springs

springs are used to absorb energy in some machines (cars, elevators and trains). They are used also to control motion of valves and followers on cams and to provide flexibility.

* Types of springs

a) Helical compression spring

b) ~ tension ~

c) Helical torsion spring (spiral spring)

d) leaf spring

* a) Helical Compression spring

L: length of spring

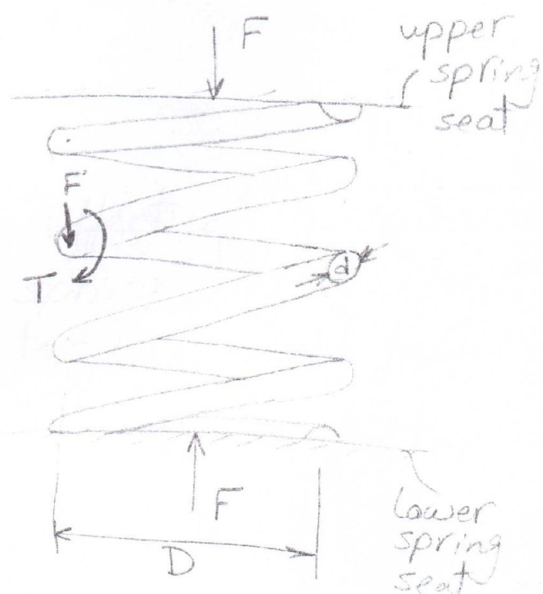
D: mean diameter

d: wire diameter

N: number of active coils = no. of coils − 2

P: pitch

F: Compression force



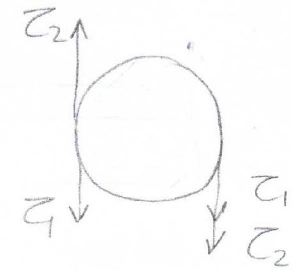
* stresses

shear stress $\tau_1 = \frac{F}{A} = \frac{F}{\frac{\pi}{4} d^2} = \frac{4F}{\pi d^2}$

torsion stress $\tau_2 = \frac{Tr}{J} = \frac{(F * \frac{D}{2}) * \frac{d}{2}}{\frac{\pi}{32} d^4}$

$$\tau_2 = \frac{8FD}{\pi d^3}$$

max. shear stress occurs at inner radius



$$\tau = \tau_1 + \tau_2$$

$$= \frac{4F}{\pi d^2} + \frac{8FD}{\pi d^3}$$

$$= \frac{8FD}{\pi d^3}\left(\frac{4F}{\pi d^2} * \frac{\pi d^3}{8FD} + 1\right) = \frac{8FD}{\pi d^3}\left(\frac{d}{2D} + 1\right)$$

let $C = \frac{D}{d}$ spring index

$$\therefore \tau = \frac{8FD}{\pi d^3}\left(1 + \frac{1}{2C}\right)$$

let $K_s = \left(1 + \frac{1}{2C}\right)$ shear factor

$$\tau = K_s \frac{8FD}{\pi d^3}$$

if we put into consideration the effect of curvature and stress concentration for the spring, the factor(K_s) can be replaced by another factor (K) wahl factor)

$$\boxed{K = \frac{4C-1}{4C-4} + \frac{0.615}{C}}$$

wahl factor

$K = K_c K_s$

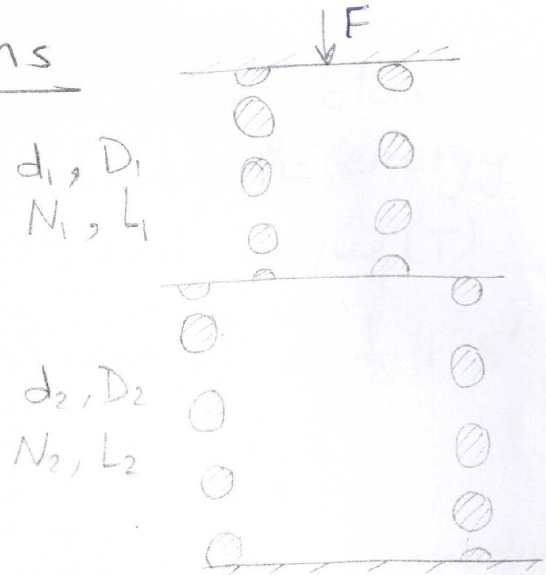
K_c: stress concentration factor

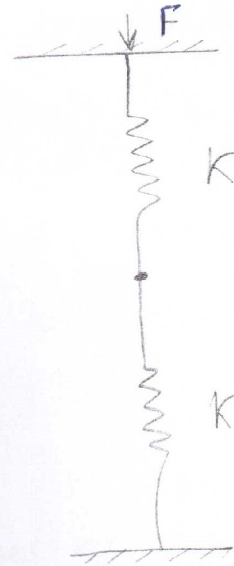
K_s: shear factor

$$\therefore \boxed{\tau = K \frac{8FD}{\pi d^3}}$$

* Connections

① series





$$\boxed{F = F_1 = F_2}$$

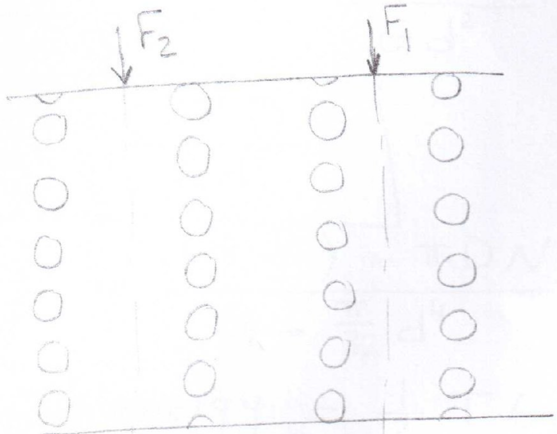
$$\boxed{\delta_1 \neq \delta_2}$$

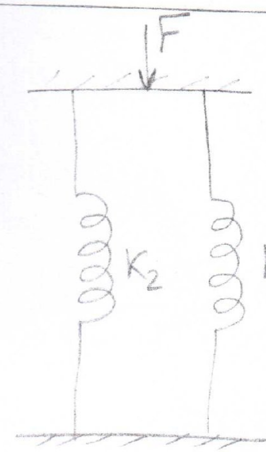
$\delta = \delta_1 + \delta_2$

$K = \dfrac{F}{\delta}$

↳ spring rate or spring stiffness (N/m)

② parallel





$F_1 \neq F_2$

$$\boxed{F = F_1 + F_2}$$

$$\boxed{\delta = \delta_1 = \delta_2}$$

Deflection

The deflection can be obtained by using Castiglia[no] theorem after determing the energy resulting from axial force (F) and Torque (T)

$$U_1 = \text{axial energy} = \frac{1}{2} F \delta'$$

$$\delta' = \frac{FL}{EA} \simeq \frac{FL}{GA}$$

modulus of elasticity for axial force only

modulus of rigidity for torsion

$$\therefore \quad \delta' = \frac{F * \pi D N}{G * \frac{\pi}{4} d^2} = \frac{4FDN}{Gd^2}$$

$$\therefore \boxed{U_1 = \frac{2F^2 DN}{Gd^2}}$$

$$\theta = \frac{TL}{GJ} = \frac{(F * \frac{D}{2}) * \pi D N}{G * \frac{\pi}{32} d^4} = \frac{16 D^2 NF}{G d^4}$$

$$U_2 = \text{torsional energy} = \frac{1}{2} T \theta$$

$$U_2 = \frac{1}{2} \left(F * \frac{D}{2}\right) * \frac{16 D^2 NF}{G d^4}$$

$$\boxed{U_2 = \frac{4F^2 D^3 N}{G d^4}}$$

total strain energy $= U = U_1 + U_2$ ——— Castigliano's theorem

$$= \frac{2F^2 D N}{G d^2} + \frac{4F^2 D^3 N}{G d^4}$$

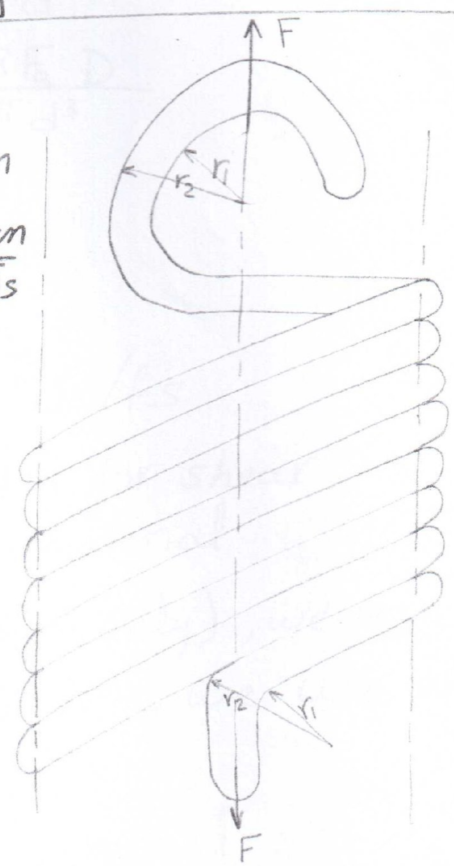
$$= \frac{4F^2 D^3 N}{G d^4} \left(\frac{d^2}{2D^2} + 1\right) = \frac{4F^2 D^3 N}{G d^4} \left(\frac{1}{2C^2} + 1\right)$$

$$\left| U = \frac{4F^2 D^3 N}{G d^4} \left(\frac{1}{2C^2} + 1\right) \right|$$

$$\delta = \frac{\partial U}{\partial F} = \frac{8FD^3N}{Gd^4}\left(\underbrace{\frac{1}{2C^2}}_{\text{neglected } D \gg d} + 1\right)$$

$$\boxed{\delta = \frac{8FD^3N}{Gd^4}}$$

* b) Helical tension spring

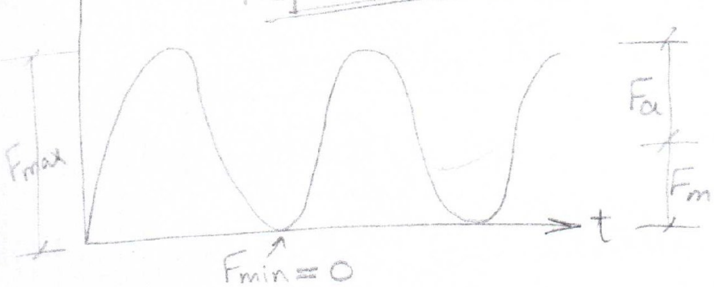


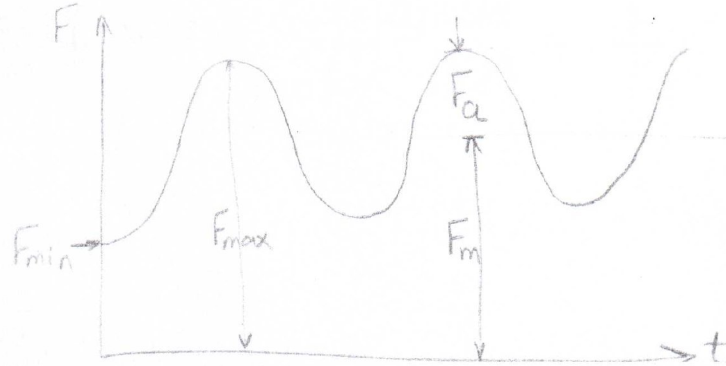
Advantages of helical compression springs with respect to tension springs

1. spring ends is less stress concentration
2. It can avoid failure by buckling by using adjacent supports.
3. It can be compressed to the solid length

* Ratio between inner radius and outer radius for ends represents stress concentration factor

Repeated load





variable load

Fatigue loads

mean force $= F_m = \dfrac{F_{max} + F_{min}}{2}$

Alternating force $= F_a = \dfrac{F_{max} - F_{min}}{2}$

mean stress $= \tau_m = K_s \dfrac{8 F_m D}{\pi d^3}$

alternating stress $= \tau_a = K_s \dfrac{8 F_a D}{\pi d^3}$

* for design

$$\tau_a = S_{es} / f.s.$$

$$\tau_{max} = \tau_m + \tau_a = S_{ys} / f.s.$$

S_{es} = corrected endurance stress for shear

S_{ys} = yield strength for spring material

- IF (F) is in one direction (tension only), we use S_y
- IF (F) is for tension and compression, we use endurance strength S'_{es} < S_{es}

S'_{es} → for speciman

S_{es} → for the actual size

$$\boxed{S_{es} = K_r K_m S'_{es}}$$

Reliability factor $= K_r$ from table

Modified stress concentrated factor $= K_m = \dfrac{1}{K_c}$

& stress concentrated factor (K_c) $= \dfrac{K}{K_s}$ → $\begin{cases} K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C} \\ K_s = \left(\dfrac{1}{2C} + 1\right) \end{cases}$

* Ultimate strength for the material $= S_{ult} = \dfrac{A}{d^m}$

A, m = are constants from table

d = wire diameter

* The previous eqns are used for infinite life (no. of cycles (N) greater than 10^6).

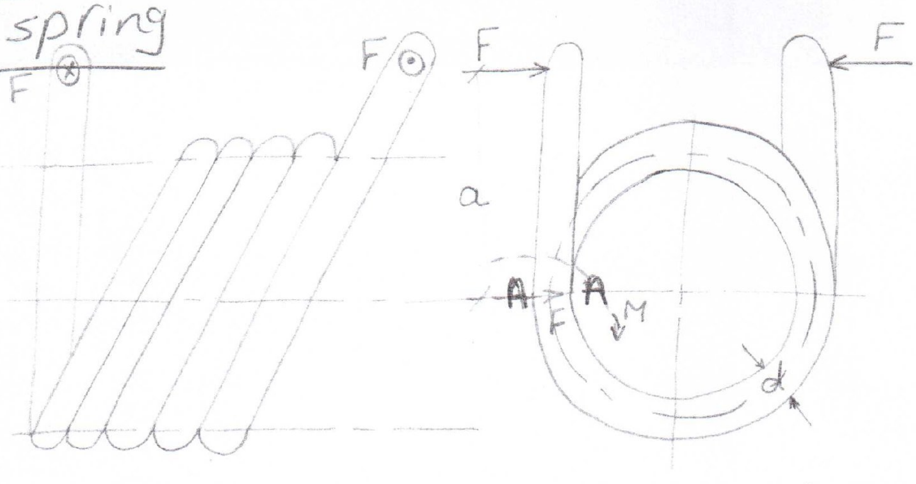
* IF no. of cycles (N) are less than 10^6, then S_{es} is replaced by another value S_{ef}

$$\boxed{S_{ef} = \frac{0.81\, S_{ults}^2}{S_{es}} \left(\frac{S_{es}}{0.8\, S_{ults}} \right)^{\log \sqrt[3]{N}}}$$

for $10^3 \leq N \leq 10^6$ & $S_{ults} = 0.6\, S_{ult}$

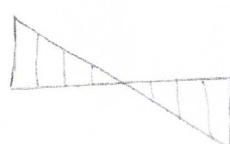
$S'_{es} = 0.5\, S_{ults}$ & $\beta = \dfrac{S_{ys}}{S_{ult}}$

c) Torsion spring



sec A – A

bending stress



σ_b

shear due to bending (neglected)

τ

Table 10–4

Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires

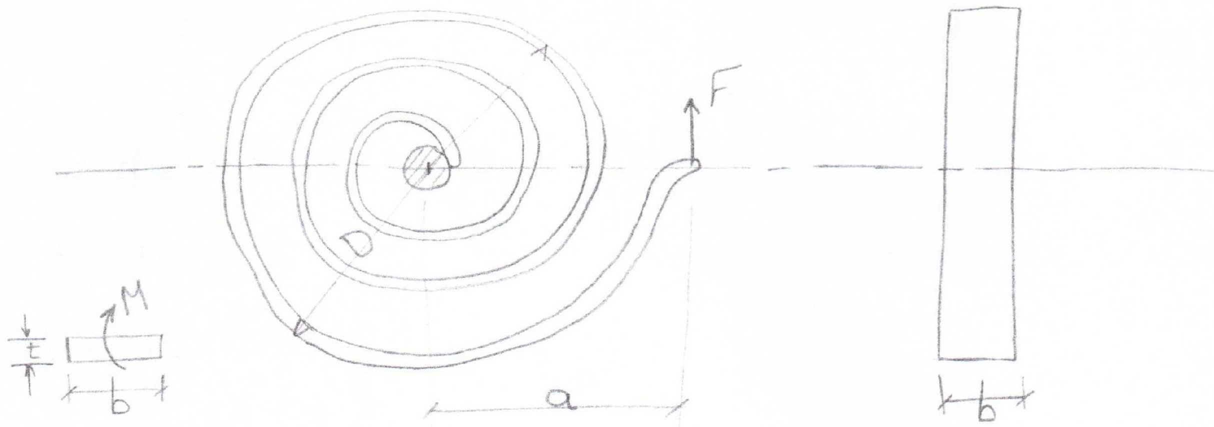
Source: From *Design Handbook*, 1987, p. 19. Courtesy of Associated Spring.

Material	ASTM No.	Exponent m	Diameter, mm	A, $MPa \cdot mm^m$	Relative Cost of wire
Music wire[*]	A228	0.145	0.10–6.5	2211	2.6
OQ&T wire[†]	A229	0.187	0.5–12.7	1855	1.3
Hard-drawn wire[‡]	A227	0.190	0.7–12.7	1783	1.0
Chrome-vanadium wire[§]	A232	0.168	0.8–11.1	2005	3.1
Chrome-silicon wire[‖]	A401	0.108	1.6–9.5	1974	4.0
302 Stainless wire[#]	A313	0.146	0.3–2.5	1867	7.6–11
		0.263	2.5–5	2065	
		0.478	5–10	2911	
Phosphor-bronze wire[**]	B159	0	0.1–0.6	1000	8.0
		0.028	0.6–2	913	
		0.064	2–7.5	932	

Table 10–5

Mechanical Properties of Some Spring Wires

Material	Elastic Limit, Percent of S_{ut} Tension	Torsion	Diameter d, mm	E GPa	G GPa
Music wire A228	65–75	45–60	<0.8	203.4	82.7
			0.8–1.6	200	81.7
			1.61–3	196.5	81.0
			>3	193	80.0
HD spring A227	60–70	45–55	<8	198.6	80.7
			0.8–1.6	197.9	80.0
			1.61–3	197.2	79.3
			>3	196.5	78.6
Oil tempered A239	85–90	45–50		196.5	77.2
Valve spring A230	85–90	50–60		203.4	77.2
Chrome-vanadium A231	88–93	65–75		203.4	77.2
A232	88–93			203.4	77.2
Chrome-silicon A401	85–93	65–75		203.4	77.2
Stainless steel					
A313*	65–75	45–55		193	69.0
17-7PH	75–80	55–60		208.4	75.8
414	65–70	42–55		200	77.2
420	65–75	45–55		200	77.2
431	72–76	50–55		206	79.3
Phosphor-bronze B159	75–80	45–50		103.4	41.4
Beryllium-copper B197	70	50		117.2	44.8
	75	50–55		131	50.3
Inconel alloy X-750	65–70	40–45		213.7	77.2



$$M = F * a$$

$$\sigma_b = \frac{My}{I}$$

$$\sigma_b = K\,\frac{My}{I}$$

for round wire

$$c = \frac{D}{d}$$

$$K = \frac{4c^2 - c - 1}{4c\,(c-1)}$$

$$\sigma_b = K\,\frac{My}{I}$$

$$= K\,\frac{(F*a)*d/2}{\frac{\pi}{64}\,d^4}$$

$$\boxed{\sigma_b = K\,\frac{32\,Fa}{\pi\,d^3}}$$

for rectangular wire

$$C = \frac{D}{t}$$

$$K = \frac{3C^2 - C - 0.8}{3C\,(C-1)}$$

$$\sigma_b = K\,\frac{(F*a)*t/2}{bt^3/12}$$

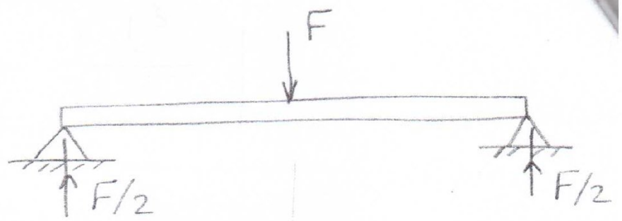
$$\boxed{\sigma_b = K\,\frac{6\,Fa}{bt^2}}$$

* twist angle in general Case $\theta = \frac{TL}{GJ}$

for spring $\theta = \frac{ML}{EI}$

in round wire $\theta = \frac{Fa(\pi DN)}{E * \frac{\pi}{64}\,d^4}$

* d) Leaf Spring

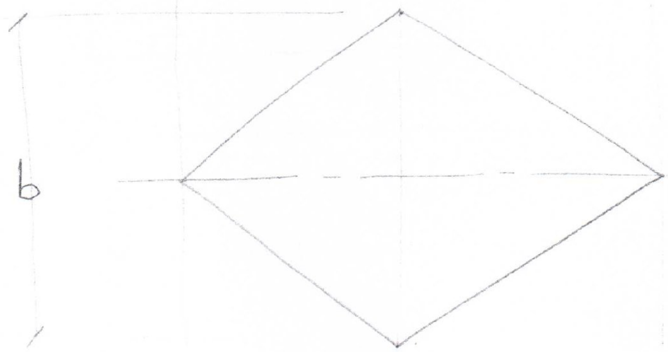


At a distance x

$$M_x = \frac{F}{2} x$$

bending stress

$$\sigma_{bx} = \frac{M_x\, y}{I_x}$$



$$= \frac{\frac{Fx}{2} * \frac{t}{2}}{b_x t^3/12} = \boxed{\frac{3Fx}{b_x t^3} = \sigma_{bx}}$$

At the mid point of the beam $(x = \frac{L}{2})$ $(b_x = b)$

$$\sigma_b = \frac{3FL}{2b\,t^3}$$

* To find the deflection, we apply double integration method

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{Fx}{2EI}$$

$$\frac{dy}{dx} = \frac{Fx^2}{4EI} + C_1 \qquad \text{eqn of slope}$$

$$y = \frac{Fx^3}{12EI} + C_1 x + C_2$$

Apply Boundary Condition to find C_1 & C_2

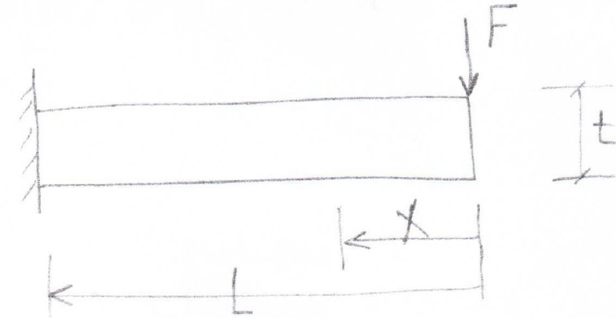
at $x = \frac{L}{2} \rightarrow \frac{dy}{dx} = 0$ $\qquad 0 = \frac{FL^2}{16EI} + C_1$ $\qquad \therefore \boxed{C_1 = \frac{-FL^2}{16EI}}$

at $x = 0 \longrightarrow y = 0$ $\qquad \therefore \boxed{C_2 = 0}$

$$\boxed{y = \frac{Fx^3}{12EI} - \frac{FL^2}{16EI}x}$$

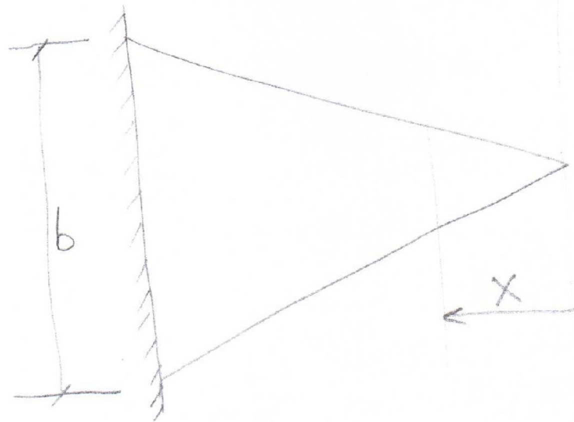
at $x = \frac{L}{2} \longrightarrow y = y_{max}$

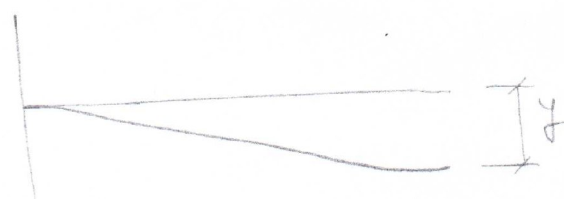
$$y_{max} = \boxed{\delta = \frac{FL^3}{96EI} - \frac{FL^3}{32EI}}$$



at $x = L \quad \frac{dy}{dx} = 0$

$y = 0$





* Multiple leaf spring