Electrical Systems

Passive circuits

- They respond to an applied voltage or current.
- They don't have any amplifiers that require an active source of power to work
- RLC circuits

Active circuits

- They have transistors and/or amplifiers that require an active source of power to work

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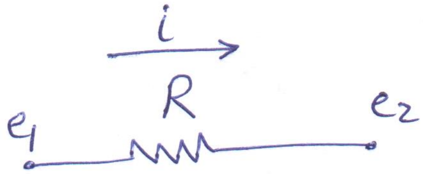
* Basic elements

1) Resistance



$$\delta e = Ri$$

$$e_1 - e_2 = Ri$$

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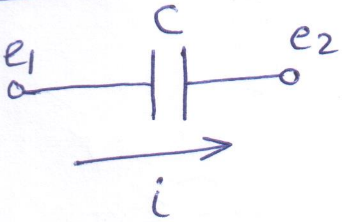
2) Capacitance



$$i = C \frac{d\delta e}{dt} \quad \text{as} \quad \delta e = e_1 - e_2$$

$$\delta e = \frac{1}{C} \int_{-\infty}^{t} i \, dt$$

$$= e_0 + \frac{1}{C} \int_{0}^{t} i \, dt$$

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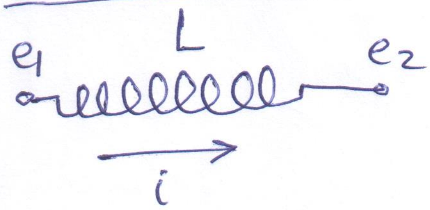
3) Inductance



$$\delta e = L \frac{di}{dt}$$

$$\text{as} \quad \delta e = e_1 - e_2$$

4) Impedance

is defined as the instantaneous ratio of the voltage difference to the current $Z = \frac{\delta e}{i}$

$$Z_r = R$$

$$Z_c = \frac{1}{CD} \qquad \frac{1}{D} \text{ integral operator}$$

$$Z_L = LD$$

5) Series and parallel Impedance Combinations

* series

$$R_{eq} = R_1 + R_2 + \cdots + R_n \qquad \text{Resistance}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} \qquad \text{Capacitance}$$

$$L_{eq} = L_1 + L_2 + \cdots + L_n \qquad \text{Inductance}$$

* parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \qquad \text{Resistance}$$

$$C_{eq} = C_1 + C_2 + \cdots + C_n \qquad \text{Capacitance}$$

$$L_{eq} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n} \qquad \text{Inductance}$$

* passive circuit analysis (RLC + voltage + current sources)

* Kirchhoff's current law

It states that the algebraic sum of all current at a node is zero.

Similarly, it is not necessary to write component equations for the components connected to current sources (either the positive or the negative node) to derive the system response equations. The only purpose for writing these equations is to determine the voltage at the current source if you need to know it.

The component equations for the *RLC* components are written in terms of the voltage drop across the component and the current through the component, using an impedance relationship. The impedance is written with *D*-operator notation so that differential equations can be derived directly.

In order to perform this type of circuit analysis, we must first draw the circuit, label the voltages at each node, and assign a variable and a polarity for the current in each component. Next, the component equations are stated in impedance form, and all of the significant node equations are written. The resulting equations are then reduced or manipulated to obtain algebraic or differential equations. This basic procedure is employed throughout the chapter.

4.3.2 Resistance Circuits

We seldom have to analyze purely resistive circuits; however, some that commonly occur are worthwhile to mention.

Voltage Divider. One of the most basic resistive circuits is the so-called voltage divider circuit, formed by two resistors in series. In the circuit shown in Figure 4.2, a voltage e_0 is applied across two resistors, R_1 and R_2, in series. Of interest is the voltage e_1 between the two resistors.

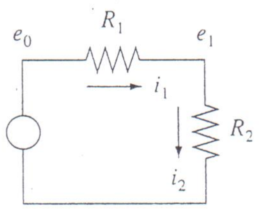


Figure 4.2 Voltage divider circuit.

The currents in the resistors are given by the basic resistance equations (voltage drop divided by resistance):

$$i_1 = \frac{e_0 - e_1}{R_1} \tag{4.52}$$

$$i_2 = \frac{e_1 - 0}{R_2} \tag{4.53}$$

The node equation at the e_1 node is

$$i_1 - i_2 = 0 \tag{4.54}$$

The two component equations for the resistors can be substituted into the node equation:

$$\frac{e_0 - e_1}{R_1} - \frac{e_1}{R_2} = 0 \tag{4.55}$$

Rearrangement of this equation yields the following equation for the voltage divider:

$$e_1 = \frac{R_2}{R_1 + R_2} e_0 \tag{4.56}$$

This equation should be memorized because you will most likely use it or a variation of it many times.

Normalization of the preceding equation reveals that the output voltage e_1 is simply a fraction of the supply voltage e_0. Thus, the output voltage can vary from zero (when R_2 is very small compared to R_1) to e_0 (when R_2 is extremely large compared to R_1):

$$e_1 = \frac{1}{1 + \frac{R_1}{R_2}} e_0 \tag{4.57}$$

The voltage divider circuit is common in volume or gain control applications, such as the volume control on a stereo, or the gain or offset in an amplifier circuit, in which case it is common to use a potentiometer, or "pot," to adjust the attenuation or gain. In a pot, a wiper is used to make contact at variable locations along a fixed resistor, as illustrated in Figure 4.3. In this case, $R_1 + R_2$ is a constant, but their ratio can be adjusted.

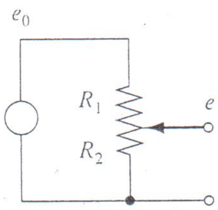


Figure 4.3 Illustration of a pot being used as a voltage divider.

Current Divider. Figure 4.4 illustrates a current divider, in which the current from the current source is split between the load resistance R_L and the shunt resistor R_s.

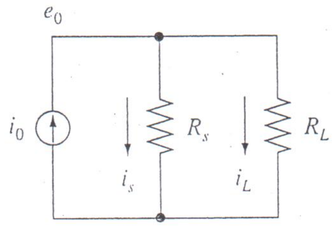


Figure 4.4 Current divider.

In this case, the node equation reveals that the source current i_0 divides between the two resistors:

$$i_0 = i_s + i_L \tag{4.58}$$

The component equations relate the actual voltage at the current source to the component currents:

$$i_s = \frac{e_0}{R_s} \tag{4.59}$$

$$i_L = \frac{e_0}{R_L} \tag{4.60}$$

Substituting these two component equations into the node equation yields

$$i_0 = \frac{R_s + R_L}{R_s R_L} e_0 \tag{4.61}$$

Again using the load current equation in this equation, we obtain the final result:

$$i_L = \frac{R_s}{R_s + R_L} i_0 = \frac{1}{1 + R_L/R_s} i_0 \tag{4.62}$$

Notice that if R_L/R_s is zero, then all of the source current will flow into the load resistor. As R_L/R_s approaches infinity, no current will go to the load resistor. If R_L is equal to the source resistance, half of the current will go to the load.

Summing Circuit. Figure 4.5 illustrates a summing circuit, in which the output voltage e_3 is the sum of two input voltages, e_1 and e_2.

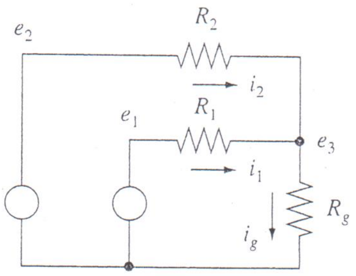


Figure 4.5 Voltage summing circuit.

The three component equations can be written in terms of the indicated voltages and currents:

$$i_1 = \frac{e_1 - e_3}{R_1} \tag{4.63}$$

$$i_2 = \frac{e_2 - e_3}{R_2} \tag{4.64}$$

$$i_g = \frac{e_3}{R_g} \tag{4.65}$$

The node equation at the interconnection of the three components follows:

$$i_1 + i_2 - i_g = 0 \tag{4.66}$$

Substituting the component equations into the node equation, we obtain

$$\frac{e_1 - e_3}{R_1} + \frac{e_2 - e_3}{R_2} - \frac{e_3}{R_g} = 0 \tag{4.67}$$

Rearrangement yields the following final result, which illustrates that the output voltage is indeed the sum of the two applied voltages:

$$e_3 = \frac{e_1}{\left(1 + \dfrac{R_1}{R2} + \dfrac{R_1}{Rg}\right)} + \frac{e_2}{\left(1 + \dfrac{R_2}{R1} + \dfrac{R_2}{Rg}\right)} \tag{4.68}$$

Notice that each voltage is attenuated from the applied value; for example, if all three resistors were equal, then the output would be one-third of the applied voltage. In practice, R_g should be smaller than R_1 and R_2 to help isolate the two input voltages, and a greater attenuation will result.

Bridge Circuit. Bridge circuits are used in many sensing applications. In a strain gauge circuit, the electrical resistance in one or more of the branches or legs of the bridge varies with the strain of the metal or surface to which the gauge is rigidly attached. This change in resistance causes a change in the voltage differential, which can then be correlated to the strain. Figure 4.6 illustrates a typical bridge circuit.

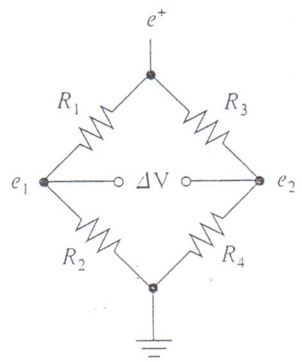


Figure 4.6 Full bridge circuit.

The bridge is composed of two voltage dividers, so the differential voltage Δe can be expressed as the difference in e_1 and e_2:

$$\Delta e = e_1 - e_2 \tag{4.69}$$

$$\Delta e = \left[\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right]e^+ \tag{4.70}$$

If we observe that the resistance R_2 is a base value R_2^*, plus a small increment in resistance, δR, then we can state that

$$R_2 = R_2^* + \delta R \tag{4.71}$$

If all four resistances are equal ($R_1 = R_3 = R_4 = R_2^* = R$), then the bridge equation reduces to

$$\Delta e = \frac{\delta R}{2R} e^+ \tag{4.72}$$

The equivalent resistance from e^+ to ground can be calculated by considering two sets of series resistors operated in parallel:

$$R_{eq} = \frac{(R_1 + R_2)(R_3 + R_4)}{(R_1 + R_2) + (R_3 + R_4)} \tag{4.73}$$

If all of the resistances are equal (with value R), then the equivalent resistance is simply R.

4.3.3 Resistance-Capacitance Circuits

Capacitors are generally used in circuits either to filter out high-frequency signals or to store energy. In these cases, the settling time or the dynamic response is of interest.

***RC* Circuit.** Consider the simple RC circuit shown in Figure 4.7 in which a resistor and capacitor are connected in series. Before analyzing this circuit, we should determine what we want from the analysis. For example, we might be interested in the voltage across the capacitor as a function of the supply voltage. If we identify the voltages and currents as shown, we can write the component and node equations as follows.
component equations:

$$i_R = \frac{e_0 - e_1}{R} \tag{4.74}$$

$$i_C = CDe_1 \qquad \text{with } e_1(0) \tag{4.75}$$

node equation:

$$i_R = i_C \tag{4.76}$$

Substituting the component equations into the node equation yields

$$\frac{e_0 - e_1}{R} = CDe_1 \tag{4.77}$$

Rearranging results in

$$RCDe_1 + e_1 = e_0 \tag{4.78}$$

This equation is a classical differential equation in e_1 as a function of the input e_0. It shows that the system has a time constant $\tau = RC$, and a steady-state value e_1 equal to the supply voltage e_0. To solve the equation, the initial voltage $e_1(0)$ on the capacitor must be known.

The differential equation can also be written as a transfer function:

$$e_1 = \frac{e_0}{RCD + 1} \tag{4.79}$$

This result could have been obtained from the voltage divider circuit equation by treating the second resistance in the voltage divider as a capacitive impedance $1/CD$.

Example 4.1 ***RC* Filter**

The circuit shown in Figure 4.7 has a resistance of 8 ohms; find the capacitance necessary to give a settling time of 2.5 milliseconds.

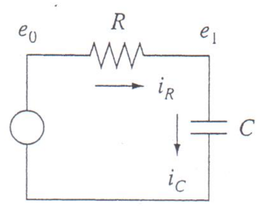


Figure 4.7 Resistor-capacitor circuit.

From the circuit, the time constant is $\tau = RC$. The settling time is 4τ; thus, $\tau = 2.5/4$ ms. Solving for the capacitance then yields

$$C = \frac{\tau}{R} = \frac{2.5/4\ 10^{-3}\,\text{s}}{8\ \text{ohm}} = 78.125\ \mu\text{f}$$

It is important to note that capacitors are not built with very precise tolerance: The value of their capacitance might be $\pm 20\%$ or worse. Therefore, don't go to the distributor and ask for a 78.125 μf capacitor. You might find a 68 μf, a 100 μf, or a 220 μf capacitor; but you won't find a 78.125 μf capacitor. If you need the settling time to be precisely 2.5 ms, then you will have to buy the closest capacitor that is smaller than the desired value and add resistance to the circuit to get the settling time exact. Note further that, since the capacitor has a wide tolerance band, you will have to trim the resistance in each circuit to get your precise settling time. However, often the settling time does not have to be exact, and a capacitor close to the desired value will be acceptable.

Dual *RC* Circuit. Figure 4.8 shows a circuit made up of two *RC* circuits. In this case, we might be interested in the voltage e_2 as a function of the input voltage e_0. The component equations are

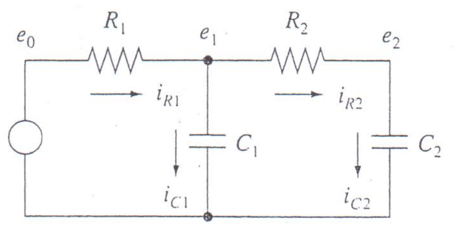


Figure 4.8 Dual *RC* circuit.

$$i_{R1} = \frac{e_0 - e_1}{R_1} \tag{4.80}$$

$$i_{C1} = C_1 De_1 \quad \text{with } e_1(0) \tag{4.81}$$

$$i_{R2} = \frac{e_1 - e_2}{R_2} \tag{4.82}$$

$$i_{C2} = C_2 De_2 \quad \text{with } e_2(0) \tag{4.83}$$

The node equations are

$$i_{R1} = i_{C1} + i_{R2} \tag{4.84}$$

$$i_{R2} = i_{C2} \tag{4.85}$$

If we substitute the component equations directly into the node equations and rearrange the result, we will have two equations involving only the voltages of the circuit:

$$[R_1C_1D + (1 + R_1/R_2)]e_1 = (R_1/R_2)e_2 + e_0 \tag{4.86}$$

$$[R_2C_2D + 1]e_2 = e_1 \tag{4.87}$$

Since we are interested in e_2 as a function of e_0, we can substitute the second equation for e_1 into the first and rearrange to obtain

$$[R_1C_1R_2C_2D^2 + (R_1C_1 + R_1C_2 + R_2C_2)D + 1]e_2 = e_0 \tag{4.88}$$

This is a second-order differential equation in e_2 as a function of the input e_0. As such, the required initial conditions are $e_2(0)$ and $De_2(0)$. Since we only know the initial charges on the capacitors, $e_1(0)$ and $e_2(0)$, we must determine the initial conditions of the differential equation. The component equation for the second capacitor gives the derivative of e_2 as a function of the voltages e_1 and e_2. Since this equation is true for all time, including time $t = 0$, the required initial condition of De_2 can be determined:

$$e_2(0) = \text{known and } e_1(0) = \text{known} \tag{4.89}$$

$$De_2(0) = \frac{e_1(0) - e_2(0)}{R_2C_2} \tag{4.90}$$

To avoid the difficulty of determining the initial conditions (which could be very difficult for higher order circuits), the governing equations for this problem could be formulated in state-space format. Notice from the original component equations that the derivative of e_1 and the derivative of e_2 appear in the equations for the capacitors. If we use e_1 and e_2 as state variables and use the two component equations for the capacitors, we have

$$De_1 = \frac{i_{C1}}{C_1} \quad \text{with } e_1(0) \tag{4.91}$$

$$De_2 = \frac{i_{C2}}{C_2} \quad \text{with } e_2(0) \tag{4.92}$$

Using the remaining two component equations and the two node equations, we can eliminate i_{C1} and i_{C2} in favor of e_0, e_1, and e_2 to obtain

$$De_1 = -\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2}\right)e_1 + \frac{1}{R_2C_2}e_2 + \frac{1}{R_1C_1}e_0 \tag{4.93}$$

$$De_2 = -\frac{1}{R_2C_2}e_2 + \frac{1}{R_2C_2}e_1 \tag{4.94}$$

In this case, the required initial conditions are $e_1(0)$ and $e_2(0)$, which are the known initial charges on the capacitors.

4.3.4 Resistance-Inductance Circuits

Solenoids, relays, automotive ignition coils, and automotive fuel injectors are all examples of components that exhibit RL circuit behavior. The inductance comes from the coil of wire that is used to make the magnetic circuit, and the resistance is usually due to the resistance of the small wire used in the coil.

Often, the solenoid is driven with a constant voltage source, in which case we would be interested in the current in the inductor and how long it would take to reach a steady state. For example, consider the circuit shown in Figure 4.9, which illustrates a solenoid with an inductance L and a parasitic resistance R.

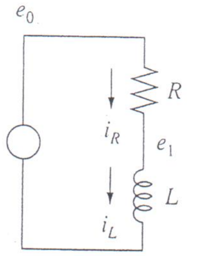


Figure 4.9 Solenoid circuit.

The equations for the components are

$$i_R = \frac{e_0 - e_1}{R} \tag{4.95}$$

$$i_L = \frac{e_1}{LD} \tag{4.96}$$

The node equation is quite simple:

$$i_R = i_L \tag{4.97}$$

Since we want to derive an equation for i_L as a function of the applied voltage e_0, we need to eliminate i_R and e_1. Substituting the node equation and the resistance equation into the inductance equation and rearranging yields

$$\left[\frac{L}{R}D + 1\right]i_L = \frac{e_0}{R} \tag{4.98}$$

This equation reveals a system time constant of $\tau = L/R$ and a steady-state current of e_0/R.

Example 4.2 Solenoid Coil Response

A coil for a solenoid valve has a resistance of 100 ohms and an inductance of 6 mh. Calculate the time constant, settling time, and current required from a 24 volt source.

The time constant is given by the previous equation:

$$\tau = \frac{L}{R} = \frac{0.006\text{ h}}{100\text{ ohm}} = 0.06\text{ ms}$$

The settling time is 4τ or 0.240 ms. The steady-state current required is

$$i = \frac{24\text{ volt}}{100\text{ ohm}} = 0.24\text{ amp}$$

The power required for this solenoid valve is almost 6 watts.

If we drive the RL solenoid of Figure 4.9 with a voltage source that has an associated source resistance R_0, the situation is described by the circuit shown in Figure 4.10.

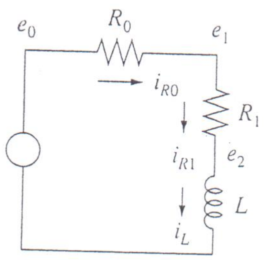


Figure 4.10 High-voltage drive for RL circuit.

The component equations are

$$i_{R0} = \frac{e_0 - e_1}{R_0} \tag{4.99}$$

$$i_{R1} = \frac{e_1 - e_2}{R_1} \tag{4.100}$$

$$i_L = \frac{e_2}{LD} \tag{4.101}$$

The node equations are

$$i_{R0} = i_{R1} \tag{4.102}$$

$$i_{R1} = i_L \tag{4.103}$$

Starting with the inductor equation, using the two resistor equations and the two node equations, and simplifying yields

$$\left[\frac{L}{R_o + R_1} D + 1\right] i_L = \frac{1}{(R_o + R_1)} e_o \tag{4.104}$$

Notice from this equation that the time constant is now $L/(R_0 + R_1)$ and the steady-state current is $e_0/(R_0 + R_1)$. If we were to increase the voltage and use a relatively high R_0, then the time constant could be improved (reduced) and the steady-state current would remain the same, resulting in a faster solenoid actuation time. The purpose of this approach is to drive the solenoid with a constant current source, in which case the electrical part of the actuation time would be very fast.

4.3.5 Resistance-Inductance-Capacitance Circuits

Even though simple passive *RC* or *RL* electrical circuits (such as that of Figure 4.8) can be represented by second or higher order differential equations, such circuits do not exhibit resonance or overshoot, nor do they induce oscillations; in other words, the roots to their characteristic equation are not complex. Complex roots are found for circuits that have a combination of *R*, *L*, and *C* components. Tuning circuits, signal filters, and similar circuits use inductors and capacitors along with resistors.

Series *RLC* Circuits. The classical *RLC* circuit is shown in Figure 4.11.

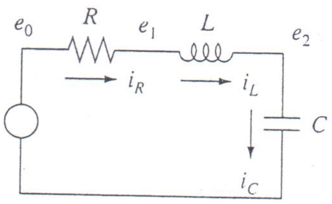


Figure 4.11 Series *RLC* circuit.

The component equations for this circuit are

$$i_R = \frac{e_0 - e_1}{R} \tag{4.105}$$

$$i_L = \frac{e_1 - e_2}{LD} \quad \text{with } i_L(0) \tag{4.106}$$

$$i_C = CDe_2 \quad \text{with } e_2(0) \tag{4.107}$$

The node equations are

$$i_R = i_L \tag{4.108}$$

$$i_L = i_C \tag{4.109}$$

Substituting the component equations into the node equations and eliminating e_1 yields the differential equation for e_2 as a function of e_0:

$$[LCD^2 + RCD + 1]e_2 = e_0 \tag{4.110}$$

This is a classical second-order differential equation, as discussed in section E.3 of Appendix E. The solution of Eq. (4.110) requires the initial conditions on e_2 and De_2, which in turn would require some manipulation of the component and node equations, since we know $i_L(0)$ and $e_2(0)$. The capacitor equation gives an expression for De_2, and by using the second node equation, we observe the following:

$$e_2(0) = \text{known and } i_L(0) = \text{known} \tag{4.111}$$

$$De_2(0) = \frac{1}{C} i_L(0) \tag{4.112}$$

This system can be put into the state-space form by noting from the component equations that the inductor current i_L and the capacitor voltage e_2 are differentiated; therefore, they can be used as state variables. Using the component equations for the inductor and the capacitor, and substituting the resistance equation and the node equations into the L and C component equations, yields the following state-space differential equations:

$$Di_L = -\frac{R}{L} i_L - \frac{1}{L} e_2 + \frac{1}{L} e_0 \tag{4.113}$$

$$De_2 = \frac{1}{C} i_L \tag{4.114}$$

The initial conditions are the natural initial conditions, $i_L(0)$ and $e_2(0)$.

Example 4.3 *RLC* Circuit

A series *RLC* circuit has an inductance $L = 1$ mh and a capacitance $C = 10\ \mu$f. Calculate the resistance R required to obtain a damping ratio of 0.707.

From the previous differential equation for this circuit, we can find the damping ratio and natural frequency as follows (see Table E–3):

$$2\frac{\zeta}{\omega_n} = RC \quad \text{and} \quad \omega_n = \frac{1}{\sqrt{LC}}$$

Thus, the resistance is

$$R = \frac{\dfrac{2\zeta}{C}}{\sqrt{LC}} = 2\zeta\sqrt{\frac{L}{C}} = 2 \times 0.707\sqrt{\frac{0.001 \text{ s ohm}}{10 \times 10^{-6}\dfrac{\text{s}}{\text{ohm}}}} = 14.14 \text{ ohms}$$

Again, you are not going to buy a resistance of 14.14 ohms; and the variance on the capacitance and inductance are so large that each circuit will require a different value of resistance to achieve a damping ratio of exactly 0.707. Therefore, you will have to settle for a damping ratio around 0.707 or trim the resistance for each circuit. You should opt for the first choice, due to the cost of trimming each circuit and the fact that the circuit responses for damping values around 0.707 are not much different from each other.

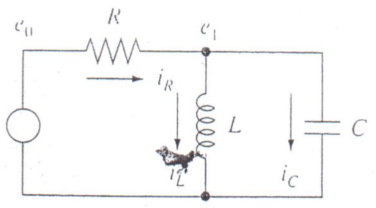


Figure 4.12 Parallel *RLC* circuit.

Parallel *RLC* Circuits. The inductor and capacitor are quite often in parallel in a circuit, as illustrated in Figure 4.12. The component equations for this circuit are

$$i_R = \frac{e_0 - e_1}{R} \tag{4.115}$$

$$i_L = \frac{e_1}{LD} \quad \text{with } i_L(0) \tag{4.116}$$

$$i_C = CDe_1 \quad \text{with } e_1(0) \tag{4.117}$$

The node equation is

$$i_R = i_L + i_C \tag{4.118}$$

Substituting the component equations into the node equation and rearranging yields the classical differential equation for the voltage e_1 as a function of e_0:

$$\left[LCD^2 + \frac{L}{R}D + 1\right]e_1 = \frac{L}{R}De_0 \tag{4.119}$$

The state-space representation of this system is found by using i_L and e_1 as state variables. Substituting the resistor equation and the node equation into the inductor and capacitor equations and rearranging, we arrive at

$$Di_L = \frac{1}{L}e_1 \tag{4.120}$$

$$De_1 = -\frac{1}{C}i_L - \frac{1}{RC}e_1 + \frac{1}{RC}e_0 \tag{4.121}$$

4.3.6 Summary of Passive Circuit Analysis Techniques

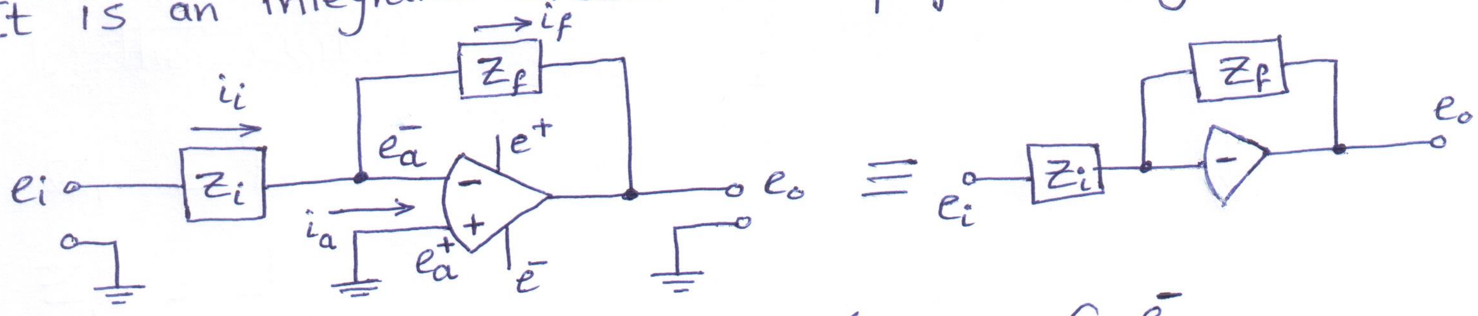
The **steps for deriving a classical differential equation** from a circuit can be stated in a simple procedure as follows:

1. Draw the schematic of the circuits, and identify each component with a unique symbol (e.g., R_1, R_2, C_1).
2. Assign a variable to represent the voltage at each node in the circuit (e.g., e_0, e_1, e_2).

* Active circuits

* operational Amplifier (op-amp)

It is an integrated circuit that amplifies voltages.



$$i_i = \frac{e_i - e_a^-}{Z_i}$$

$$i_f = \frac{e_a^- - e_o}{Z_f}$$

$$\begin{cases} e_o = -G\, e_a^- \quad \text{(amplifier equation)} \\ i_i - i_f = i_a \simeq 0 \end{cases}$$

Z_i : input impedance

Z_f : feedback ~

$$i_i = i_f$$

$$\frac{e_i - e_a^-}{Z_i} = \frac{e_a^- - e_o}{Z_f} \qquad e_a^- = -\frac{e_o}{G}$$

$$\frac{e_i}{Z_i} + \frac{e_o}{G Z_i} = \frac{-e_o}{G Z_f} - \frac{e_o}{Z_f}$$

$$\frac{e_i}{Z_i} = -e_o\left[\frac{1}{G Z_i} + \frac{1}{G Z_f} + \frac{1}{Z_f}\right]$$

$$\frac{e_i}{\cancel{Z_i}} = -e_o\left(\frac{Z_f + Z_i + G Z_i}{G \cancel{Z_i} Z_f}\right)$$

$$e_o = \frac{-G Z_f e_i}{Z_f + Z_i + G Z_i} \qquad \div Z_i G$$

$$\boxed{e_o = \frac{-\frac{Z_f}{Z_i}\, e_i}{1 + \frac{1 + \frac{Z_f}{Z_i}}{G}}}$$

$$e_o = -Ge_a \qquad \text{(amplifier equation)} \tag{4.124}$$

The node equation at the input port e_a of the amplifier is

$$i_i - i_f = i_a \approx 0 \tag{4.125}$$

Substituting the component equations into the node equation and rearranging yields

$$e_o = \frac{-\dfrac{Z_f}{Z_i} e_i}{1 + \dfrac{1 + \dfrac{Z_f}{Z_i}}{G}} \tag{4.126}$$

If the impedance ratio Z_f/Z_i is small (compared to G, which is about 10^6), then the op-amp equation reduces to the classic result,

$$e_o = -\frac{Z_f}{Z_i} e_i \tag{4.127}$$

The reader should memorize this equation, since it is used often.

4.4.2 Typical Circuits

Table 4.1 illustrates a variety of uses for the op-amp with different impedances. Notice that resistive input and feedback impedances result in a voltage amplifier, a resistive input and a capacitive feedback impedance results in an integrator, and a capacitive input and a resistive feedback impedance results in a differentiator. Other combinations of series and parallel impedances in the input and feedback impedances result in a variety of interesting transfer functions.

The op-amp circuits described so far in this section can be used in more complex circuits. The transfer functions for the op-amp circuits can be inserted in the circuit analysis, since the op-amp acts as a voltage source that provides an isolation in voltages. For example, consider an op-amp driving a solenoid, as shown in Figure 4.14. The circuit analysis for this system can be divided into two parts, since the op-amp acts as a voltage driver for the solenoid RL circuit.

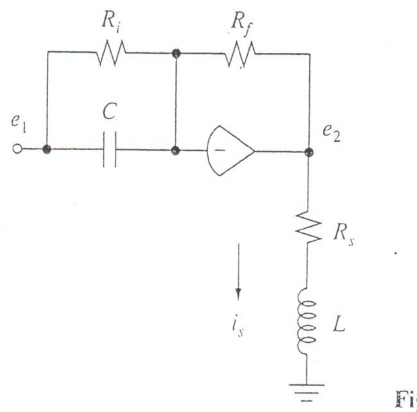
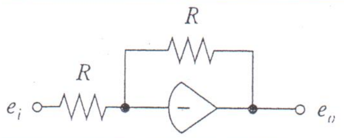
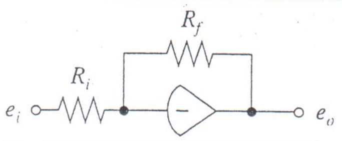
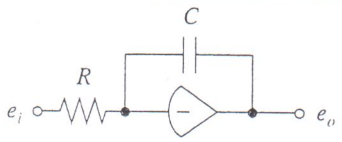
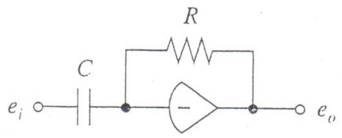
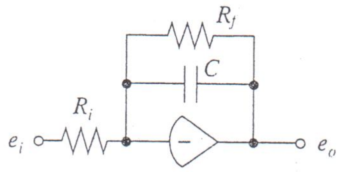
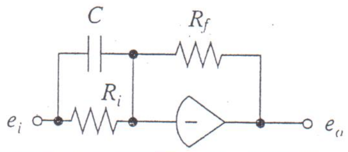
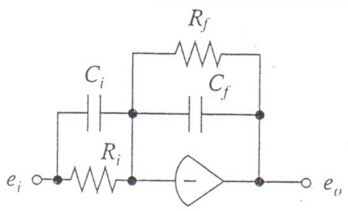
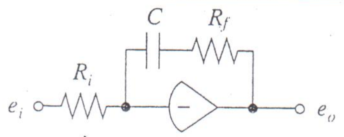


Figure 4.14 Op-amp solenoid driver.

TABLE 4.1 Op-Amp Circuits.

Description	Transfer Function	Circuit
Sign Changer	$e_o = -e_i$	
Amplifier	$e_o = -\frac{R_f}{R_i} e_i$	
Integrator	$e_o = \frac{-e_i}{\tau D}$ $\tau = RC$	
Differentiator	$e_o = -\tau D e_i$ $\tau = RC$	
Lag	$e_o = \frac{-\frac{R_f}{R_i} e_i}{(\tau D + 1)}$ $\tau = R_f C$	
Lead	$e_o = -\frac{R_f}{R_i}(\tau D + 1)\, e_i$ $\tau = R_i C$	
Lead-Lag or Lag-Lead	$e_o = -\frac{R_f}{R_i} \frac{(\tau_i D + 1) e_i}{(\tau_f D + 1)}$ $\tau_i = R_i C_i$ $\tau_f = R_f C_f$	
Bandwidth-Limited Integrator	$e_o = \frac{-(\tau_f D + 1)\, e_i}{\tau_i D}$ $\tau_f = R_f C$ $\tau_i = R_i C$	
Bandwidth-Limited Differentiator	$e_o = \frac{-\tau_f D e_i}{(\tau_i D + 1)}$ $\tau_f = R_f C$ $\tau_i = R_i C$