CHAPTER 1

Introduction to Modeling and Simulation

1.1 DYNAMIC SYSTEMS

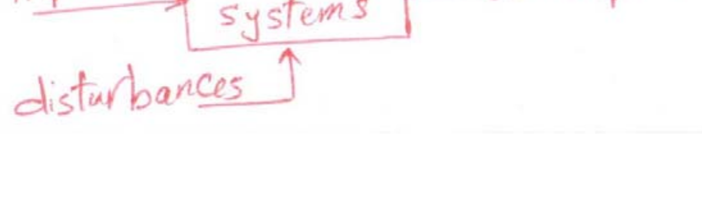
this course

This text discusses both the modeling of dynamic systems as found in the major engineering disciplines and solutions of the resulting differential equations by analytical and computational means. The book is intended for use at the introductory level, but serves the practicing engineer as a reference source as well. It presents modeling of the engineering disciplines using a unified approach. Since the equations that represent the dynamics of a physical system can take several different forms, we emphasize selecting the form most compatible with the mathematical method or numerical process that is ultimately to be used in solving the equations. This chapter presents general considerations that will be used throughout the text.

1.1.1 Examples of Dynamic Systems

Static systems have an output response to an input that does not change with time; i.e., the input is held constant. This means that the output always has the same instantaneous relationship with the input. **Dynamic systems** have a response to an input that is not instantaneously proportional to the input or disturbance and that may continue after the input is held constant. Dynamic systems can respond to input signals, disturbance signals, or initial conditions.

Examples of dynamic systems are all around us. They may be observed in common devices employed in everyday living, Figure 1.1, as well as in sophisticated engineering systems such as those in spacecraft that took astronauts to the moon. Dynamic systems are found in all major engineering disciplines and include mechanical,



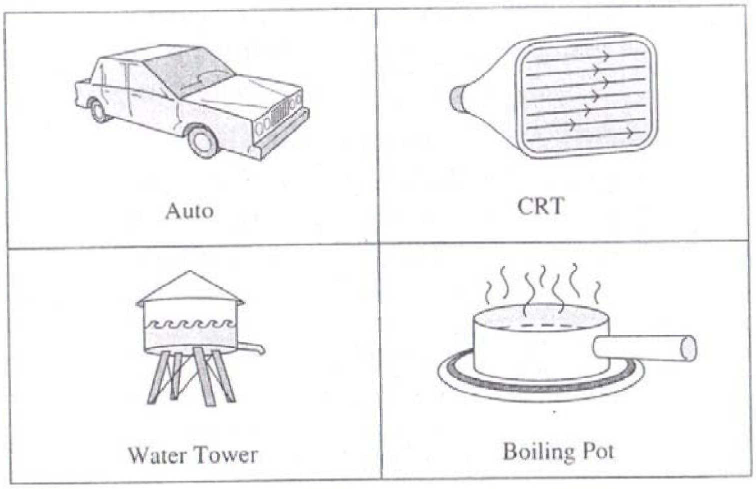


Figure 1.1 Examples of dynamic systems.

electrical, fluid, and thermal systems. They can be observed as well in natural systems (ecological, biological, economic, traffic, etc.); but while these have a dynamic behavior that is similar to that of engineering systems, they are not treated here.

1 - **Mechanical Systems.** Systems that possess significant mass, inertia, and spring and energy dissipation components driven by forces, torques, specified displacements are considered to be **mechanical systems**. An automobile is a good example of a dynamic mechanical system. It has a dynamic response as it speeds up, slows down, or rounds a curve in the road. The body and the suspension system of the car have a dynamic response of the position of the vehicle as it goes over a bump. An airplane in flight has a dynamic response of its speed and altitude as it maneuvers in the air. A paint shaker at the hardware store, with its unbalanced motor suspended on springs, provides a dynamic response of the position of the frame when the device is in use. A musical drum has a dynamic response or vibration of the position of the membrane. The structural frame of a building may have a dynamic response or vibration due to external loading, such as wind forces or ground motions. dif Ex.

2 - **Electrical Systems.** **Electrical systems** include circuits with resistive, capacitive, or inductive components excited by voltage or current. Electronic circuits can include transistors or amplifiers. We need not look far to find good examples of electrical systems with important dynamic response characteristics. A television receiver has a dynamic response of the beam that traces the picture on the screen of the set. The TV tuning circuit, which allows you to select the desired channel, also has a dynamic response, and a simpler, though no less important, example is the dynamic voltage and current responses that occur when you switch a light on or off.

3 - **Fluid Systems.** **Fluid systems** employ orifices, restrictions, control valves, accumulators (capacitors), long tubes (inductors), and actuators excited by pressure

or fluid flow. A city water tower has a dynamic response of the height of the water as a function of the amount of water pumped into the tower and the amount being used by the citizens. If a garden hose is suddenly blocked at its end when water is flowing through it, the pressure in the hose will have a dynamic response. Airflow across a cavity in a tube will cause a dynamic response (generate an acoustic tone) in an organ pipe. A water pump with an accumulator to damp out pulsations will have a dynamic response of the output pressure when in use.

4- **Thermal Systems.** **Thermal systems** have components that provide resistance (conduction, convection, or radiation) and capacitance (mass and specific heat) when excited by temperature or heat flow. A heating system warming a house has a dynamic response as the temperature rises to meet the set point on the thermostat. Placing a pot of water over a burner to boil has a dynamic response of the temperature. The size of the pot, the material it is made of, the amount of water in the pot, and the size of the burner all play a role in how quickly the water comes to a boil.

5- **Mixed Systems.** Some of the more interesting dynamic systems use two or more of the previously mentioned engineering disciplines, with energy conversion between the various components. Figure 1.2 shows several examples.

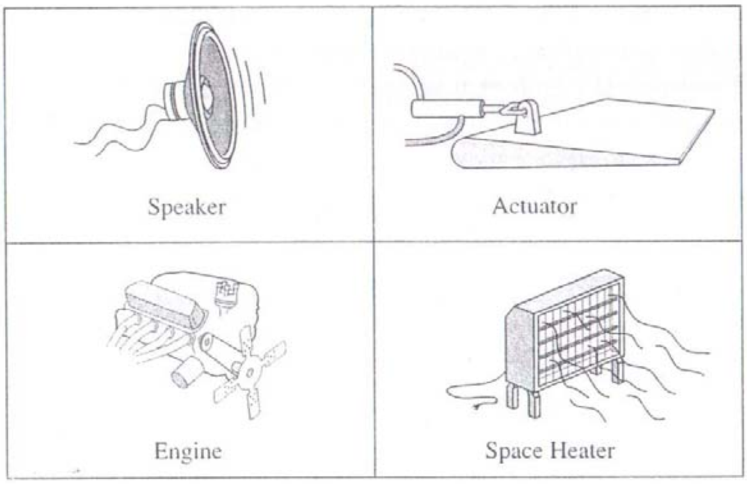


Figure 1.2 Examples of mixed systems.

5-1 ***Electro-Mechanical.*** Systems employing an electromagnetic component that converts a current into a force generally have a dynamic response. Examples are a loudspeaker in a stereo system, a solenoid actuator, and electric motors. In a loudspeaker, electrical current from the amplifier is transformed into movement of the speaker cone and the subsequent air pressure fluctuations that cause us to hear the amplified sound.

5-2 ***Fluid-Mechanical.*** Hydraulic or pneumatic systems with fluid-mechanical conversion components exhibit dynamic behavior. Examples are a hydraulic pump, a valve-controlled actuator, and a hydraulic motor drive. A hydraulic servo system used for flight control in an airplane is a good example of a common electro-fluid-mechanical dynamic system.

S-3 ***Thermo-Mechanical.*** A combustion engine used in a car, truck, ship, or airplane is a thermo-fluid-mechanical (or simply, thermomechanical) device, since it converts thermal energy into fluid power and then into mechanical power. Thermal dynamics, fluid dynamics, and mechanical dynamics are all involved in the process.

S-4 ***Electro-Thermal.*** A space heater that uses electric current to heat a filament, which in turn warms the air, has a dynamic response to the surrounding environment. An electric water heater is another common example of an electrothermal dynamic system.

1.1.2 Definitions Related to Dynamic Systems

Modeling is the process of identifying the principal physical dynamic effects to be considered in analyzing a system, writing the differential and algebraic equations from the conservation laws and property laws of the relevant discipline, and reducing the equations to a convenient differential equation form.

A **system** is a set of interacting components connected together in such a way that the variation or response in the state of one component affects the state of the others. In this text, "system" refers to a collection of components from the major engineering disciplines.

The **major disciplines** of engineering systems are mechanics, electricity and electronics, fluid mechanics and fluid controls (including hydraulics and pneumatics), and thermodynamics. Magnetism and optics also involve dynamic systems, but are not covered here.

The behavior of a system is characterized by its **response** to external inputs, disturbances, and initial conditions. Figure 1.3 shows this relationship. By **outputs,** we mean the dependent variables of the differential equation that represent the response of the system. By **inputs,** we mean functions of the independent variable of the differential equation, the excitation, or the forcing function to the system. By **external disturbances** or **perturbations,** we mean those external environmental effects

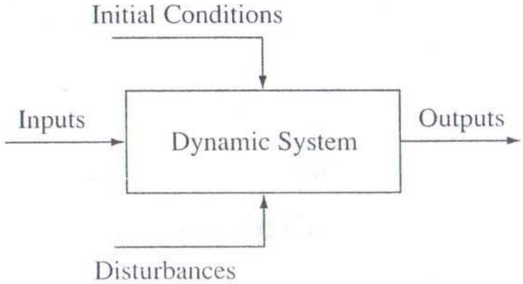


Figure 1.3 Excitation and response of a system.

that may occur randomly or unexpectedly. The **initial conditions** are the initial values of the dynamic variables of the system. The **dynamic variables** of a system are those variables whose time derivatives appear in the governing equations.

As an example of a system and its response, consider a vehicle traveling down the road and passing over a bump. The positions of the wheel and the body of the vehicle relative to the ground could be the system variables. Their differential equations could be written from a knowledge of the spring rates, mass, and damping values of the vehicle's components. The initial conditions would be the values of these variables just before the vehicle hits the bump. The bump would be the input to the system, and any aerodynamic turbulence could be considered a disturbance. From these considerations, it is possible to develop equations and solve them for the outputs, i.e., the displacements of the wheel and body. The maximum stresses in the springs could then be found, as could other critical performance parameters necessary in the design of the vehicle.

A **dynamic system** is described by time-differential equations; therefore, the future response of the system is determined by the present state of the system (the initial conditions) and the present input. Thus, a dynamic system may continue to have a time-varying response after the inputs are held constant. In contrast, a **static system** is described by algebraic equations, so that the present response of the system is totally determined by the present value of the input, and there will be no change in the response in the future if the input is held constant.

The **transient response** of a dynamic system to an external input refers to the behavior of the system as it makes a transition from the initial condition to the final condition. The transient response is expressed as a function of time. Figure 1.4 shows a typical dynamic response of a system. A dynamic system will reach a **steady state** after all of the transients have died out. The time it takes to reach the steady state is

settling time

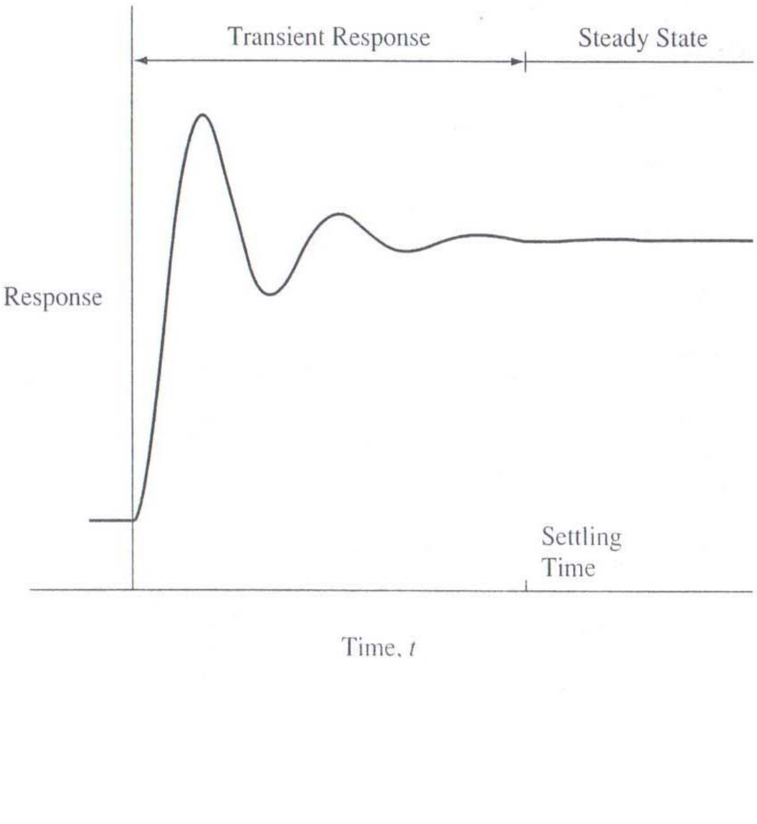


Figure 1.4 Typical dynamic response and settling of a system.

Differential Equations

* classical differential equations

$$\frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = f(t)$$

response variable

external input

$x(0) = x_0$
$\frac{dx}{dt}(0) = \dot{x}_0$ } Initial Values

$$\ddot{x} + a_1 \dot{x} + a_0 x = f(t)$$

Let $D = \frac{d}{dt}$ $\quad \left[D^2 + a_1 D + a_0\right] x(t) = f(t)$

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* Transfer function

It is the ratio of output to input at zero initial Conditions

$$\left[s^2 + a_1 s + a_0\right] x(s) = F(s)$$

$$\text{transfer function} = \frac{O/P}{I/P} = \frac{X(s)}{F(s)} = \frac{1}{s^2 + a_1 s + a_0} \quad \text{in s-domain}$$

$$= \frac{x(t)}{f(t)} = \frac{1}{D^2 + a_1 D + a_0} \quad \text{in t-domain}$$

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* state space differential equations

They form a set of simultaneous first order differential eqns

The state variables are dependent variables

They represent the dynamic response variables of the system

$$\dot{x} = a_1 x + a_2 y + f(t)$$

$$\dot{y} = a_3\, x y \sin(x) + g(t)$$

$$\dot{z} = a_4\, x z + a_5 e^{-yt} + h(t)$$

$$x(0) = x_0 \; , \quad y(0) = y_0 \quad , \quad z(0) = z_0$$

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* The order of D.E. or dynamic system

It's given by the number of independent derivatives in the system [in all equations].

$\ddot{x} + a_1 \dot{x} + a_0 x = f(t)$ ⟶ 2nd order $\ddot{x}, \dot{x}$

previous state space eqns ⟶ 3rd order $\dot{x}, \dot{y}, \dot{z}$

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* A linear Function

It has two properties

① $f(ax) = a\, f(x)$

② $f(x_1 + x_2) = f(x_1) + f(x_2)$

$\hookrightarrow f(a_1 x_1 + a_2 x_2) = a_1 f(x_1) + a_2 f(x_2) \Longrightarrow$

* A straight line through the origin is a linear function. But $y = mx + b$ is not a linear function.

* A linear Combination $L = ax + by$

* A linear differential equation $L = ax + b\dot{x} + c\ddot{x}$ if a, b, c are Constants

A **linear system** is described by linear algebraic and differential equations. By contrast, a **nonlinear system** has nonlinear combinations of the variables and their derivatives. Examples of nonlinear functions are the product of two variables, the square of a variable, a trigonometric function of a variable, and so on. Equations (1.8b) and (1.8c) are examples of nonlinear differential equations.

The **analytic solution** of a differential equation is the mathematical expression of the dependent variable as a function of time and may include exponentials, sinusoids, and/or other mathematical functions. An analytic solution of a given differential equation requires knowledge of the initial conditions and the inputs as explicit functions of time. Analytic solutions are found by employing techniques for solving classical differential equations or by using Laplace transform techniques. (See Appendices E and F for a discussion of these methods.)

Linear differential equations are well understood, and their analytical solutions usually can be obtained by applying the widely accepted methods that are discussed in a course in elementary differential equations. Nonlinear systems, on the other hand, with the exception of a few first-order systems and a limited number of second- or higher order systems, do not have known analytic solutions. If an analytic solution is not possible for a nonlinear system, a numerical approximation to the solution of the nonlinear differential equation might be found by using appropriate simulation methods. We call such an approximation a computational solution.

Computational solutions to differential equations can be found by numerical integration, using a digital computer. Numerical integration is the process of computing an approximate solution to the integral of a derivative function by a numerical algorithm. The algorithm propagates the solution of the differential equation by using small increments in time. Thus, the solution of the differential equation is known only at certain discrete times. Computational methods commonly employ the state-space representation of differential equations. Calculation of the response of a dynamic system in this way is commonly called **digital simulation**.

In **analog computation,** the differential equation is represented by an interconnection of linear and/or nonlinear electrical components and electronic integrators (operational amplifiers with capacitive feedback). Since the equations that govern the electrical system are the same as the equations that govern the dynamic system under consideration, an analogy between the two systems is formed. The electronic integrators then "solve" the differential equation by executing an electrical dynamic behavior corresponding to that of the system being studied. Many systems can be simulated by making an "analogy" between the voltages displayed by an analog computer and the variables of the equations being solved.

1.2 MODELING OF DYNAMIC SYSTEMS

1.2.1 Steps in Modeling and Representing Dynamic Systems

Modeling sequence and levels of representation.