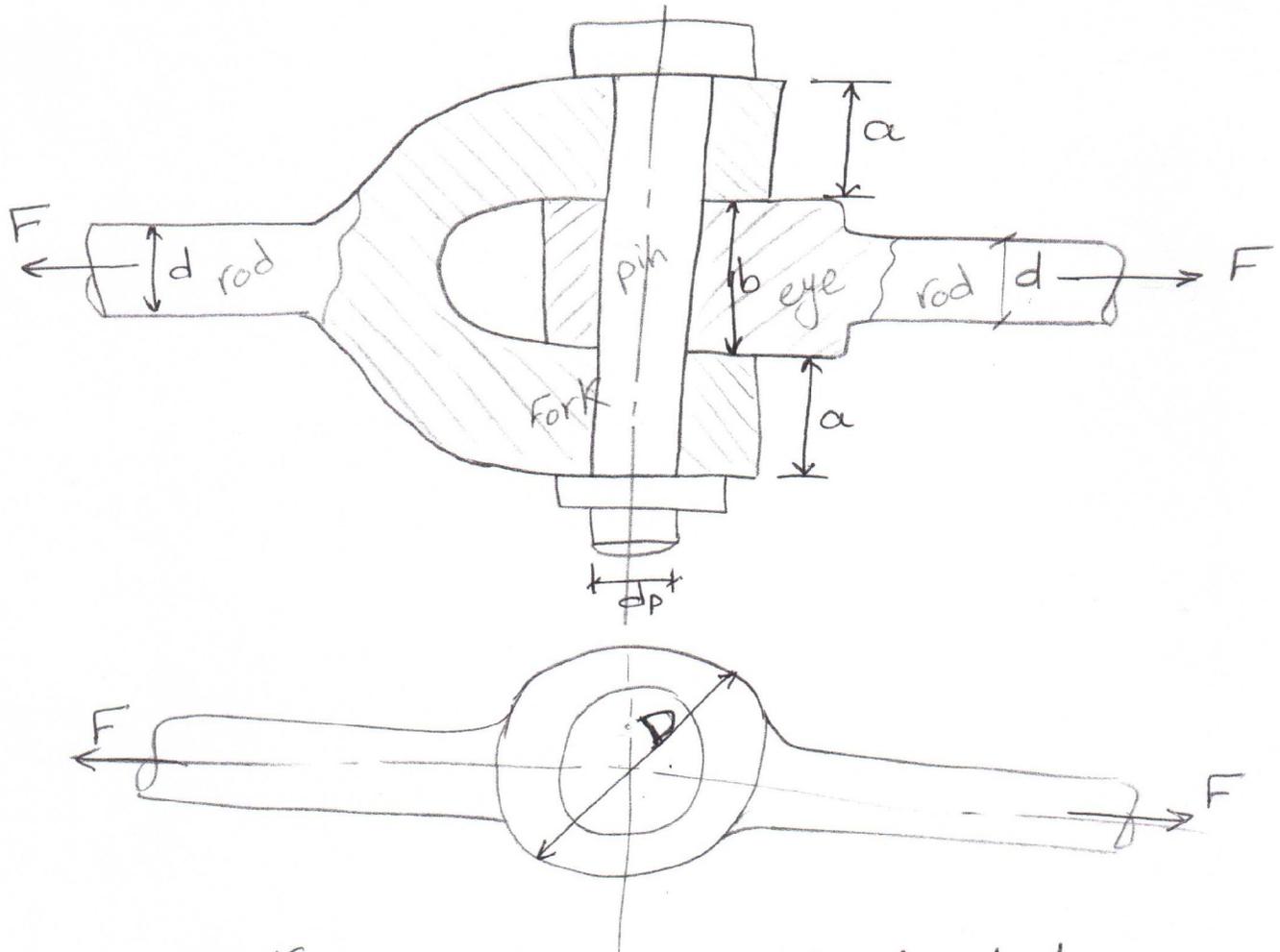
* Factor of safety > 1

It depends on the judgement of the designer.
It should depend on the following considerations:-

- ① mechanical properties aren't known exactly.
- ② Effect of size, heat treatment and type of loading
- ③ Effect of wear, time and environment on m/c life
- ④ assumptions may not be exact.
- ⑤ overall concern for human safety.
- ⑥ shutdown and maintenance may make loss of

* Const. load - ductile Material f.s. = 1.2 - 2 - brittle f.s. = 5-6
 Repeated ~ ~ ~ f.s. = 3-4 - ~ f.s. = 8-11
 shock ~ ~ ~ f.s. = 5-7 - ~ f.s. = 15-20

Design of Knuckle Joint



Fork, eye, pin are made of steel $S_y = 420 \text{ MPa}$
use factor of safety of 3

If $F = 20 \text{ kN}$, Find suitable dimensions.

$$d = ??$$

$$a = ??$$

$$b = ??$$

$$d_p = ??$$

allowable bearing
stress $\sigma_{br} = 100 \text{ MPa}$

① Design of rod (tension stress)

$$\sigma = \frac{F}{A} = \frac{S_y}{F.S.}$$

$$\frac{20 \times 10^3}{\frac{\pi}{4} d^2} = \frac{420}{3}$$

$$\Rightarrow d = 13.5 \text{ mm}$$

يتم التقريب للأعلى

$$\text{take } d = 14 \text{ mm}$$

② Design of pin (double shear)

$$\tau = \frac{F}{nA} = \frac{0.5 S_y}{F.S.}$$

$$\frac{20 \times 10^3}{2 \times \frac{\pi}{4} d_p^2} = \frac{0.5 \times 420}{3}$$

$$\Rightarrow d_p = 13.5$$

$$\text{take } d_p = 14 \text{ mm}$$

③ Design of Fork

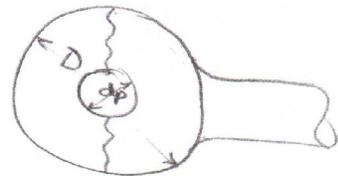
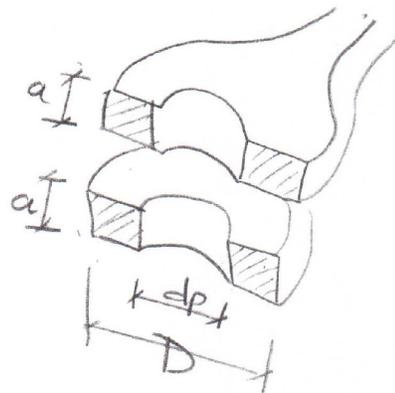
* tension stress

$$\sigma = \frac{F}{A} = \frac{S_y}{F.S.}$$

$$A = 2(D - d_p) \cdot a$$

$$\frac{20 \times 10^3}{2a(D - d_p)} = \frac{420}{3}$$

$$a = \frac{20 \times 10^3 \cdot 3}{2(D - 14) \cdot 420} \rightarrow \text{①}$$

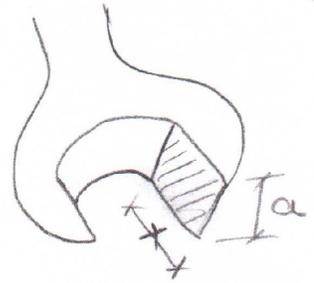


* Double shear

$$\tau = \frac{F}{A} = \frac{0.5 S_y}{f.s.} \rightarrow (2)$$

$$A = 4 \times a$$

$$\text{from (1) \& (2)} \quad = 4a \sqrt{\left(\frac{D}{2}\right)^2 - \left(\frac{d_p}{2}\right)^2}$$



$$\frac{20 \times 10^3}{4 \times \frac{20 \times 10^3 \times 3}{2(D-14) \times 420}} \times \sqrt{\left(\frac{D}{2}\right)^2 - \left(\frac{14}{2}\right)^2} = \frac{0.5 \times 420}{3}$$

$$D - 14 = \sqrt{\left(\frac{D}{2}\right)^2 - 7^2}$$

$$D^2 - 28D + (14)^2 = \left(\frac{D}{2}\right)^2 - 7^2$$

$$\frac{3}{4} D^2 - 28D + 245 = 0$$

بالترتيب

$$a = 3/4$$

$$b = -28$$

$$c = 245$$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+28 \pm \sqrt{(-28)^2 - 4 \times \frac{3}{4} \times (245)}}{2 \times \frac{3}{4}}$$

$$D = \frac{28 \pm 7}{1.5} = 23.3 \text{ mm or } 14 \text{ mm}$$

take $D = 24 \text{ mm}$ take the higher value

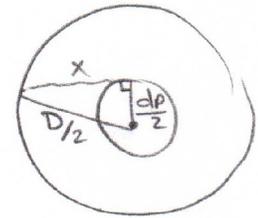
sub. in (1)

$$a = \frac{20 \times 10^3 \times 3}{2(24-14) \times 420} = 7.14 \text{ mm}$$

take $a = 8 \text{ mm}$

$$x = \sqrt{\left(\frac{D}{2}\right)^2 - \left(\frac{d_p}{2}\right)^2}$$

$$= \sqrt{\left(\frac{24}{2}\right)^2 - \left(\frac{14}{2}\right)^2} = 9.75 \text{ mm}$$



④ Design of eye

* tension stress

$$\sigma = \frac{F}{A} = \frac{S_y}{f.s.}$$

$$\frac{20 \times 10^3}{b(D - d_p)} = \frac{420}{3} \Rightarrow b = 14.3 \text{ mm}$$

take $b = 15 \text{ mm}$

* shear stress

$$\tau = \frac{F}{A} = \frac{0.5 S_y}{f.s.}$$

$$\frac{20 \times 10^3}{2bx} = \frac{0.5 \times 420}{3}$$

$$\therefore x = 9.75 \text{ mm}$$

$$\therefore b = 14.7 \text{ mm}$$

take $b = 15 \text{ mm}$

as before \therefore O.K.

* Bearing stress betⁿ fork & pin

$$\sigma_{br} = \frac{F}{2adp} = 100$$

$$\Rightarrow a = 8 \text{ mm}$$

as before
 \therefore O.K.

* Bearing stress betⁿ eye & pin

$$\sigma_{br} = \frac{F}{bdp}$$

$$\Rightarrow b = 15 \text{ mm}$$

as before
 \therefore O.K.