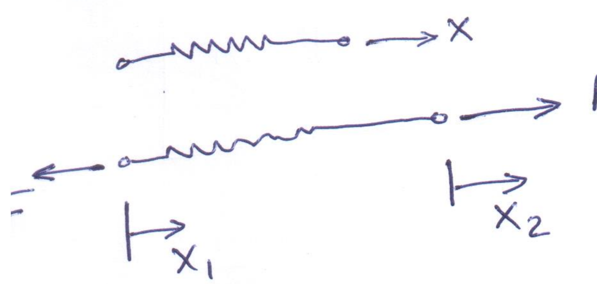
chapter 3

Modeling of mechanical systems

The physical behavior of each mechanical element, together with newtons laws of motion, provides the fundamental principles governing the development of suitable models for mechanical systems.

Translational systems

* springs

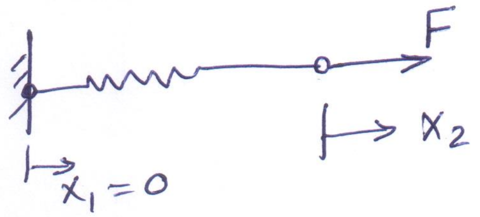


spring constant (N/m)

$$\boxed{F = K(X_2 - X_1)}$$

for a linear springs

EX.
A tensile force of 350N is applied statically to the free end of a linear spring that is fixed at the other end as illustrated in the figure below. The spring constant is 2000 N/m. Find the resulting deflection



$$F = K(X_2 - X_1)$$

$$350 = 2000(X_2 - 0)$$

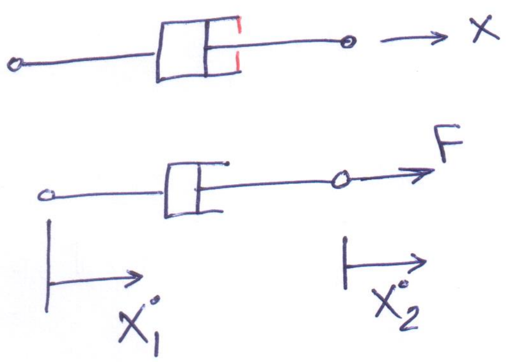
$$\therefore X_2 = \frac{350}{2000} = 0.175 \text{ m}$$

* for non-linear springs

* a hardening spring $\quad F = K(X_2 - X_1)^3$

K: spring constant (N/m^3)

* Dampers (dashpot)



damping Constant (N.s/m)

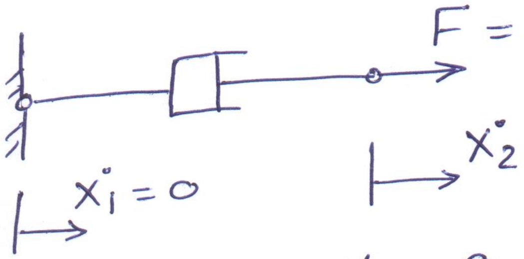
$$\boxed{F = b(\dot{x}_2 - \dot{x}_1)}$$

for a linear damper

In viscous dampers, the generated force is due to pressure drop across a fluid resistor.

Ex.

A viscous damper element is initially at rest, with its left end constrained from motion. A Constant force of 340 N is applied to the other end. what displacement has occured at the right end after this force has been applied for 1.5 sec if the viscous damping Constant is 60 N.s/mm ??



$b = 60$ N.s/mm

$x_2 = ??$ after $t = 1.5$ sec

$x_2 = 0$

at $t=0$ $\quad x_1 = 0$, $\quad \dot{x}_1 = 0$

$$F = b(\dot{x}_2 - \cancel{\dot{x}_1}^{\,0})$$

$$340 = 60\, \dot{x}_2$$

$\therefore \dot{x}_2 = 5.667$ mm/sec

$$\dot{x} = \frac{dx}{dt}$$

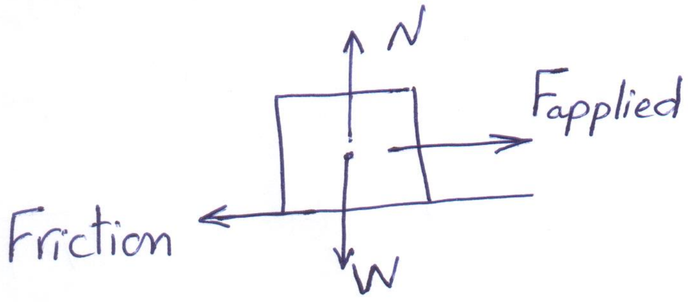
$$\int dx = \int \dot{x}_2\, dt$$

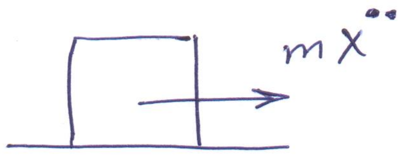
$\therefore x_2 = \dot{x}_2 \Delta t$

$= 5.667 * 1.5 = 8.5$ mm

Couloumb friction represents the sliding friction between two surfaces. This friction depends on the Coefficient of friction and the normal force that posses the two surfaces together.

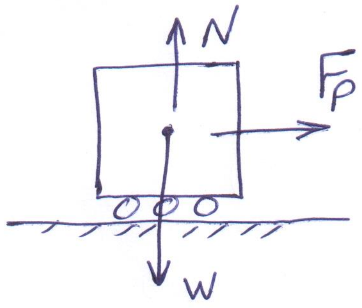
* Discrete Mass

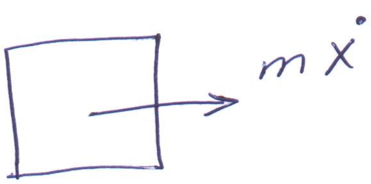




Ex.

A Carge Container is being loaded onto an airplane. The Container has a mass of 95 Kg. Find its accel. if a force $F_p = 12\,N$ is applied to push it. How far would the Container move if the Constant pushing force applied for 5 sec ? It's reasonable to neglect the force of friction action on the Container because it is supported by rollers.





$m = 95\,Kg$

$F_p = 12\,N$

$X = ??$

$t = 5\,sec$

$Friction = 0$

$\ddot{X} = ??$

$$\Sigma F = m\ddot{X}$$

$$F_p = m\ddot{X}$$

$$12 = 95\,\ddot{X}$$

$$\therefore \ddot{X} = \frac{12}{95} = 0.126\ m/s^2$$

$$\frac{d\dot{x}}{dt} = \ddot{x}$$

$$\int d\dot{x} = \int \ddot{x}\, dt$$

$$\dot{x} = 0.126\,t + C_1$$

at $t = 0$ $\quad \dot{x} = 0 \quad \therefore C_1 = 0$

$$\frac{dx}{dt} = \dot{x}$$

$$\int dx = \int 0.126\,t\, dt$$

$$x = 0.126\,\frac{t^2}{2} + C_2$$

at $t = 0$ $\quad x = 0 \quad \therefore C_2 = 0$

$$\therefore \boxed{x = 0.0632\,t^2}$$

at $t = 5$ sec $\quad x = 1.5789$ m

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* Modeling Translational systems

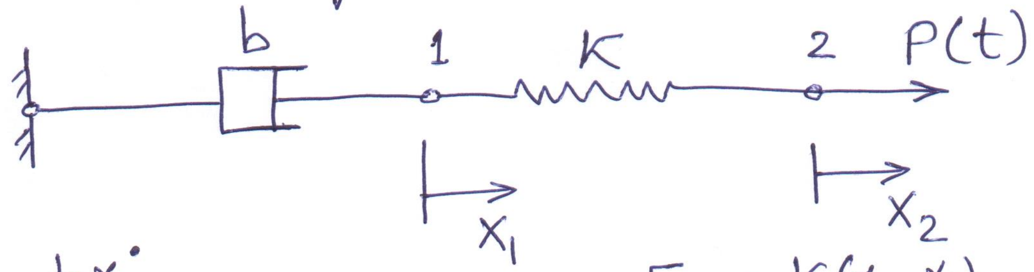
systems with Combinations of :-

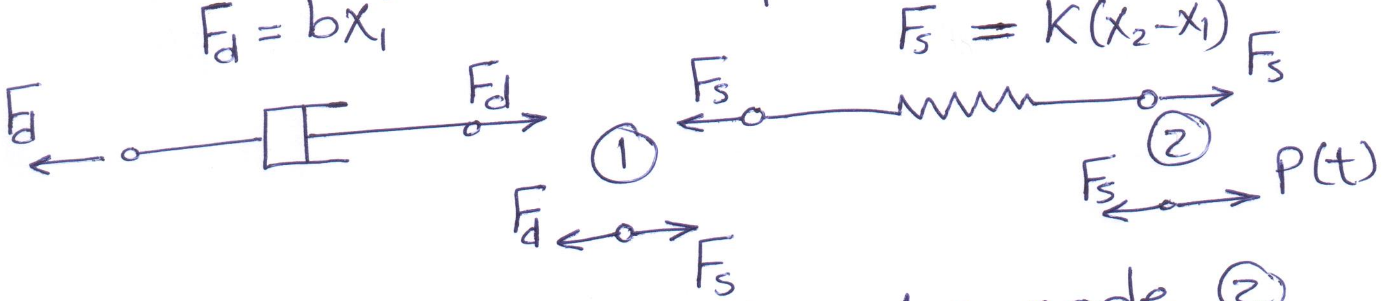
* spring
* dissipation
* discrete mass elements.

EX.

A Component of a photocopy machine is modeled as a viscous damper connected to a linear spring, as shown in figure. The mass of the part is judged to be negligible. The spring has a displacement $x_2(t)$ prescribed at its free end. Find the equations governing the displacement of the node at which the spring and damper joint together and expression for the force p(t) that must be applied to cause the motion of the components.

soln





masses are neglected

for node 1

$$\Sigma F = m\ddot{x}$$

$$F_s - F_d = 0$$

$$F_s = F_d$$

$$K(x_2 - x_1) = b\dot{x}_1 \quad \therefore b\dot{x}_1 + Kx_1 = Kx_2$$

$$\therefore \left(\frac{b}{K} D + 1\right) x_1 = x_2 \longrightarrow (2)$$

for node 2

$$\Sigma F = m\ddot{x}$$

$$P(t) - F_s = 0$$

$$P(t) = F_s$$

$$\therefore P(t) = K(x_2 - x_1) \longrightarrow (1)$$

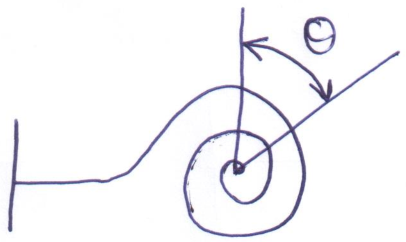
as $x_2(t)$ is a known quantity

sub. in (2) to get $x_1(t)$

sub. in (1) to get P(t)

* Rotational Systems

* Rotational springs



- "clock spring"
- "fixed end shafts"

for linear torsional springs

$$\boxed{T = K\,\theta}$$

T: torque (N.m)

K: torsional spring Constant $\left(\frac{N.m}{\cancel{rad}}\right)$

θ: deflection angle of twist (radians)

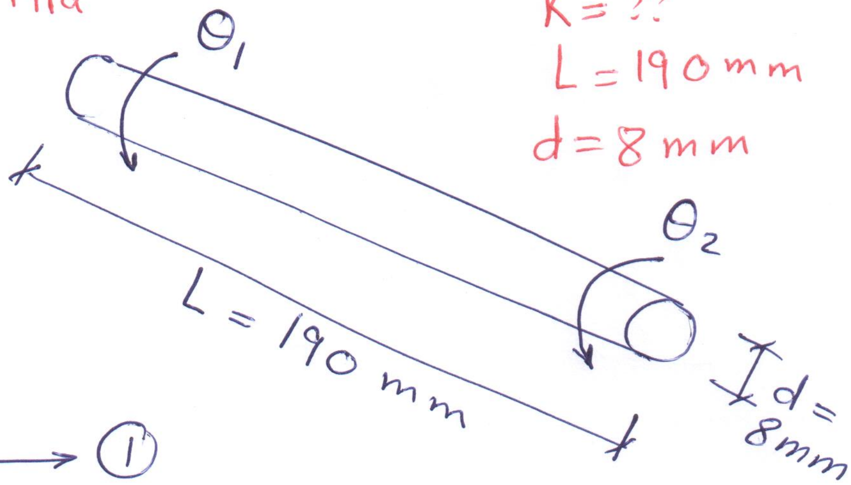
Ex.

Find the torsional spring Constant for a steel shaft that is 190 mm long and has a circular cross-section that is 8 mm diameter. The shear modulus of steel is 85 GPa. $G = 85 * 10^3\ MPa$

$K = ??$

$L = 190\ mm$

$d = 8\ mm$



soln

$$\boxed{\theta = \frac{TL}{GJ}}$$

$$K = \frac{T}{\theta} = \frac{GJ}{L} \longrightarrow (1)$$

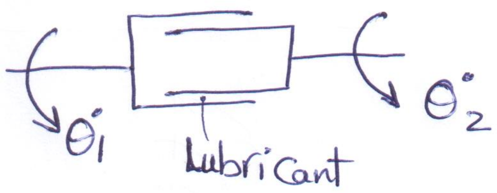
$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(8) = 128\pi\ mm^4$$

sub. in (1)

$$K = \frac{85 * 10^3 * 128\pi}{190} = 179897.516$$

$$\simeq 180 * 10^3\ N.mm/\cancel{rad}$$

* Rotational dampers



$$\boxed{T = b\,(\dot{\theta}_2 - \dot{\theta}_1)}$$

b: torsional damping coefficient $\left(\frac{N.m.s}{\cancel{rad}}\right)$

* Discrete Inertias

It is the resistance of an object to angular accel. It depends on the mass and the geometry of the object.

$$\boxed{\Sigma M_x = J_x\,\ddot{\theta}_x}$$ Newton's 2nd law

ΣM_x: sum of moments

J_x: discrete inertia $\left(\frac{N.m.s^2}{\cancel{rad}}\right) = Kg\,m^2$

$\ddot{\theta}_x$: angular accel.

Ex.

A uniform brass ($\rho = 8.5 * 10^3\ Kg/m^3$) disc of 250 mm diameter and 125 mm length is supported on a shaft as shown in figure below. The disk is spinning at a constant angular rate of 42 rad/sec when a constant breaking torque of 1.2 N.m is applied. what is the resulting angular accel?? what is the angular speed of the disk if the torque is held constant for 7.5 sec??

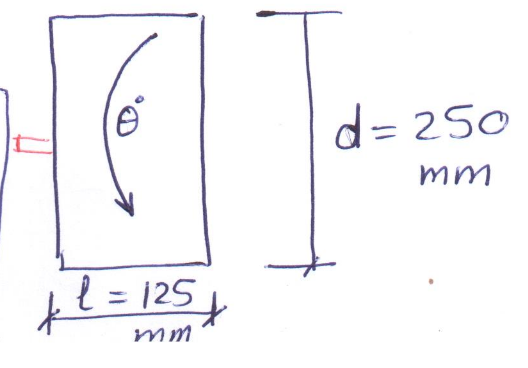
soln

$\dot{\theta}_1 = 42$ rad/sec

$T_b = 1.2$ N.m

$\ddot{\theta} = ??$

$\dot{\theta} = ??$ at $t = 7.5$ sec



$$\rho = \frac{m}{V}$$

$$m = \rho V = \rho A l$$

$$= 8.5 * 10^3 * \frac{\pi}{4}\left(\frac{250}{1000}\right)^2 * \frac{125}{1000}$$

$$= 52.16 \text{ Kg}$$

$$J_x = \frac{1}{2} m r^2 = \frac{m d^2}{8} = \frac{52.16 * (0.250)^2}{8}$$

$$= 0.4075 \text{ Kg.m}^2$$

$$\boxed{\Sigma M_x = J_x \ddot{\theta}}$$

$$\ddot{\theta} = \frac{T_b}{J_x} = \frac{-1.2}{0.4075} = -2.945 \text{ rad/sec}^2$$

$$\frac{d\dot{\theta}}{dt} = \ddot{\theta}$$

$$\int d\dot{\theta} = \int \ddot{\theta}\, dt$$

$$\dot{\theta}_2 - \dot{\theta}_1 = -2.945 \,\Delta t$$

$$\dot{\theta}_2 = \dot{\theta}_1 - 2.945 * 7.5$$

$$= 42 - 22.08 = 19.92 \text{ rad/sec}$$

$$\dot{\theta}_2 = 19.92 * \frac{60}{2\pi} = 190.2 \text{ rpm}$$