Thermal Systems

They transfer or store thermal energy by virtue of temp. and heat flow rate. The thermal effects are ① conduction, ② convection, ③ radiation and ④ heat storage capacity. Although thermal systems exhibit resistance and capacitance, there is no thermal inductance.

Basic Effects

⊛ Thermal Conduction

It is the ability of solid or continuous media to conduct heat.

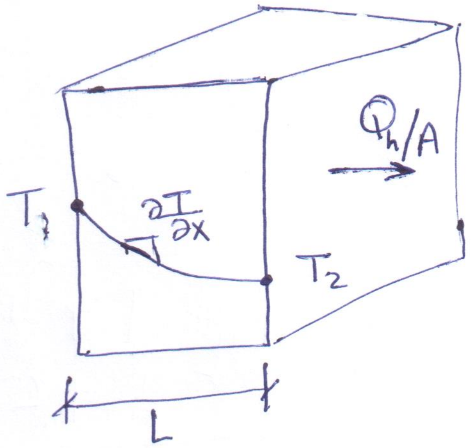
heat transfer

$$\boxed{\frac{Q_h}{A} = -K_t \frac{dT}{dx}}$$

Fourier's law of heat conduction

thermal conductivity

temp. gradient in the direction of heat flow



$$Q_h = -K_t A \frac{\partial T}{\partial x}$$

$$= -K_t A \frac{\Delta T}{\Delta x}$$

$$= \frac{-K_t A}{L} \Delta T$$

$$= \frac{-K_t A}{L} (T_2 - T_1)$$

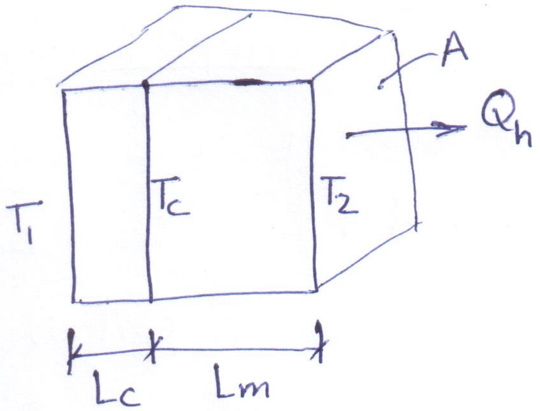
* Conduction through a flat plate

$$\therefore \boxed{Q_h = \frac{\delta T}{R}}$$

as $\delta T = T_1 - T_2$

$$R = \frac{L}{K_t A}$$

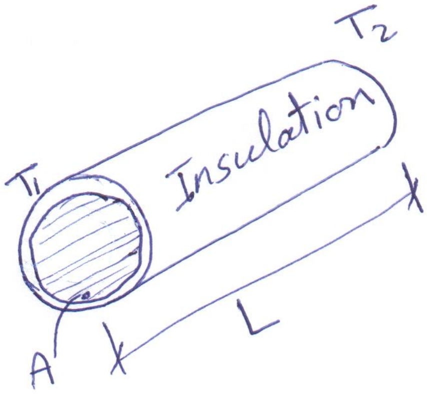
R : thermal resistance due to conduction



$$\boxed{Q_h = \frac{T_1 - T_2}{R_c + R_m}}$$

as $R_c = \dfrac{L_c}{K_c A}$ & $R_m = \dfrac{L_m}{K_m A}$

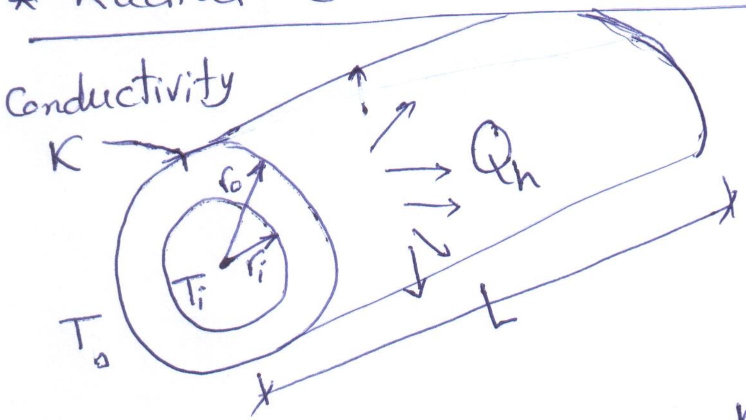
* Axial Conduction in a rod



$$\boxed{Q_h = \frac{K_t A}{L}(T_1 - T_2)}$$

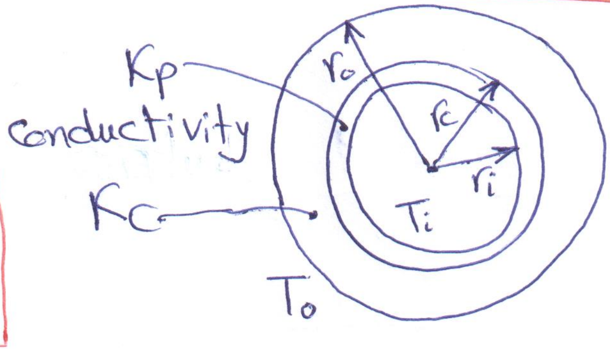
only axial heat transfer.
No radial.

* Radial Conduction in a cylinder



$$Q_h = -K_t (2\pi r L)\frac{dT}{dr}$$
$$= -K_t\, 2\pi L \frac{dt}{dr/r}$$

$$\boxed{Q_h = \frac{2\pi K_t L}{\ln(r_o/r_i)}(T_i - T_o)}$$

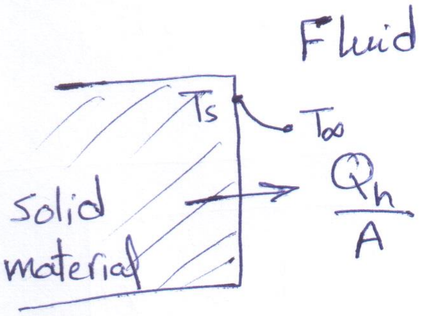


$$\boxed{Q_h = \frac{T_i - T_o}{\dfrac{\ln(r_c/r_i)}{2\pi L K_p} + \dfrac{\ln(r_o/r_c)}{2\pi L K_c}}}$$

⊛ Thermal Convection

It's the process of heat transfer between a surface of a solid material and a fluid that is exposed to the solid surface.

once the fluid has been subjected to the heat transfer; the fluid in the vicinity of the surface can move to another location and be replaced with fresh fluid, and the process can repeat itself.



$$\boxed{\frac{Q_h}{A} = h\,(T_s - T_\infty)}$$

Newton's law of cooling

h: Convection coefficient

* Convection → free convection, forced convection

* $h_{cooling} \neq h_{heating}$

* $Q = h A \delta T = \dfrac{\delta T}{R}$ ∴ $\boxed{R = \dfrac{1}{hA}}$

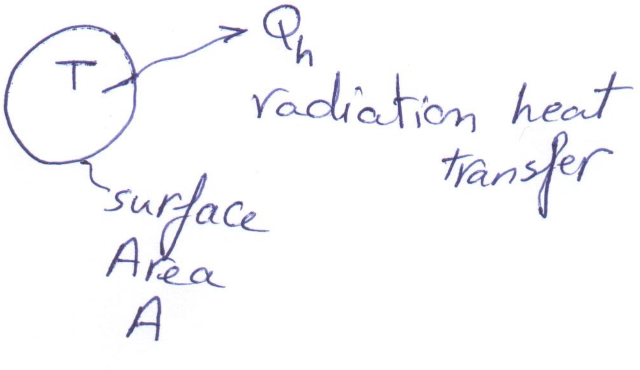
R: thermal resistance due to convection

⊛ Thermal Radiation

It's the process of heat transfer in which the energy is high enough to transfer heat without the presence of a surrounding medium, such as fluid or solid.

Thermal radiation can take place in vacuum.

$$\boxed{\frac{Q_h}{A} = \sigma T^4}$$



as $\sigma = 56.68 * 10^{-9}\ \dfrac{\text{Watt}}{m^2 K^4}$

stefan-Boltzmann Constant

$$\boxed{Q_h = F_e F_v \sigma A (T_H^4 - T_L^4)}$$

F_e: emissivity factor

$F_e = 1$ for black body

F_v: view factor

account lost radiation betn two bodies

T_H: high temp.

T_L: Low temp.

$$\boxed{Q_h = F_e F_v h_{eq} A_h \Delta T}$$

$\Delta T = T_H - T_L$ as $T_H > T_L$

as $h_{eq} = 4\sigma T_H^3 \left[1 - \frac{3}{2}\frac{\Delta T}{T_H} + \left(\frac{\Delta T}{T_H}\right)^2 - \frac{1}{4}\left(\frac{\Delta T}{T_H}\right)^3\right]$

h_{eq}: equivalent Convection Coefficient.

(*) Thermal Capacitance

It's the behavior of the material when it holds or store heat.

$$\boxed{Q_h = \underbrace{C_p m}_{\text{total heat storage (heat capacity)}} \frac{dT}{dt}}$$

dynamic eqn (diff. eqn)

flow $\propto \frac{d\text{effort}}{d\text{time}}$

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static thermal systems

* Thermal Conduction circuits

ex. 6-1. Conduction in a rod

* Thermal Conduction and Convection circuits

ex. 6-2. plexiglas plate

ex. 6-3. steam pipe

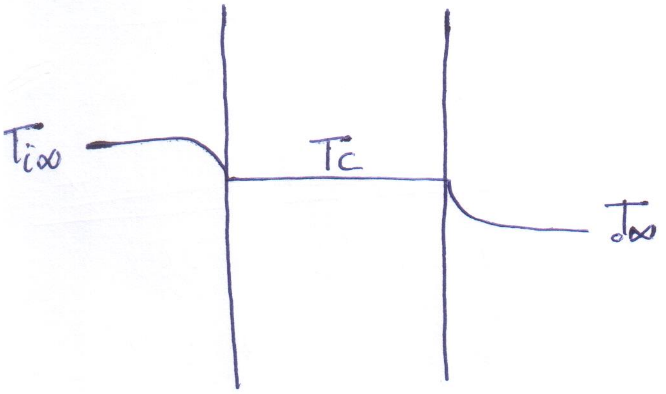
* Dynamic thermal systems

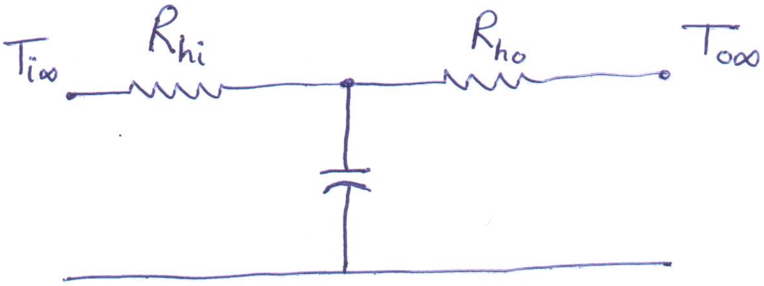
- In these systems, the heat capacity is large enough or the considered time is short enough for there to be a rate of change of temp. with time during a process.
- Diffusion equation, 2nd order diff. eqn, is the actual modeling eqn.
- Lumped-parameter model is used to solve dynamic systems.
- Biot number $N_b = \frac{R_{cond.}}{R_{conv.}} = \frac{h L_c}{k}$

  L_c : a characteristic length of the solid materials

$$L_c = \frac{\text{Volume}}{\text{surface area}} = \text{thickness for a plate}$$
$$= \frac{\text{thickness}}{2} \text{ for a fin}$$
$$= \frac{\text{diameter}}{4} \text{ for a long cylinder}$$
$$= \frac{\text{diameter}}{6} \text{ for a sphere}$$

- IF N_b is small ($N_b < 0.1$) $\therefore R_{cond.} < R_{conv.}$, then we can consider a single lump of capacitance and only consider the heat capacity of the solid material and the heat transfer due to convection. Thermal conduction resistance isn't considered.





IF N_b is large ($N_b > 0.1$) ∴ $R_{cond.}$ is significant compared to $R_{conv.}$
there will be temp. diff. inside the solid materials.
In this Case, we must consider several lumps of Capacitance.

* Single-Lumped Capacitance Modeling

ex. 6-4. Watermelon warming

* Multiple-Lumped Capacitance Modeling

The thermal conductivity elements are divided into n elements as defined by:

$$Q_h = \frac{1}{R} \delta T \qquad \text{as } R = \frac{L/n}{KA}$$

the capacitive elements :-

& $$Q_h = C_n \frac{dT}{dt} \qquad \text{as } C_n = C_p \frac{M}{n}$$

① constant surface temp. source

convection coefficient on each side of the wall are very good.
∴ Convection resistance will be small
surface temp. will equal free stream fluid temp.

② Convection surface temp. source

Convection Coefficient on each side of the wall are not very good
∴ Convection resistance will be large
there will be a convection resistance between the free stream fluid temp. and the temp. of the surface.