Fluid Systems

They range from simple systems involving fluid flow and lines to hydraulic and pneumatic control systems.

* properties of fluids

Density

$$\rho = \frac{m}{V}\Big|_{P_o, T_o}$$

mass per unit volume under specified conditions of pressure and temperature.

* Equation of state

⊛ Liquids

$$\rho = \rho_o + \frac{\partial \rho}{\partial P}\Big|_{P_o, T_o} (P - P_o) + \frac{\partial \rho}{\partial T}\Big|_{P_o, T_o} (T - T_o)$$

$$\rho = \rho_o \left[1 + \frac{1}{\beta}(P - P_o) - \alpha (T - T_o) \right]$$

Bulk modulus

$$\beta = \rho_o \frac{\partial P}{\partial \rho}\Big|_{P_o, T_o}$$

$$= \frac{\partial P}{\partial \rho / \rho_o}\Big|_{P_o, T_o}$$

* For a fixed mass of fluid

$$\beta = -\frac{\partial P}{\partial V / V_o}$$

isothermal bulk modulus

* Adiabatic ~ ~

$$\beta_a = \frac{C_p}{C_v} \beta$$

C_p: specific heat at const. pressure

C_v: sp. heat at const. Volume

β on the order of 5000 to 15,000 bar

thermal expansion coefficient

$$\alpha = \frac{-1}{\rho_o} \frac{\partial \rho}{\partial T}\Big|_{P_o, T_o}$$

* For a fixed mass

$$\alpha = \frac{\partial V / V_o}{\partial T}\Big|_{P_o, T_o}$$

* It has an approximate value of α

$$\alpha = 0.5 \times 10^{-3} / {}^\circ F$$

for most liquids.

⊛ Gases

$$\rho = \frac{P}{RT}$$

as P, T are absolute
R is gas Constant

$$\frac{P}{\rho^n} = \text{Const} = C$$

$n = 1$ isothermal process
$n = k$ adiabatic process
(k = ratio of specific heats)
$n = 0$ isobaric process
$n = \infty$ isovolumetric ~

$$\therefore P = C\rho^n$$

$$\beta = \rho_o \frac{\partial P}{\partial \rho}\Big|_{P_o, T_o} = \rho_o \left[n C \rho^{n-1}\right]\Big|_{P_o, T_o} = \rho_o n \frac{C\rho^n}{\rho}\Big|_{P_o, T_o} = n P_o$$

β on the order of 1 to 10 bar

Viscosity

$$\mu = \frac{\text{shear stress}}{\text{shear rate}} = \frac{\tau}{\partial u / \partial y}$$

* Newtonian fluid: a fluid in which the absolute viscosity is independent of the shear rate.
* Non-Newtonian fluid has a variable viscosity, depending upon the shear rate.
* Kinematic viscosity $\nu = \frac{\mu}{\rho}$

⊛ liquids

$\mu \downarrow$ as Temp. $\uparrow$ exponential decay

$$\mu = \mu_o e^{-\lambda_L (T - T_o)}$$

as: μ_o, T_o values at reference Conditions

λ_L Const. depends on the liquid

⊛ Gas

$\mu \uparrow$ as Temp. $\uparrow$ straight line relation

$$\mu = \mu_o + \lambda_G (T - T_o)$$

λ_G → Const. depends on the gas

propagation speed

- speed of sound $C_o = \sqrt{\frac{\beta}{\rho}}$ — β: bulk modulus, ρ: density

- at high speeds $\beta = KP$ as $K = \frac{C_p}{C_v}$ specific heat ratio

$$\boxed{C_o = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{KP}{\rho}} = \sqrt{KRT}} \quad \text{as: } P = \rho RT$$

- $\therefore C_o = \sqrt{KRT}$

- specific heat of a fluid is the amount of heat required to raise the temp. of a unit of mass of fluid by 1 degree.

$K(\text{for liquids}) \simeq 1.04 \qquad K(\text{air}) = 1.4$

Reynolds number effects

$$N_r = \frac{\text{inertial flow forces}}{\text{viscous flow forces}} = \frac{\propto \rho A v^2}{\propto \mu d v} = \frac{\rho d^2 v^2}{\mu d v} = \frac{\rho v d}{\mu} = \frac{v d}{\nu}$$

* for small N_r viscous flow forces $\uparrow\uparrow$
* As N_r increases, the effect of the inertial forces becomes sufficiently great to break up streamlined flow.
* In laminar flow, the pressure loss due to friction just like electrical resistance (Volt $\propto$ current) easy to deal with
* In turbulent flow, pressure loss $\propto$ flow^2 $\left(\frac{\rho V^2}{2}\right)$

Derivation of passive Components

Capacitance

* Fluid Capacitance:

It relates how fluid energy Can be stored by virtue of pressure

$$Q = \frac{V}{\beta} \dot{P}_{cv}$$

$$\boxed{i = c \frac{de}{dt}}$$

$$\therefore \; C_f = \frac{V}{\beta}$$

⊛ Inductance

$$\boxed{e = L \frac{di}{dt}}$$

$$L = \frac{\rho l}{A} \qquad \text{as } \delta P = \frac{\rho l}{A} \dot{Q}$$

Fluid Inductance is often called fluid "inertance", since its effect is due to the inertia of a moving incomp. fluid in a fluid line of constant area.

$$\delta P = P_1 - P_2$$

⊛ Resistance

A fluid resistor dissipates power and can have a large variety of forms: laminar flow resistance, orifice type or head loss resistance and compressible flow resistance.

* laminar flow resistance

$$\boxed{e = i R}$$

$$\delta P = R Q \qquad \therefore R = \frac{32 \mu L}{A d_h^2}$$

d_h = hydraulic diameter $= \dfrac{4 \text{ area}}{\text{perimeter}}$

$R = \dfrac{128 \mu L}{\pi d^4}$ for a circular section with dia. d

$= \dfrac{32 \mu l}{w^4}$ for a square section with width w