6.3.1 Thermal Conduction Circuits

Example 6.1 Conduction in a Rod

Consider the system shown in Figure 6.12 in which a 0.25 inch diameter steel rod serves as a structural member to locate a steam boiler and a water tank. The one foot long rod is insulated along its length to prevent convection from the surface; however, there can be conduction along the axis of the rod. The boiler temperature is 220°F, and the water tank is 100°F. The question is: How much heat will be transferred from the boiler to the water supply tank?

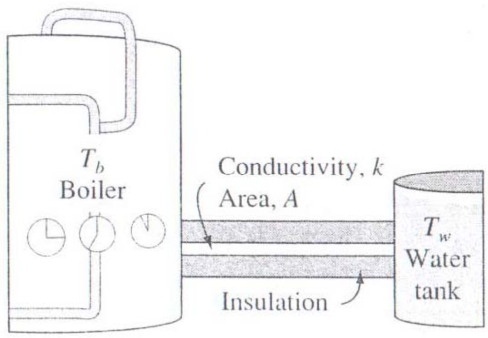

Figure 6.12 Conductivity in a structural rod.

Solution For this problem, we can use the conduction equation, Eq. (6.7), and the properties of the steel rod to obtain

$$Q_h = \frac{k_t A}{L}(T_b - T_w) \tag{6.22}$$

From Table 6.2, we find that $k_t = 23$ Btu/(hr ft °F). Therefore, the heat loss is

$$Q_h = \frac{23 \dfrac{\text{Btu}}{\text{hr ft °F}} \pi 0.25^2 \text{ in}^2 (220 - 100)\text{°F}}{4\,(1.0 \text{ ft})\, 144 \dfrac{\text{in}^2}{\text{ft}^2}} \tag{6.23}$$

or

$$Q_h = 0.941 \frac{\text{Btu}}{\text{hr}} = 0.277 \text{ watt} \tag{6.24}$$

This loss is quite small.

6.3.2 Thermal Conduction and Convection Circuits

Example 6.2 Plexiglas® plate

Consider the system shown in Figure 6.13, in which a plate of Plexiglas® that is a wall of a container is exposed to an internal temperature $T_i = 50°\text{C}$ on one side and is subjected to free convection to room temperature, 25°C, on the other side. We want to know how much heat is lost and what will be the outer surface temperature T_s. The plate is 100 mm by 100 mm and is 6 mm thick. The thermal conductivity of Plexiglas® is 0.195 W/(m °K).

Solution Since there is some very slight air motion on the outer surface, we will assume that the convection coefficient is 20 W/(m² °K).

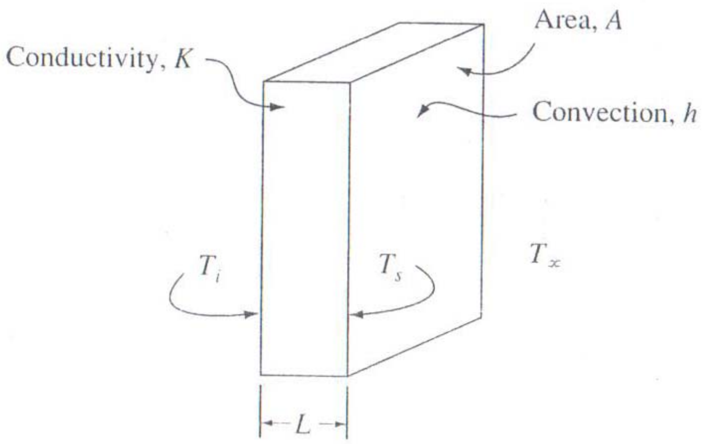

Figure 6.13 Conduction and convection from a plate.

The resistances for the conduction and convection can be calculated from Eq. (6.5) and Eq. (6.15) as follows:

$$R_{plex} = \frac{L}{kA} = \frac{0.006 \text{ m}}{0.195 \frac{W}{\text{m °K}} 0.10 \text{ m } 0.10 \text{ m}} = 3.08 \frac{°\text{K}}{\text{W}} \tag{6.25}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{20 \frac{\text{W}}{\text{m}^2 \text{ °K}} 0.10 \text{ m } 0.10 \text{ m}} = 5.00 \frac{°\text{K}}{\text{W}} \tag{6.26}$$

Therefore, the heat flow can be calculated as follows (note that $\Delta T\,°\text{C} = \Delta T\,°\text{K}$):

$$Q_h = \frac{\Delta T}{(R_{plex} + R_{conv})} = \frac{(50 - 25)\ °\text{K}}{(3.08 + 5.00) \frac{°\text{K}}{\text{W}}} \tag{6.27}$$

$$Q_h = 3.1 \text{ watts} = 1.05 \text{ Btu/hr} \tag{6.28}$$

The outer surface temperature can be calculated by first equating the heat flows from the Plexiglas® and the convection and then solving for T_s:

T⇒K

$Q_h = \frac{T_i - T_s}{R_{plex}} = \frac{T_s - T_o}{R_{conv.}}$

$$T_s = \frac{R_{conv} T_i + R_{plex} T_\infty}{R_{plex} + R_{conv}} = \frac{5.00[50°C] + 3.08[25°C]}{3.08 + 5.00} \tag{6.29}$$

$$T_s = 30.9°C + 9.5°C = 40.4°C \tag{6.30}$$

Therefore, the outer surface temperature, is closer to the internal temperature, since the resistance of the Plexiglas® is lower.

Example 6.3 Steam Pipe

Consider the system shown in Figure 6.14 in which steam at 220°F is flowing in a 6.0 foot steel pipe (2.0 inch ID, 2.25 inch OD) with free convection to room temperature, 75°F. We are considering whether we should insulate the pipe.

Solution The uninsulated pipe has three resistances to heat flow from the inside to the outside: the convection on the inside of the pipe h_i; the conduction through the steel k_s; and the convection on the outside of the pipe h_o. These resistances can be calculated from Eq. (6.15) and Eq. (6.10) as follows:

$$R_i = \frac{1}{h_i A_i} = \frac{12\frac{\text{in}}{\text{ft}}}{25\frac{\text{Btu}}{\text{hr ft}^2\,°\text{F}}\,\pi 2.0\text{ in }6\text{ ft}} = 0.0127\frac{°\text{F}}{\text{Btu/hr}} \tag{6.31}$$

$$R_s = \frac{\ln(d_o/d_i)}{2\pi k L} = \frac{0.1178}{2\pi 30\frac{\text{Btu}}{\text{hr ft }°\text{F}}\,6\text{ ft}} = 0.000104\frac{°\text{F}}{\text{Btu/hr}} \tag{6.32}$$

$$R_o = \frac{1}{h_o A_o} = \frac{12\frac{\text{in}}{\text{ft}}}{5\frac{\text{Btu}}{\text{hr ft}^2\,°\text{F}}\,\pi 2.25\text{ in }6\text{ ft}} = 0.0566\frac{°\text{F}}{\text{Btu/hr}} \tag{6.33}$$

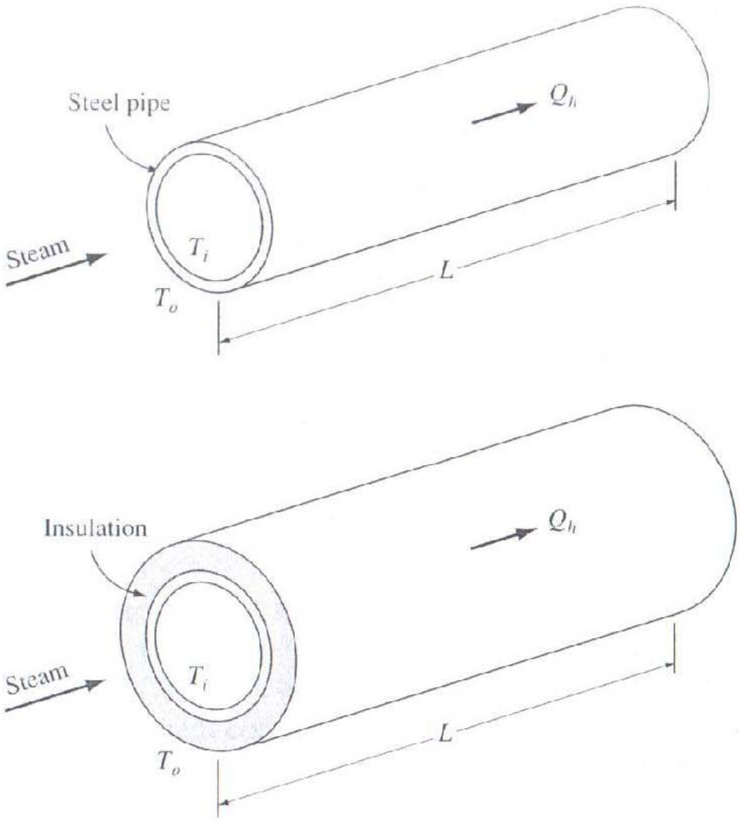

Figure 6.14 Steam pipe convection/conduction example.

Since the total thermal resistance is the sum of the individual resistances, the total heat flow can be calculated as follows:

$$Q_h = \frac{(220 - 75)°F}{(0.0127 + .000104 + .0566)} \frac{Btu/hr}{°F} \tag{6.34}$$

$$Q_h = 2089 \text{ Btu/hr} = 614 \text{ watts} \tag{6.35}$$

Now consider insulation that could be added to the outside of the pipe. If we use a wrap that has 0.50 inch thickness (3.25 OD, 2.25 ID) and a thermal conductivity of 0.06 Btu (hr ft °F), the resistance is

$$R_c = \frac{\ln(d_c/d_o)}{2\pi kL} = \frac{0.3677}{2\pi 0.06 \frac{\text{Btu}}{\text{hr ft °F}} 6 \text{ ft}} = 0.1626 \frac{°F}{\text{Btu/hr}} \tag{6.36}$$

Since the resistances are in series, they are additive, and the resulting heat transfer can be calculated as follows:

$$Q_h = \frac{(220 - 75)°F \text{ Btu/hr}}{(0.0127 + 0.000104 + 0.0566 + 0.1626)°F} \tag{6.37}$$

$$Q_h = 625 \text{ Btu/hr} = 184 \text{ watts} \tag{6.38}$$

We note that the insulation provides a 70% reduction in heat loss.

$$\frac{614 - 184}{614} = 0.7$$

6.4.1 Single-Lumped Capacitance Modeling

Many applications in engineering in which the heat capacity is obviously significant can be treated by the single-lumped capacitance analysis technique. In these cases, the Biot number is small.

Example 6.4 Watermelon Warming

Suppose that we are interested in predicting how long a watermelon, such as the one depicted in Figure 6.16, will maintain its temperature at a picnic. A 4 kg watermelon was initially cooled to 5°C, but is exposed to 30°C with free convection at the picnic. How long will it take to get to 63% of the temperature rise from 5°C to 30°C, i.e., to 20.75°C?

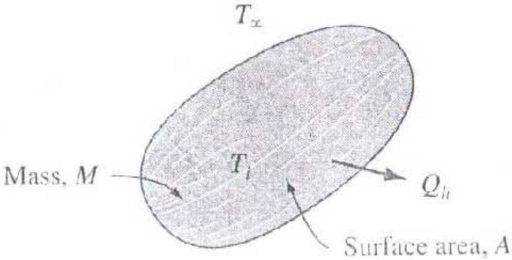

Figure 6.16 Watermelon-warming problem.

Solution The inside of a watermelon is basically water and so should have a specific heat about equal to that of water. Therefore, we will assume that the specific heat C_p is 4200 J/(kg°C) and the density is 1000 kg/m³. With free convection, h will be about 10 W/(m²°C.) Based upon the mass and density of the watermelon, the diameter of the watermelon is 0.20 m, and the surface area of the watermelon is approximately 0.12 m².

The first thing to do is calculate the Biot number. The thermal conductivity of water is 0.6 W/(m°C), so

$$N_b = \frac{hL_c}{k} = \frac{10\,\frac{W}{m^2\,°C}\,\frac{0.20\,m}{6}}{0.6\,\frac{W}{m\,°C}} = 0.547 \tag{6.41}$$

Although this Biot number is above 0.1, we still want to use a single-lumped capacitance model just to get a good estimate of time.

The mass of the melon acts as a thermal capacitor. From Eq. (6.21), the heat transfer from the capacitor is

$$Q_h = C_p M \frac{dT_i}{dt} \tag{6.42}$$

To solve this problem, we must equate the heat transfer released by the thermal capacitance to the convection heat transfer:

$$Q_h = MC_p DT_i = hA(T_i - T_\infty) \tag{6.43}$$

Equation (6.43) can be rearranged to produce a differential equation in T_i, namely,

$$(\tau D + 1)T_i = T_\infty \tag{6.44}$$

where

$$\tau = \frac{MC_p}{hA} \tag{6.45}$$

This is a first-order differential equation with a time constant of τ. The system will reach 63% of its response in one time constant. Using the earlier-given numerical values, we can calculate the time constant as follows:

$$\tau = \frac{MC_p}{hA} = \frac{5\ \text{kg}\ 4200\ \dfrac{\text{J}}{\text{kg}\ ^\circ\text{K}}}{15\ \dfrac{\text{W}}{\text{m}^2\ ^\circ\text{K}}\ 0.15\ \text{m}^2} = 9333\ \text{s} = 2.6\ \text{hr} \tag{6.46}$$

4 10 0.12 3.889 hr

Therefore, it will take 2.6 hours for the watermelon to warm up to 20.75°C.