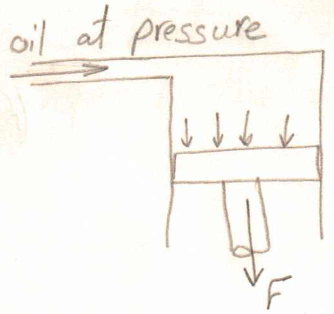
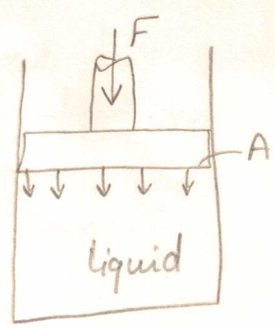
Fluid statics

* **pressure** : is the Normal force per unit area

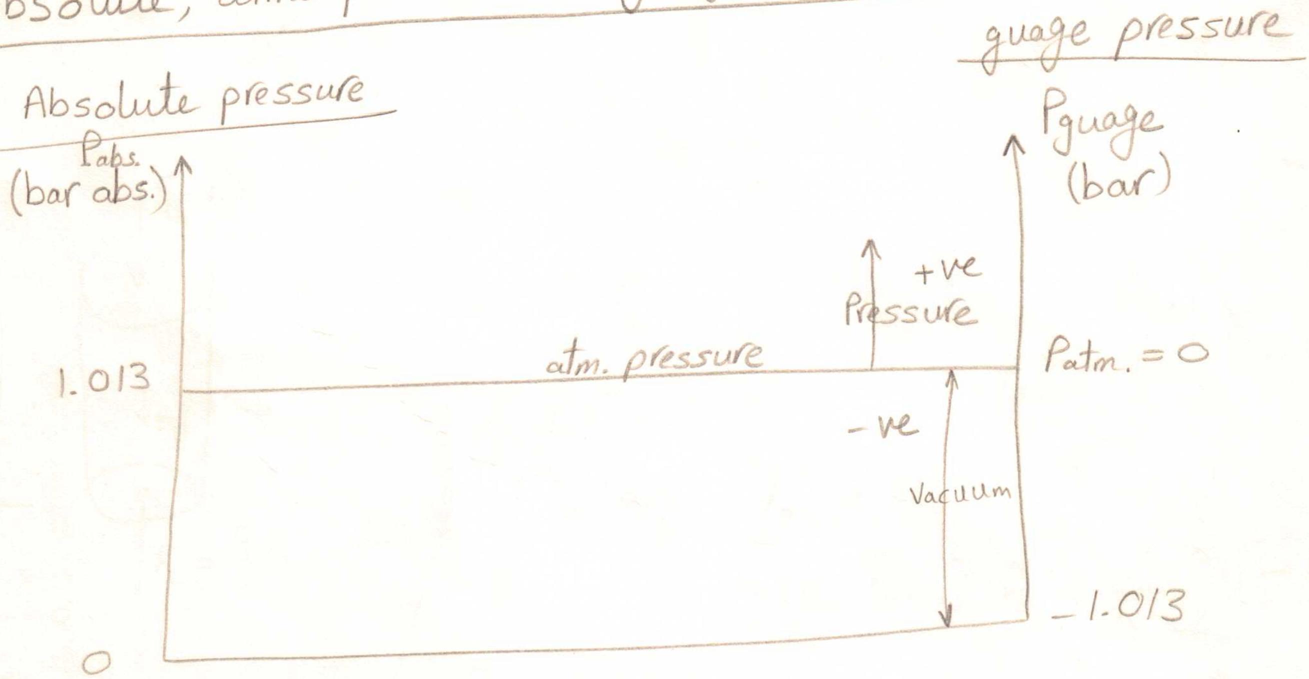
$$P = \frac{F}{A}$$

$$Pa = \frac{N}{m^2}$$





* Absolute, atmospheric and guage pressure



Absolute pressure = true pressure

$$P_{abs.} = P_{guage} + P_{atm.}$$

* All given values for pressure are guage except if :-
  1. (abs.) is mentioned beside the unit.
  2. dealing with atmospheric pressure.
  3. dealing with vapour pressure. ⟹ In Hydraulic circuits: $P_{min} > P_{vap.} - P_{atm}$

* No pressure guage value less than -1.013 bar

* $1 \text{ bar} = 10^5$ Pascal

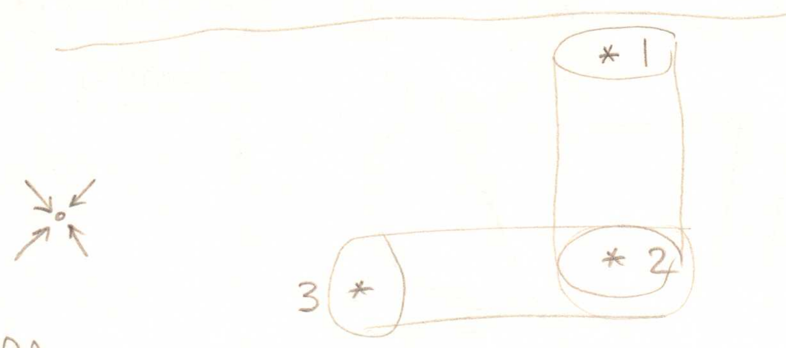
* $-ve$ pressure is called vacuum.

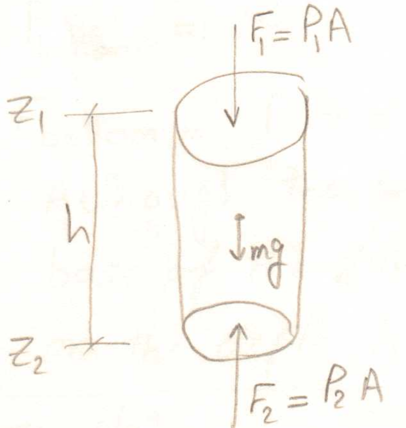
* In a static liquid

① The pressure at a certain point is the same in all direc

② The pressure is Constant in the same horizontal plane.

③ The pressure changes in the vertical direction.





$$\rho = \frac{m}{V} \quad \therefore \quad m = \rho V = \rho A h$$

$\Sigma F = 0$ in static

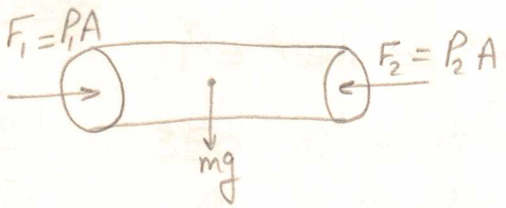
vertical:

$$F_1 + mg - F_2 = 0$$

$$P_1 A + \rho A (z_1 - z_2) g - P_2 A = 0 \qquad \boxed{\div A}$$

$$P_1 + \rho h g - P_2 = 0$$

$$\boxed{P_2 - P_1 = \rho g h} \quad \text{or} \quad \boxed{P_2 - P_1 = wh}$$



$\Sigma F = 0$ in static

horizontal:

$$F_1 - F_2 = 0$$

$$P_1 A - P_2 A = 0 \qquad \boxed{\div A}$$

$$P_1 - P_2 = 0$$

$$\boxed{P_1 = P_2}$$

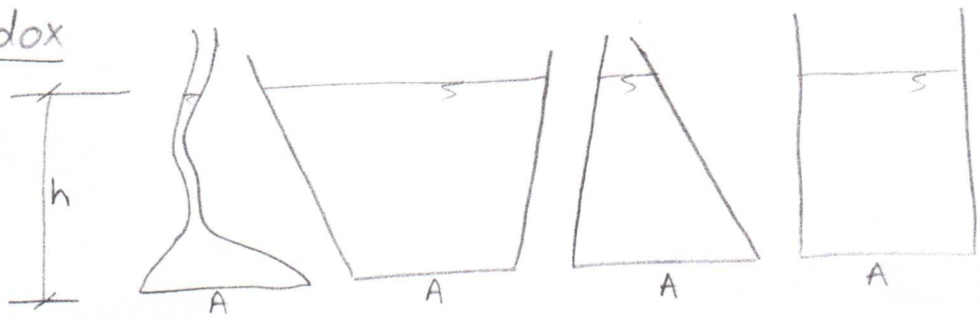
* pressure and head

$$P = wh$$

pressure (N/m^2) ← P; wh → head (m)

* head: is the vertical length that can define the pressure at const. g

* The hydrostatic paradox



$P_{bottom} = wh$

$F_{bottom} = PA = whA$

Although the weight of fluid is different, the force in the base of the four vessels is the same. This force depend on the depth h and the base area A.

Example

A cylinder contains a fluid at a pressure of $350\ KN/m^2$.

- Express the pressure in terms of a head of
  (a) water $\rho_w = 1000\ Kg/m^3$ (b) mercury $\gamma = 13.6$
- Determine the absolute pressure if $P_{atm} = 101.3\ KN/m^2$?

Soln

$$P = wh = \rho g h$$

(a) $350 * 10^3 = 1000 * 9.81 * h_w$ $\therefore h_w = 35.68\ m$

(b) $P = \gamma \rho_w g h$

$350 * 10^3 = 13.6 * 1000 * 9.81 * h_m$ $\therefore h_m = 2.62\ m$

- $P_{abs.} = P_{gage} + P_{atm}$

$= 350 + 101.3$

$= 451.3\ KN/m^2$

Example

if $h_{atm} = 76$ cm Hg, determine P_{atm}.

$$P_{atm} = \omega_m h \qquad \gamma_m = \frac{\omega_m}{\omega_w}$$

$$= \gamma_m \omega_w h \qquad \omega_m = \gamma_m \omega_w$$

$$= 13.6 * 9800 * (76 * 10^{-2})$$

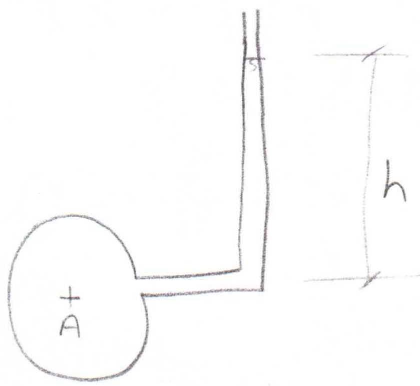
$$= \quad N/m^2$$

* pressure measurements by manometers

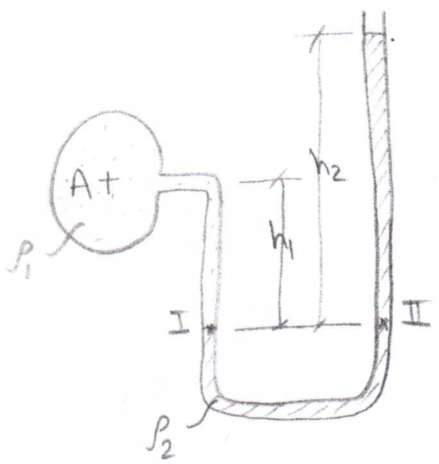
* piezometer

pressure tube or piezometer Consists of a single vertical tube.

$$P_A = \omega h$$



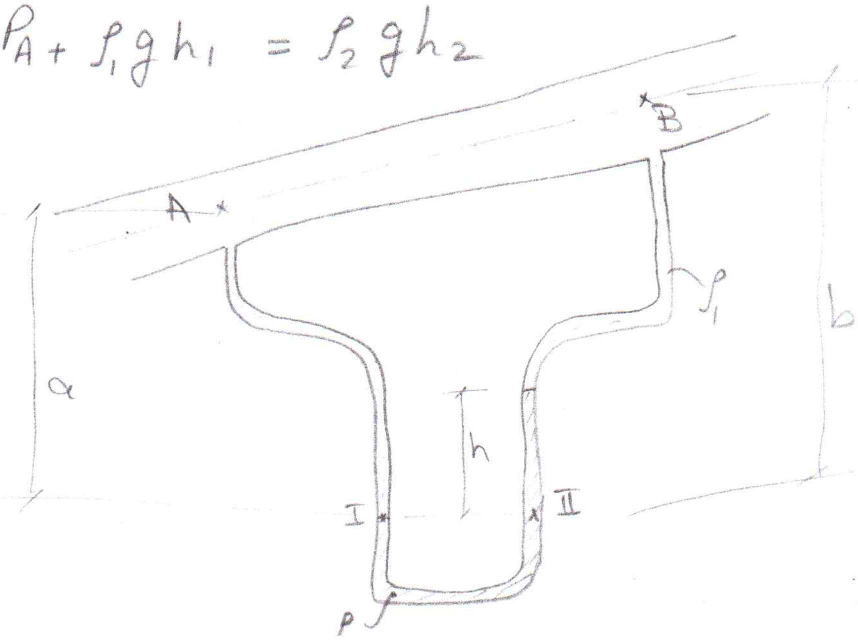
* U- tube manometer



* to make pressure equivalence
  1. still liquid
  2. Continued liquid
  3. same liquid

$$P_I = P_{II}$$

$$P_A + \rho_1 g h_1 = \rho_2 g h_2$$



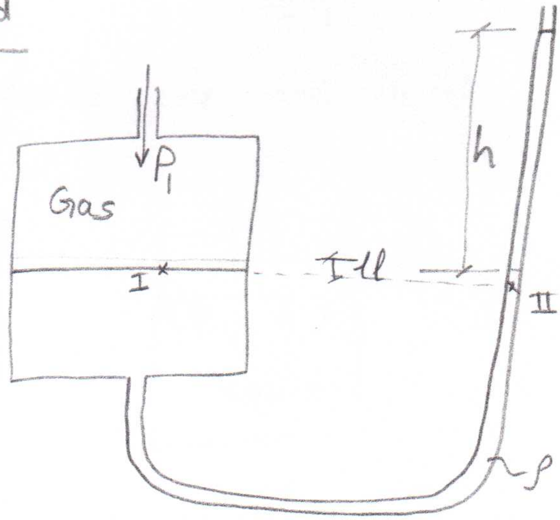
$$P_I = P_{II}$$

$$P_A + \rho_1 g a = P_B + \rho_1 g (b-h) + \rho_2 g h$$

$$P_A - P_B = \rho_1 g (b-h) + \rho_2 g h - \rho_1 g a$$

$$\Delta P =$$

* U-tube with one leg enlarged



Volume = Volume

$$A * ll = a * h$$

$$ll = \frac{a}{A} * h$$

$$= \frac{\pi/4\, d^2}{\pi/4\, D^2} * h$$

$$\boxed{ll = \frac{d^2}{D^2} * h}$$

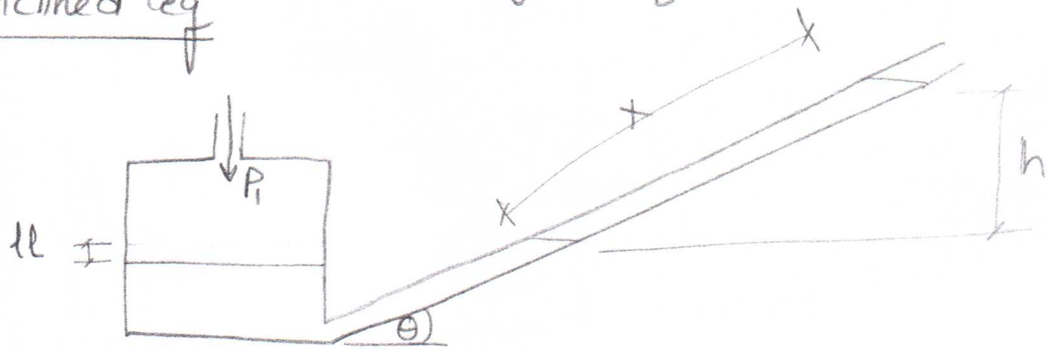
$$P_I = P_{II}$$

$$P_1 = \rho g\, ll + \rho g h$$

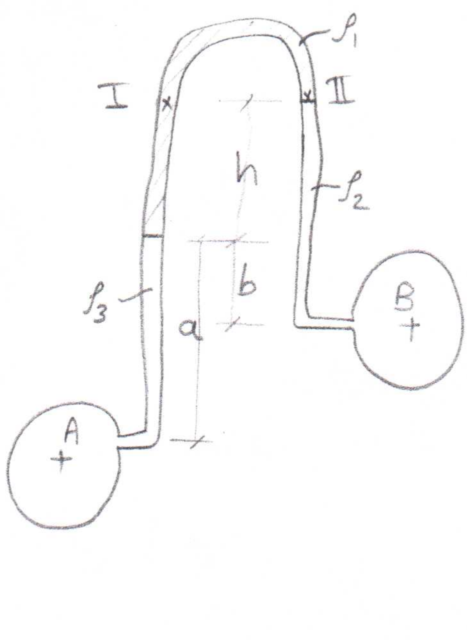
$$= \rho g * \frac{d^2}{D^2} h + \rho g h$$

$$= \rho g h \left(\frac{d^2}{D^2} + 1\right)$$

* U-tube with an inclined leg



* Inverted U-tube



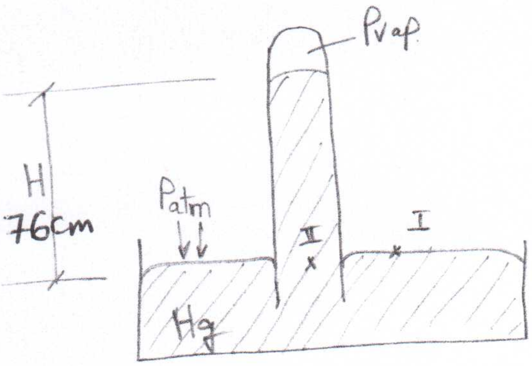
$$P_I = P_{II}$$

$$P_A - \rho_3 g a - \rho_1 g h = P_B - \rho_2 g (h+b)$$

$$P_A - P_B = \rho_3 g a + \rho_1 g h - \rho_2 g (h+b)$$

$$\Delta P = \;\ldots$$

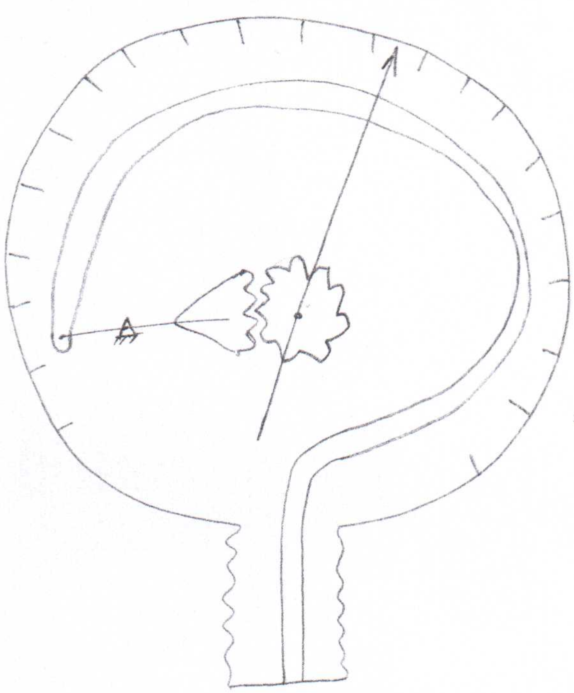
* Atmospheric pressure (Barometric pressure)



$$P_{vap_{Hg}} = 1.7 * 10^{-5} \text{ bar}$$
$$= 1.7 \frac{N}{m^2} = 0$$
neglected

$$P_I = P_{II}$$
$$P_{atm} = P_{vap_{Hg}}{}^{\nearrow 0} + \rho_m g H$$
$$= 13600 * 9.8 * 0.76$$
$$= 1.013 * 10^5 \ N/m^2$$
$$= 1.013 \text{ bar}$$
$$= 14.7 \frac{lbf}{in^2} \Rightarrow Psi$$
$$= 1.03 \ Kg_f/cm^2$$

* Bourdon tube gauge



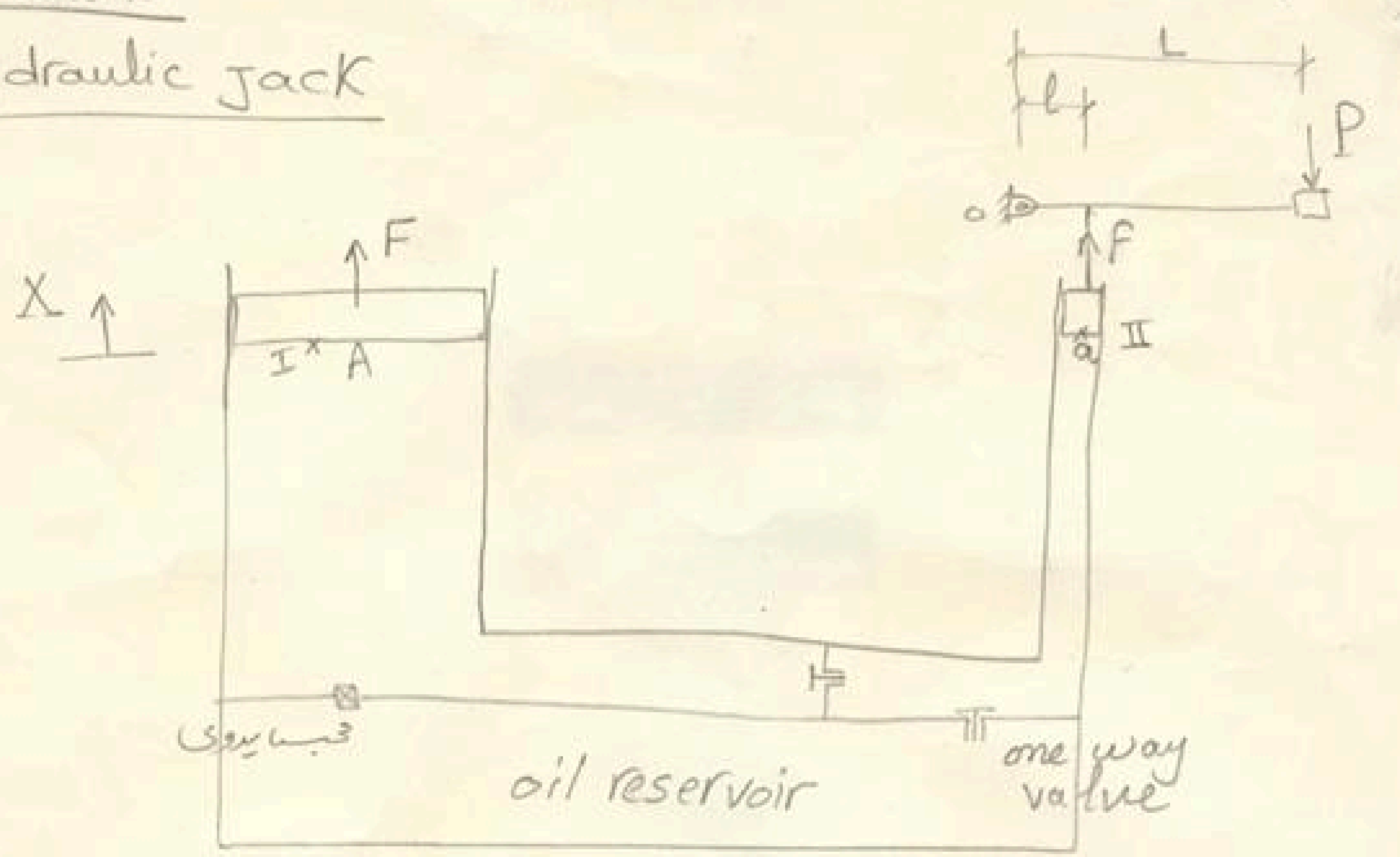
* For measuring pressur in almost all ranges except minutely small press

* Disadvantages

1- Needs Calibration on dead weight tester.
2- acuracy is less tha liquid Columns.

Applications

① Hydraulic jack



$$P_I = P_{II}$$

$$\frac{F}{A} = \frac{f}{a} \qquad \therefore F = f\frac{A}{a} \longrightarrow ①$$

$$\Sigma M_o = 0$$

$$f l = P L \qquad \therefore f = P\frac{L}{l} \longrightarrow ②$$

from ① & ②
$$\boxed{F = P * \frac{L}{l} * \frac{A}{a}}$$

n: no. of strockes to lift the load distance x

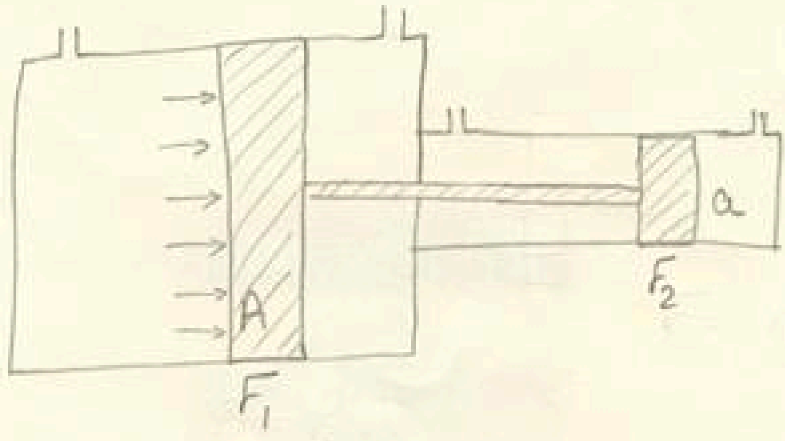
$$n = \frac{\text{total volume of oil}}{\text{volume / stroke}}$$

② Hydraulic press

$$\eta = \frac{o/p}{i/p} < 1 \qquad o/p = \frac{F * X}{t} \text{ watt}$$

$$i/p = \frac{o/p}{\eta} = \frac{F\,X}{t \cdot \eta}$$

③ pressure intensifier



$$F_1 = F_2$$

$$P_1 * A = P_2 * a \qquad \therefore \boxed{P_2 = \frac{P_1 A}{a}}$$

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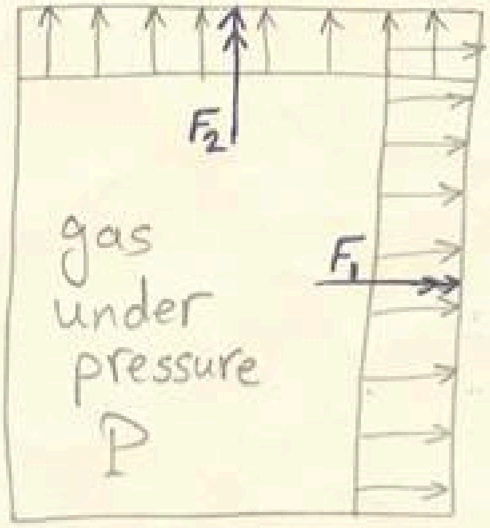
* Forces due to Fluid pressure
  * on Flat surface
  * for gases

F_1 & F_2 perpendicular ($\perp$) on the surface & acts at the center of area subjected to pressure

$$F_1 = PA_1 \quad \& \quad F_2 = PA_2$$



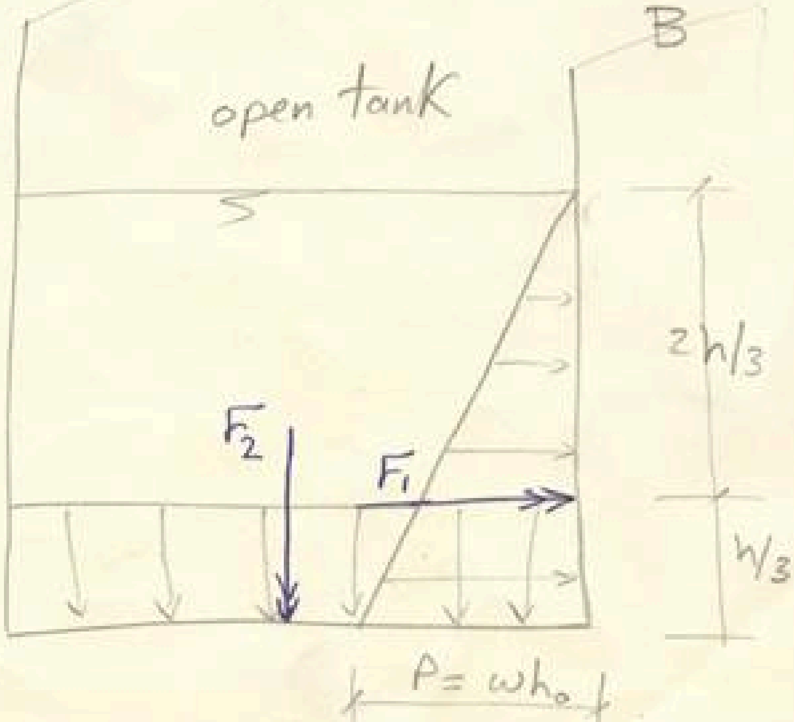
* for liquids

* F_1 = volume of pressure prisme

$$= wh_o * \frac{h_o}{2} * B$$

$$= \frac{wh_o^2 B}{2}$$

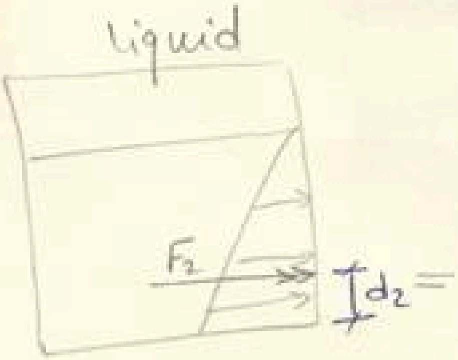
* F_1 acts at the center of volume of the prisme $\perp$ to the surface.

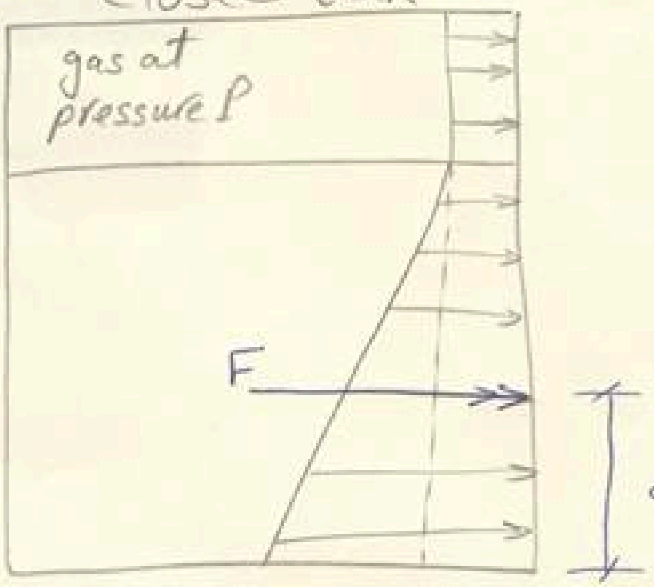


* F_2 (on bottom) $= P A$

$= \text{who} * A$

* F_2 acts $\perp$ on bottom and at the center of area.





$$F_1 d_1 + F_2 d_2 = F d$$

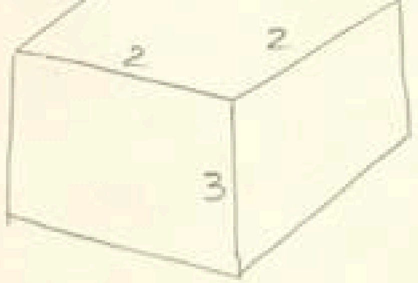
$$F_1 + F_2 = F$$

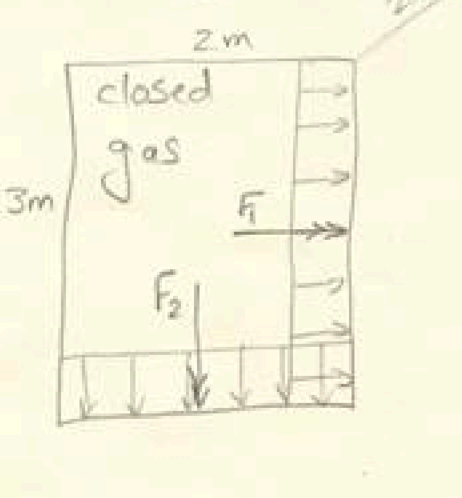
Example

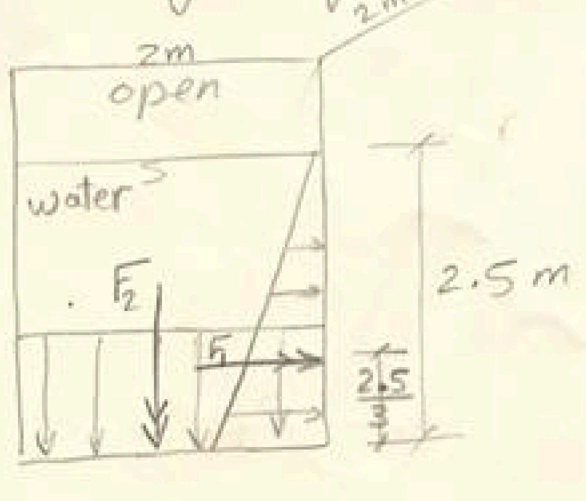
A square tank $(2 \times 2) \times 3$ m high. Calculate the force on one of the vertical sides of the tank and on its bottom on the following cases:-

1- Tank is closed containing gas of pressure 5 bar.

2- Tank is opened containing water to height of 2.5 m







① ②

① gas at 5bar

ⓐ side $F_1 = P A_1$

$= 5*10^5 * 2*3$

$= 3*10^6 \, N = 3 \, MN$ ⊥ side at center

ⓑ bottom $F_2 = P A_2$

$= 5*10^5 * 2*2$

$= 2*10^6 \, N = 2 \, MN$ ⊥ bottom at center

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② water with 2.5 m height

a) side $F_1 = w_w h_o * \frac{h_o}{2} * B$

$= 9800 * \frac{(2.5)^2}{2} * 2$

$=$ N ⊥ side at $\frac{2.5}{3}$ from bottom

ⓑ bottom $F_2 = w_w h_o * A$

$= 9800 * 2.5 * (2*2)$

$=$ N ⊥ bottom at center