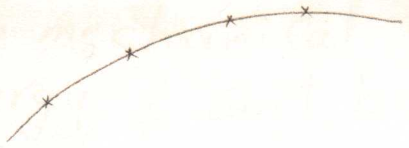
Flow in motion
(Hydrodynamics)

* **Definitions**

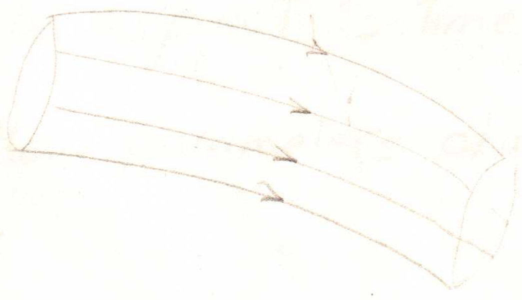
* **streamline:** is a smooth imaginary curve represents one particle in the flow. The tangent of this line gives the direction of velocity at any point.



- streamlines can never intersect
- They can never have sudden change in direction.

Intersection or sudden change in direction means that there is a point where the velocity vector has two directions in the same time which is impossible.

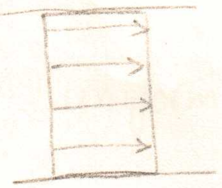
* **stream tube:** is a tube formed of an infinite number of streamlines which are drawn passing through a closed curve in the flow.



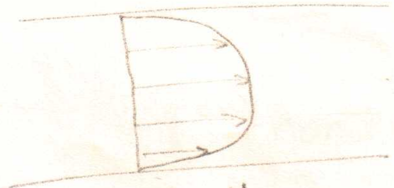
No flow can go in or out of the sides of this tube.

* **Types of flow**

① **Ideal and real flow**



Ideal flow



Real flow

* <u>Ideal flow</u>: means frictionless flow, no energy is lost, the viscosity is considered zero.

* <u>Real flow</u>: viscosity can't be neglected, there is friction. Friction causes some of the mechanical energy to be converted into heat energy & can't be restored.

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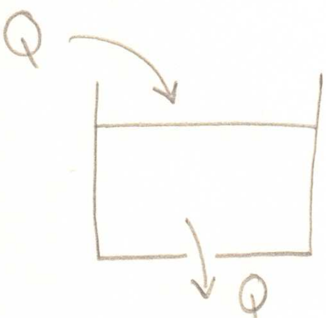
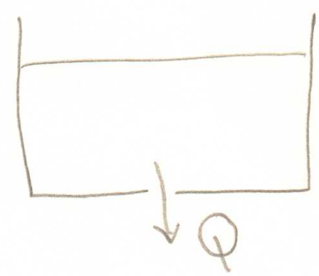
② steady and unsteady flow (with respect to time)
[from time to time)

$\frac{\partial}{\partial t} = 0$



steady flow



$\frac{\partial}{\partial t} \neq 0$

unsteady flow

* <u>steady flow</u>: pressure, velocity, flow rate (flow parameters) are constant with respect to time.

* <u>unsteady flow</u>: any of the flow parameters change with time.

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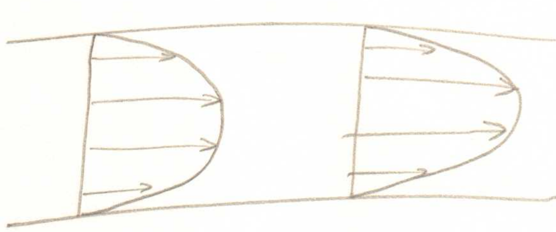
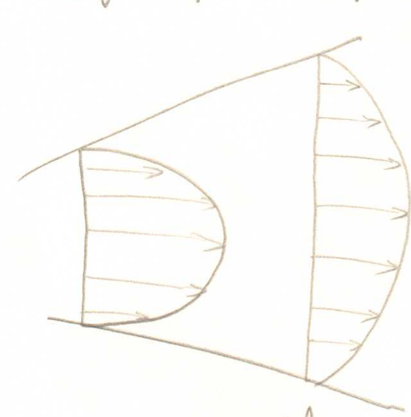
③ uniform and non-uniform flow
[from point to point]



uniform flow

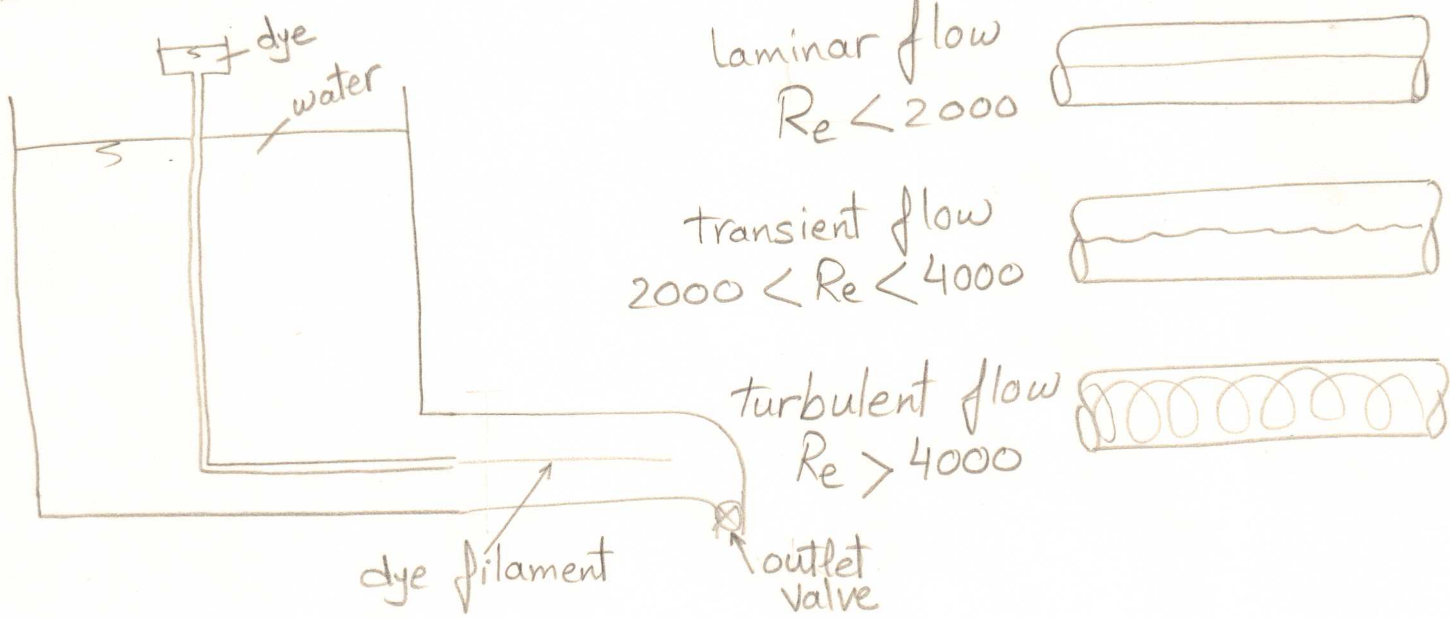


non-uniform flow

* uniform flow: The velocity at a given instant is the same in magnitude and direction at every point in the fluid.

* non-uniform flow: The velocity at a given instant changes from point to point.

④ Laminar, transient and turbulent flow



* laminar flow: (viscous flow, streamline flow)
  The particles move in parallel lines (layers).

* Transient flow at which the dye filament begin to oscillate.
* Turbulent flow the dye color is diffused over the whole cross-section.

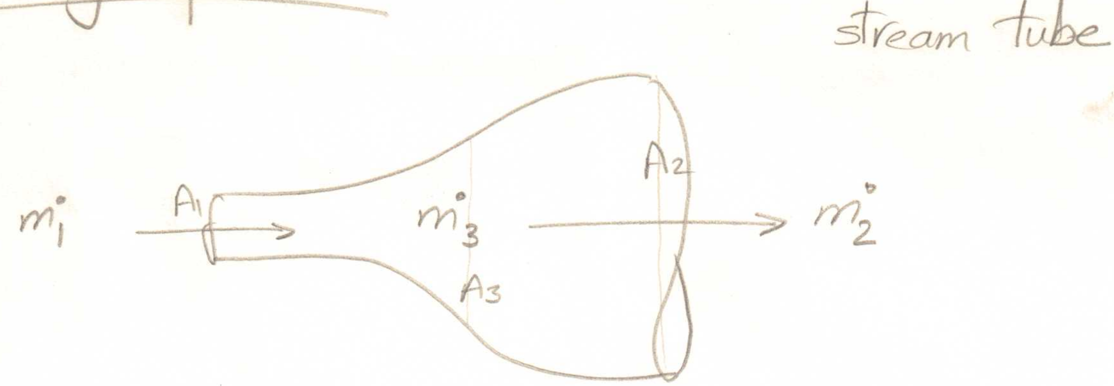
$$R_e = \frac{\rho V d}{\mu} \qquad \text{Reynolds number}$$

Equation of motion

① Continuity equation

② Bernoulli's equation

① Continuity equation



Mass of fluid entering per unit time = Mass of fluid leaving per unit time

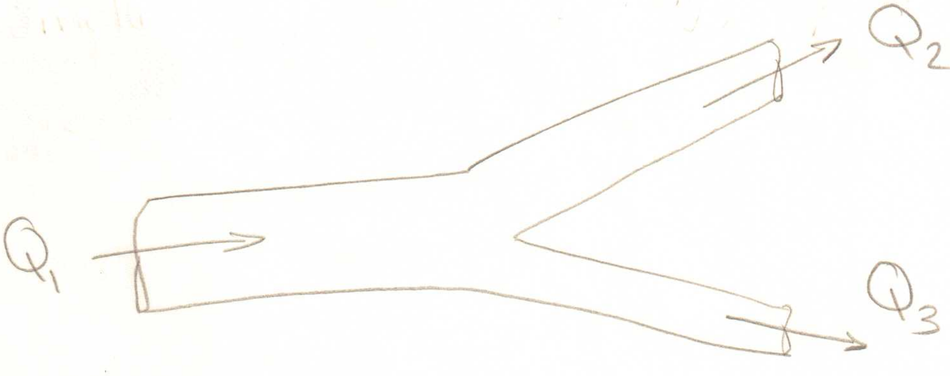
$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \text{Const.} \Rightarrow \text{mass flow rate}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \rho_3 A_3 V_3$$

* For liquids (incompressible fluid) $\rho_1 = \rho_2 = \rho_3 = \text{Const.}$

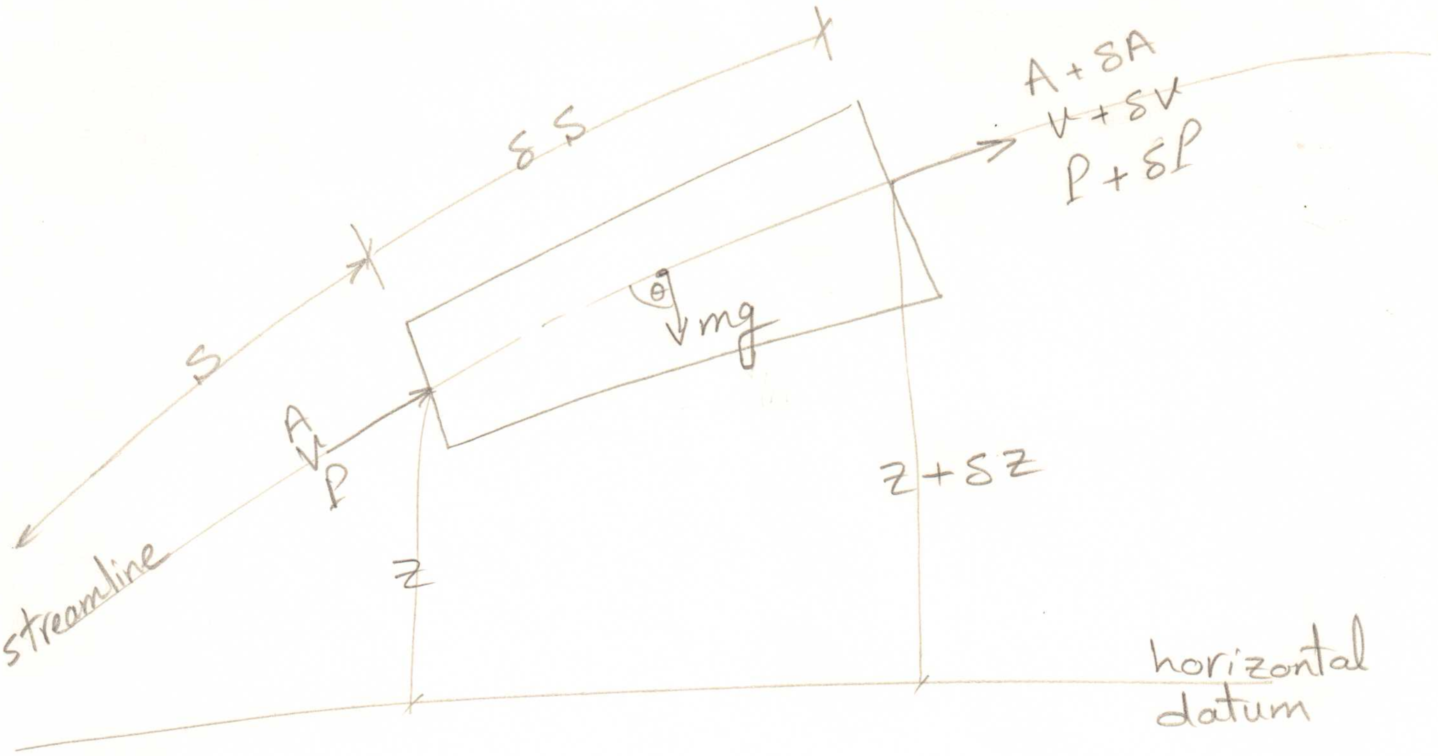
$$\boxed{A_1 V_1 = A_2 V_2 = A_3 V_3 = Q}$$ discharge (volume flow rate)

Continuity eqn



$$Q_1 = Q_2 + Q_3$$

②: Bernolli's equation [energy equation]



$$E = Z + \frac{P}{w} + \frac{V^2}{2g} = Const.$$

E: total energy per unit weight (m)

$\frac{N.m}{N}$ = energy per unit weight (m)

Z: potential energy per unit weight (m)

$\frac{P}{w}$: pressure energy per unit weight (m)

$\frac{V^2}{2g}$: Kinetic energy (velocity energy) (m) per unit weight

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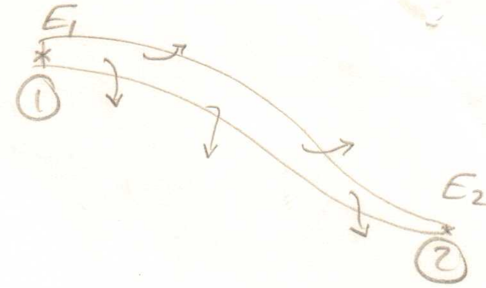
* For Ideal flow

$$E_1 = E_2$$

$$Z_1 + \frac{P_1}{w} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{w} + \frac{V_2^2}{2g}$$

* For Real flow

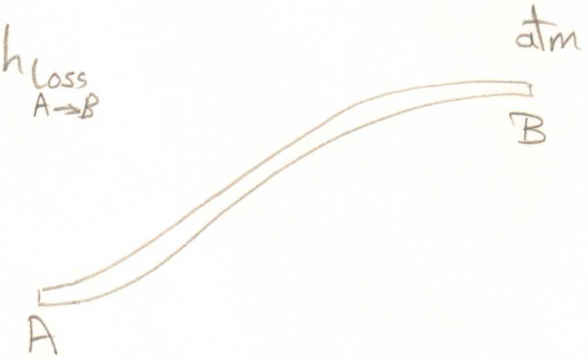
$$E_1 - h_{loss} = E_2$$



$$E_1 = E_2 + h_{loss}$$

$$\frac{P_1}{w} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{w} + Z_2 + \frac{V_2^2}{2g} + h_{loss_{1\to 2}}$$

$$\frac{P_A}{w} + Z_A + \cancel{\frac{V_A^2}{2g}} = \underbrace{\cancel{\frac{P_B}{w}}}_{\substack{=0 \\ \text{atm}}} + Z_B + \cancel{\frac{V_B^2}{2g}} + h_{loss_{A\to B}}$$



$A_A = A_B$

$\therefore V_A = V_B$

$$\frac{P_A}{w} = (Z_B - Z_A) + h_{loss_{A\to B}}$$

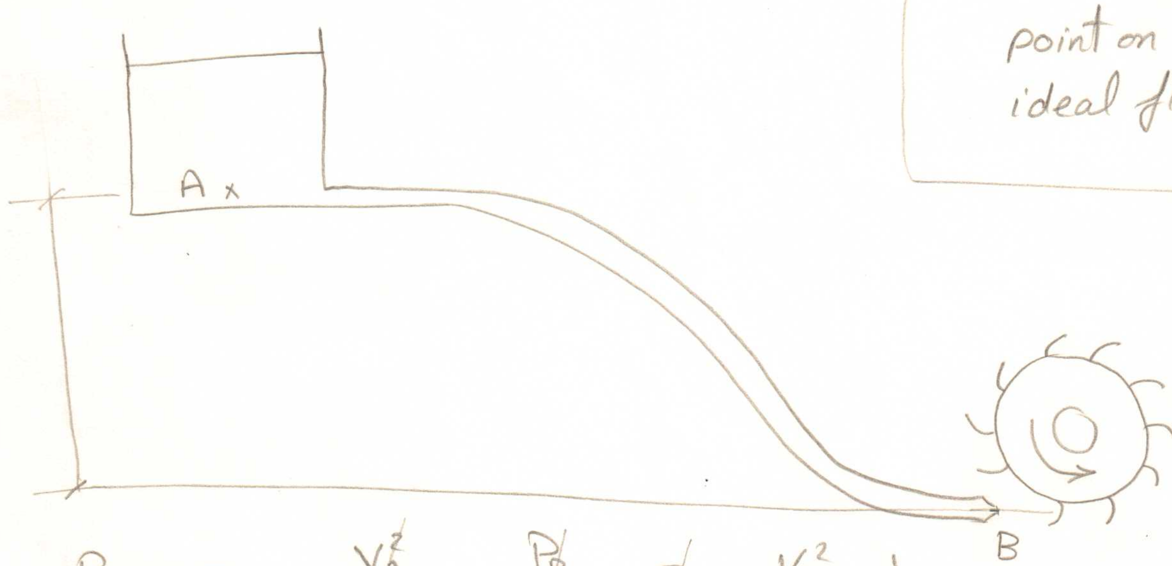
$P_{atm} = 0$

same dia. $\therefore$ same vel

vel. inside tank $= 0$ *

point on datum $Z = 0$ *

ideal flow $h_{loss} = 0$ *



$$\frac{P_A}{w} + Z_A + \underbrace{\frac{V_A^2}{2g}}_{\substack{\to 0 \\ \text{vel. inside} \\ \text{tank}}} = \underbrace{\frac{P_B}{w}}_{\substack{\to 0 \\ \text{atm}}} + \underbrace{Z_B}_{\substack{\to 0 \\ \text{datum}}} + \frac{V_B^2}{2g} + h_{loss_{A\to B}}$$

$$\frac{P_A}{w} + Z_A = \frac{V_B^2}{2g} + h_{loss_{A\to B}}$$

Flow in Motion

Hydrodynamics

* Continuity Equation

$$Q = A_1 V_1 = A_2 V_2$$

Q: discharge or volume flow rate

* Bernoulli's equation (Energy equation)

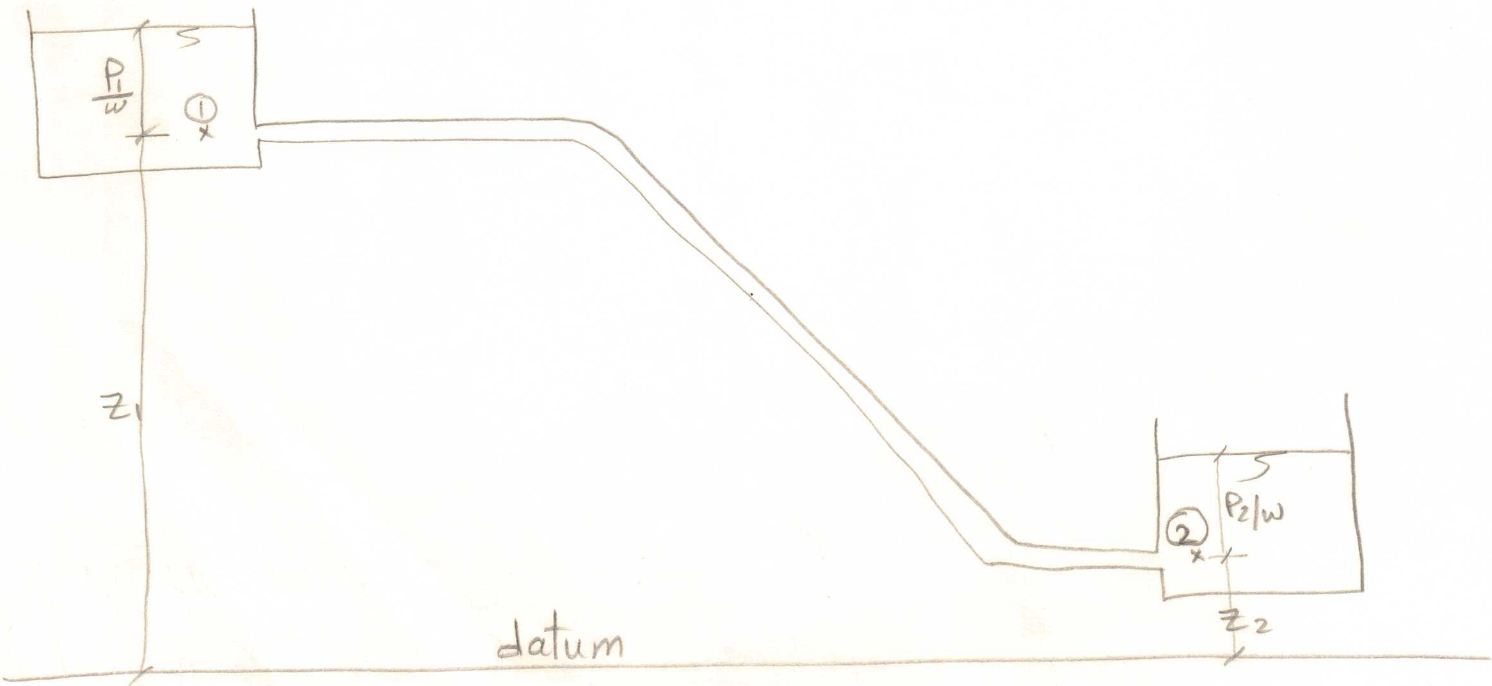
*** for ideal flow** $\quad E_1 = E_2 = E_3 = E$ (Constant)

$$Z_1 + \frac{P_1}{w} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{w} + Z_2 = \_\_\_\_\_\_ = E$$

*** for real flow** $\quad E_1 = E_2 + h_{loss_{1\to 2}}$

$$Z_1 + \frac{P_1}{w} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{w} + \frac{V_2^2}{2g} + h_{loss_{1\to 2}}$$

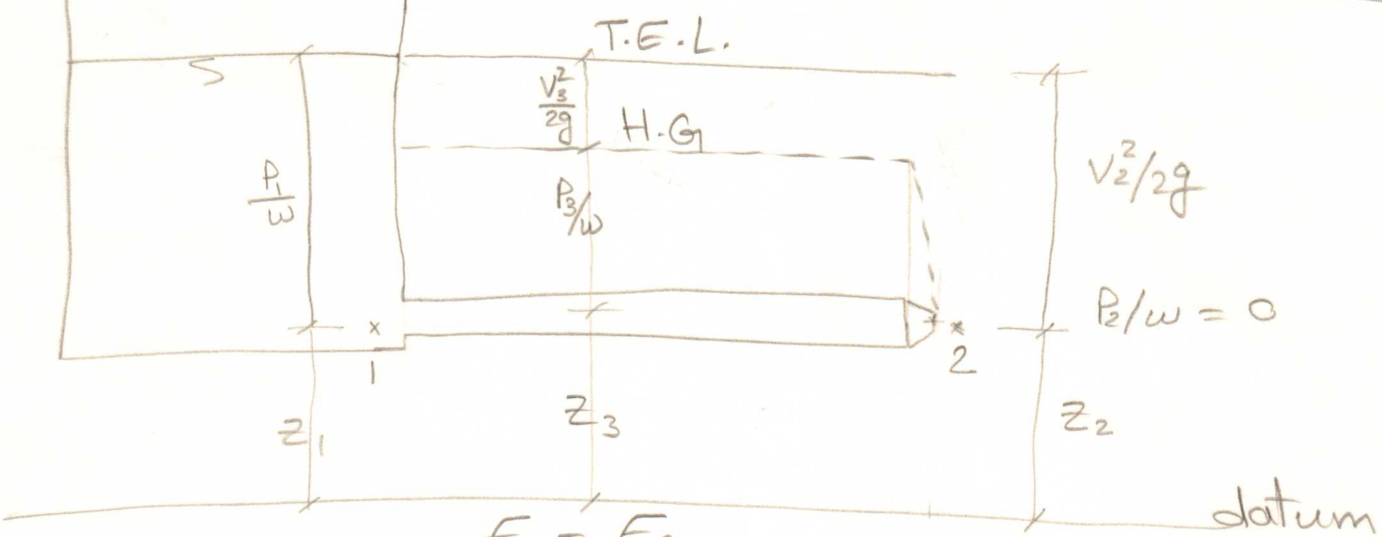
from energy eqn, the flow is always from point of higher total energy to point of lower total energy.
[from higher $Z_1 + \frac{P_1}{w} + \frac{V_1^2}{2g}$ to lower $Z_2 + \frac{P_2}{w} + \frac{V_2^2}{2g}$]



Total Energy and Hydraulic gradient lines

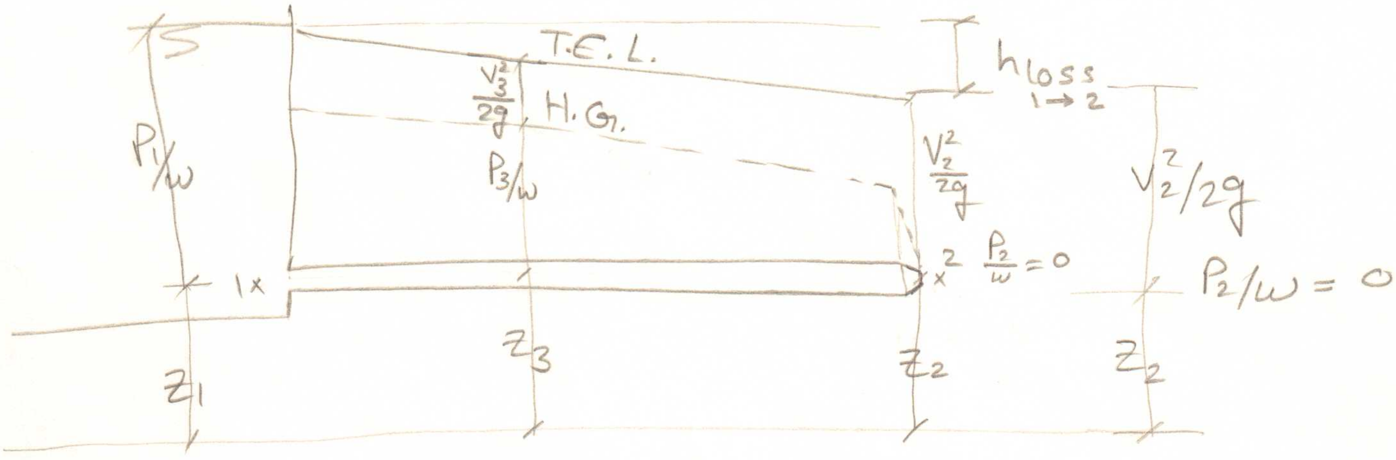
$$E = Z + \frac{P}{\omega} + \frac{V^2}{2g}$$ (Graphical representation of Bernolli's equation)

* Ideal flow



$$E_1 = E_2$$

Real flow



$$E_1 = E_2 + h_{loss\ 1\to 2}$$

* T.E.L. in the reservoir is the liquid surface.

* vertical distance betn datum & T.E.L. represents total energy

$$E = Z + \frac{P}{\omega} + \frac{V^2}{2g}$$

* vertical distance betn T.E.L. & H.G. represents K.E. $= \frac{V^2}{2g}$

* ~ ~ ~ H.G. & pipe Centerline represents head energy at any point $= P/w$

* For ideal flow T.E.L. is horizontal & $h_{loss} = 0$ & $E_1 = E_2 = \ldots$

* For real flow T.E.L. drops continuously in the direction of flow & $h_{loss} = \text{—}$ & $E_1 > E_2 > E_3$ ---
* H.G is parallel to T.E.L. if the area of pipe is constant & start to deviate if the area of pipe is changing.
* at atmospheric pressure ($P = 0$), H.G intersects the pipe center line.

Application

① Changes of pressure in a tapering pipe

Example Pg 159

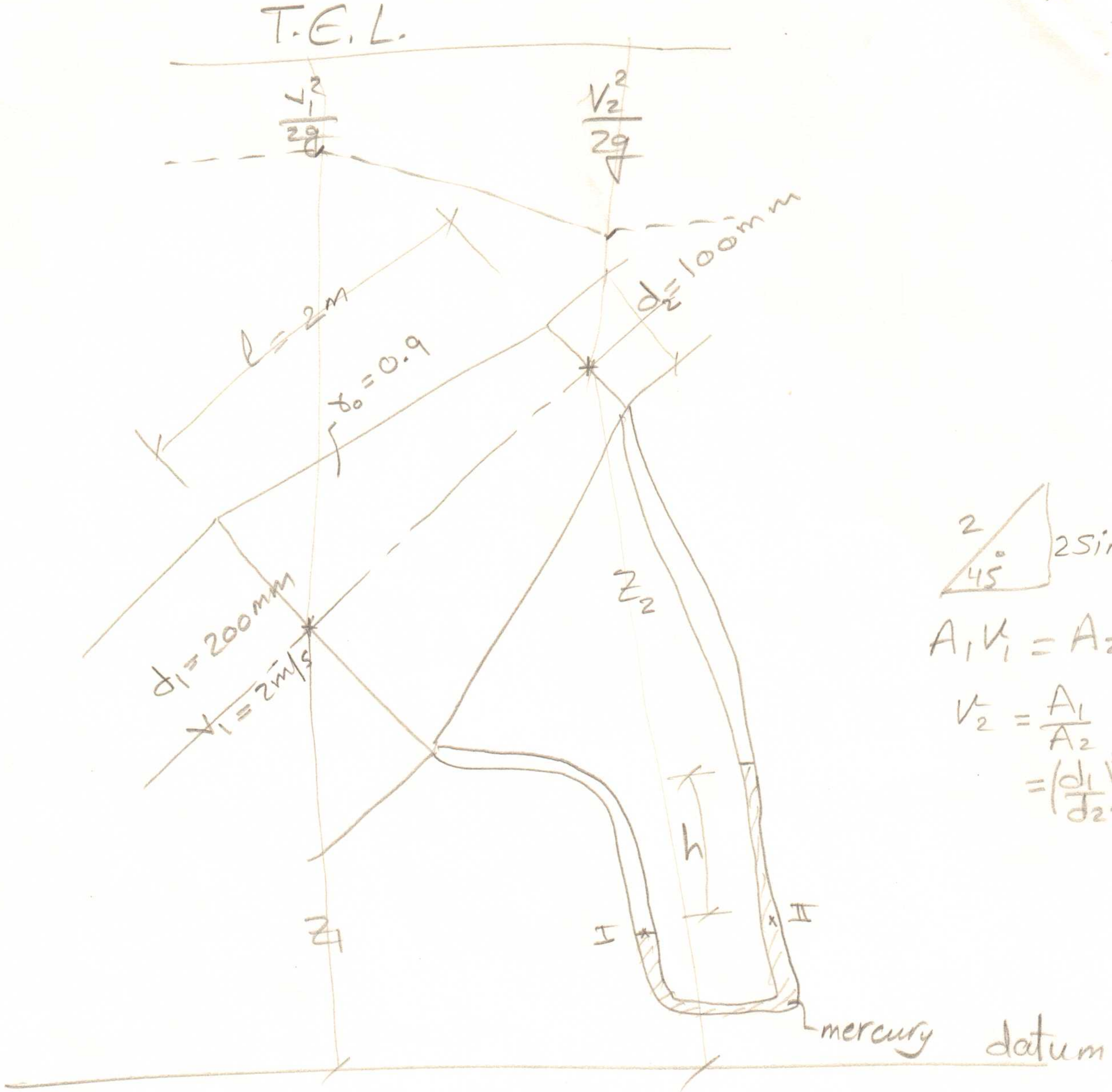
A pipe inclined at 45° to the horizontal converges over a length ℓ of 2m from diameter d_1 of 200mm to a diameter d_2 of 100mm at the upper end. Oil of relative density 0.9 flows through the pipe at a vel. V_1 at the lower end of 2m/s. Find the pressure difference across the 2m length ignoring any loss of energy, and the difference in level that would be shown on a mercury manometer connected across this length. The relative density of mercury is 13.6 and the leads to the manometer are filled with the oil.

Soln $h_{loss} = 0$, $\Delta P = P_2 - P_1 = ??$, $h = ??$

$$E_1 = E_2$$

$$\frac{P_1}{w} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{w} + Z_2 + \frac{V_2^2}{2g}$$

$$\frac{P_2 - P_1}{w} = (Z_1 - Z_2) + \frac{V_1^2 - V_2^2}{2g}$$



$$A_1V_1 = A_2V_2$$

$$V_2 = \frac{A_1}{A_2}V_1 = \left(\frac{d_1}{d_2}\right)^2 V_1$$

$$\frac{P_2 - P_1}{0.9 * 9800} = -2\sin 45 + \frac{(2)^2 - \left(\left(\frac{2}{1}\right)^2 * 2\right)^2}{2 * 9.8}$$

$$= -39\,484 \ N/m^2$$

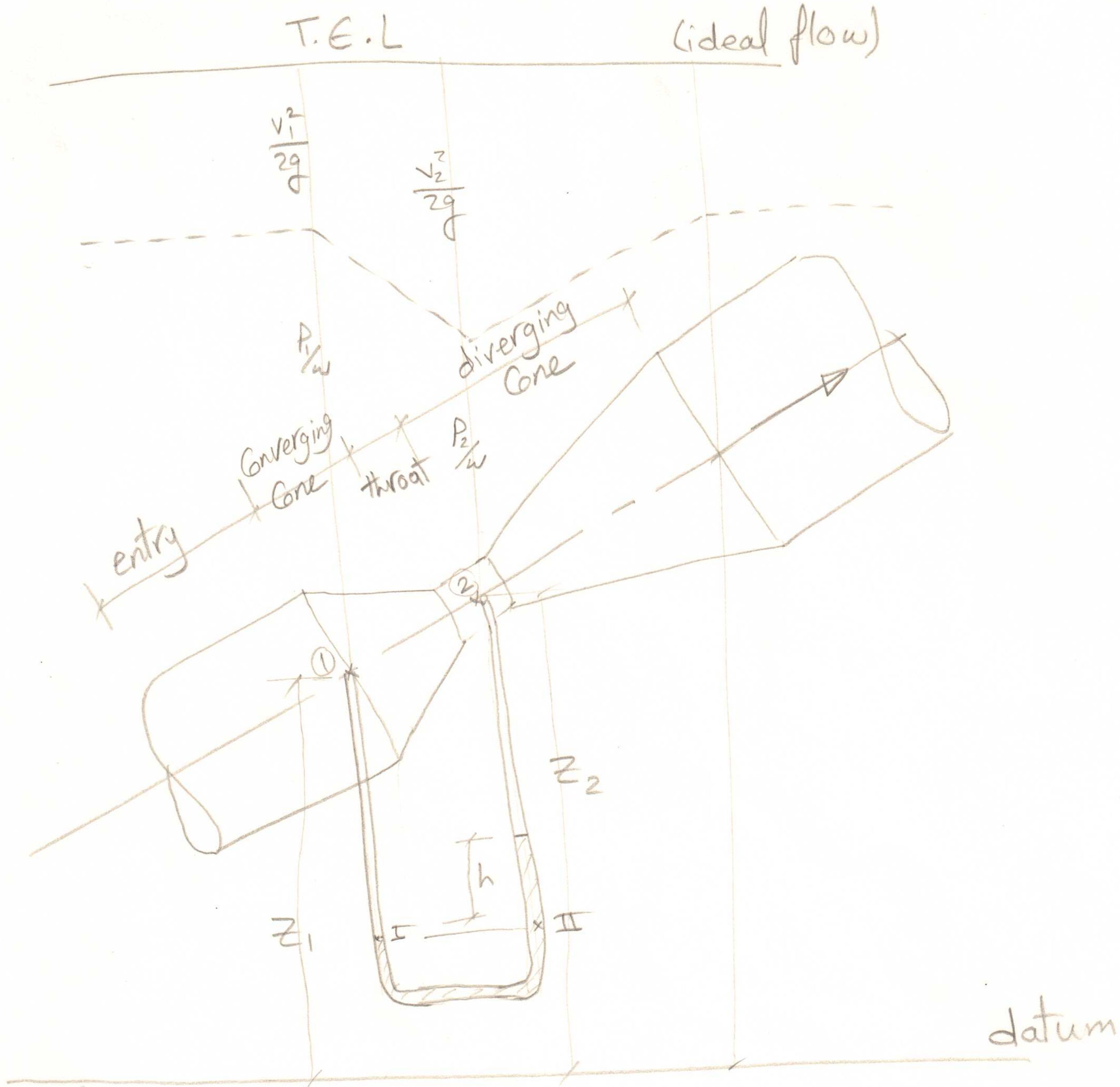
$$P_I = P_{II}$$

$$w_o h + \cancel{w_o l} + P_1 = w_m h + \cancel{w_o l} + w_o(Z_2 - Z_1) + P_2$$

$$(w_o - w_m)h = P_2 - P_1 + w_o(Z_2 - Z_1)$$

$$h = \underline{0.217} \ m \text{ of oil}$$

② principle of venturi meter



$$E_1 = E_2$$

$$Z_1 + \frac{P_1}{w} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$\frac{V_2^2 - V_1^2}{2g} = Z_1 - Z_2 + \frac{P_1 - P_2}{w}$$

$$V_2^2 - V_1^2 = 2g\left[(Z_1 - Z_2) + \frac{P_1 - P_2}{w}\right] \longrightarrow ①$$

$$Q = A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2}$$

sub. in ①

$$\left(\frac{A_1}{A_2}\right)^2 V_1^2 - V_1^2 = 2g\left[(Z_1 - Z_2) + \frac{P_1 - P_2}{\omega}\right]$$

$$V_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] = 2g\left[(Z_1 - Z_2) + \frac{P_1 - P_2}{\omega}\right]$$

$$V_1 = \sqrt{\frac{2g\left[(Z_1 - Z_2) + \frac{P_1 - P_2}{\omega}\right]}{\left(\frac{A_1}{A_2}\right)^2 - 1}} = \sqrt{\frac{2gH}{m^2 - 1}}$$

$$\boxed{\begin{array}{l} Q = A_1 V_1 = A_1\sqrt{\frac{2gH}{m^2-1}} \\ Q_{actual} = C_d\, Q_{th} \qquad C_{dv} = 0.97 \end{array}}$$

theoretical discharge

C_d: Coefficient of discharge < 1

According to U-tube manometer

$$P_I = P_{II}$$

$$P_1 + \cancel{\omega l} + \omega h = P_2 + \omega(Z_2 - Z_1) + \cancel{\omega l} + \omega_{man} h$$

$$P_1 - P_2 = \omega(Z_2 - Z_1 - h) + \omega_{man} h$$

$\div \omega$

$$\frac{P_1 - P_2}{\omega} = Z_2 - Z_1 - h + \frac{g * \rho_{man}}{g * \rho} h$$

$$H = \frac{P_1 - P_2}{\omega} + Z_1 - Z_2 = h\left(\frac{\rho_{man}}{\rho} - 1\right)$$

$$\boxed{Q_{act} = C_d A_1 \sqrt{\frac{2gh\left(\frac{\rho_{man}}{\rho} - 1\right)}{\left(\frac{A_1}{A_2}\right)^2 - 1}}}$$

A_1 : Area of pipe

A_2 : Area of throat

Example Pg 162

A venturi meter having a throat diameter d_2 of 100 mm is fitted into a pipeline which has a diameter d_1 of 250 mm through which oil of specific gravity 0.9 is flowing. The pressure difference between the entry and throat tapping is measured by a U-tube manometer, containing mercury of specific gravity 13.6, and the connections are filled with the oil flowing in the pipeline. If the difference of level indicated by the mercury in the U-tube is 0.63 m, Calculate the theoretical volume rate of flow through the meter.

Sol^n

$$d_2 = 100\ mm$$

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.1)^2 = 0.00785\ m^2$$

$$d_1 = 250\ mm$$

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(0.25)^2 = 0.0491\ m^2$$

$$m = \frac{A_1}{A_2} = 6.25$$

$\gamma = 0.9$, $\gamma_{man} = 13.6$, $h = 0.63$, $Q_{th} = ??$

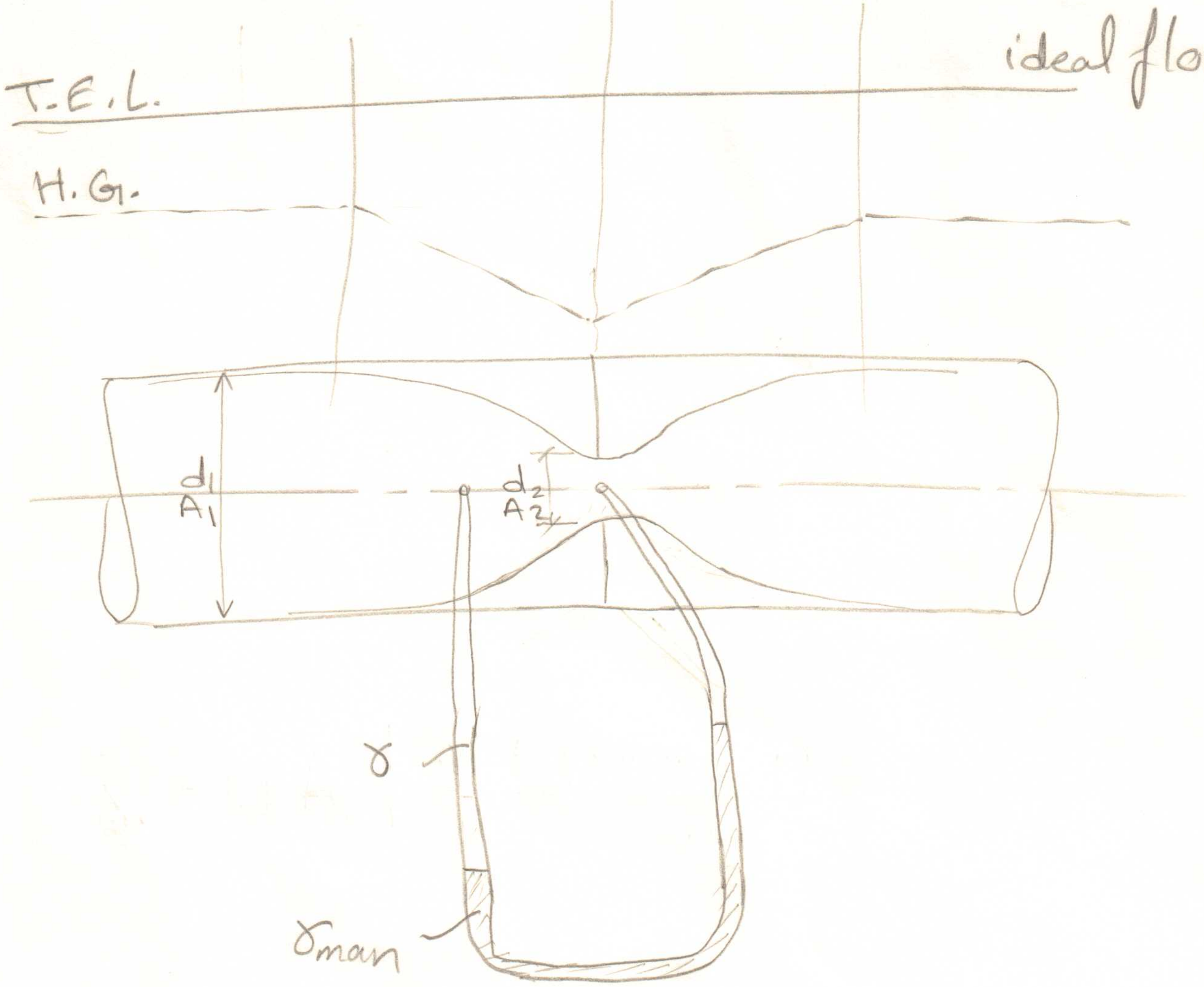
$$Q_{th} = A_1 \sqrt{\frac{2gh\left(\frac{\gamma_{man}}{\gamma} - 1\right)}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

$$= 0.0491 \sqrt{\frac{2 * 9.8 * 0.63\left(\frac{13.6}{0.9} - 1\right)}{(6.25)^2 - 1}}$$

$$= 0.105\ m^3/sec$$

③ pipe orifices

ideal flow



$$Q_{act.} = C_d \; A_1 \sqrt{\frac{2gh\left(\frac{\gamma_{man}}{\gamma}-1\right)}{\left(\frac{A_1}{A_2}\right)^2-1}}$$

$$C_{d_o} = 0.65$$