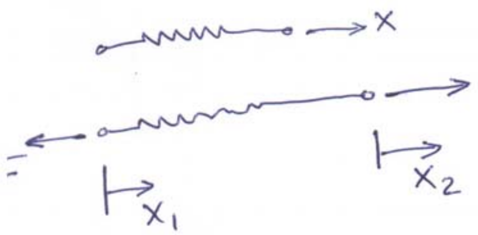
Alexandria Higher Institute of Engineering & Technology (AIET)			
Department of: Mechatronics		Fourth Year	4th Year
EME403	Dynamic System Analysis		Final, Jan., 21, 2015
Examiners:	Dr. Rola Afify and committee		Time: 3 hour

Answer the following questions:

Question one (15 marks)

A) For mechanical systems, define: Spring, damper, and discrete mass.

* springs

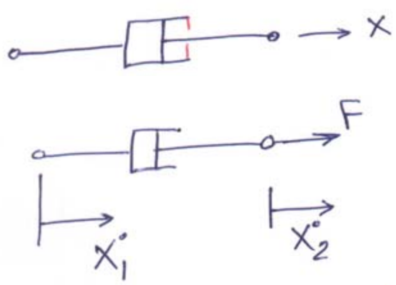


spring Constant (N/m)

$$\boxed{F = K(x_2 - x_1)}$$

for a linear springs

* Dampers (dashpot)



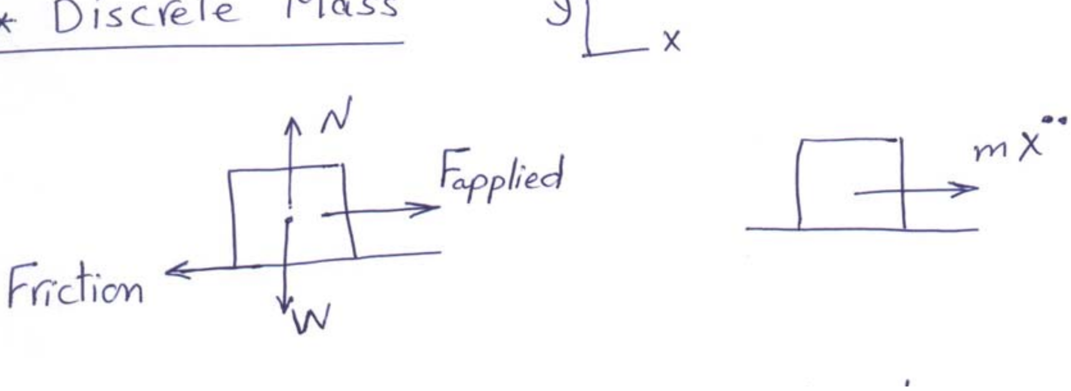
damping Constant (N·s/m)

$$\boxed{F = b(\dot{x}_2 - \dot{x}_1)}$$

for a linear damper

In viscous dampers, the generated force is due to pressure drop across a fluid resistor.

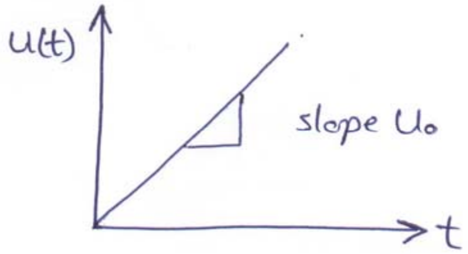
* Discrete Mass



B) Prove that the solution of the first order differential equation using ramp input $u(t) = u_o$ will be in this form $x(t) = x_o e^{-t/\tau} + Gu_o\left[t - \tau(1 - e^{-t/\tau})\right]$.

1.3. Ramp input response $u(t) = u_o t$

$$\boxed{\tau \dot{x} + x = G u_o t}$$



as $x(t) = x_h(t) + x_p(t)$

$x_h = Ae^{-t/\tau}$ & $x_p = Rt + Q$

$\dot{x}_p = R$ as Q & R are constant

/* to find R & Q, solve 1st order differential eqn for x_p only

$$\tau \dot{x}_p + x_p = G u_o t$$

$$\tau R + (Rt + Q) = G u_o t$$

$$\tau R + Q + Rt = G u_o t$$

at $t=0$ $\qquad \boxed{R = G u_o}$ as $\tau R + Q = 0$

$$\tau R + Q = 0$$

$$Q = -\tau R \Rightarrow \boxed{Q = -\tau G u_o}$$

$\therefore x_p = Rt + Q = G u_o t - \tau G u_o = G u_o (t - \tau)$

/* to find the constant A apply the initial conditions

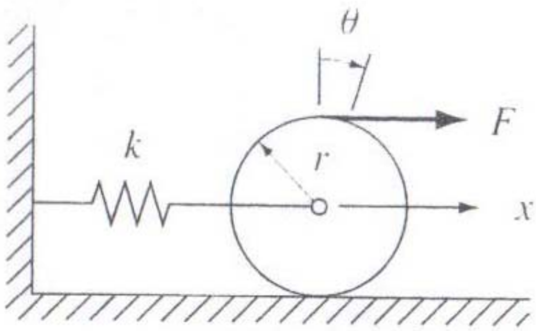
$$x_o = A e^{0} + G u_o t - \tau G u_o$$

$$x_o = A - \tau G u_o \qquad \therefore \boxed{A = x_o + \tau G u_o}$$

$\therefore x(t) = x_o e^{-t/\tau} + \tau G u_o e^{-t/\tau} + G u_o t - \tau G u_o$

$$\boxed{x(t) = x_o e^{-t/\tau} + G u_o \left[t - \tau\left(1 - e^{-t/\tau}\right)\right]}$$

C) A portion of a mechanical device may be idealized as a uniform, homogeneous wheel rolling without slipping on a horizontal surface, as shown in figure. The center of the wheel is fastened to the frame of the device by a linear spring, and a force is applied at the top of the wheel. Find the equation of motion that governs the horizontal position of the center of the wheel.



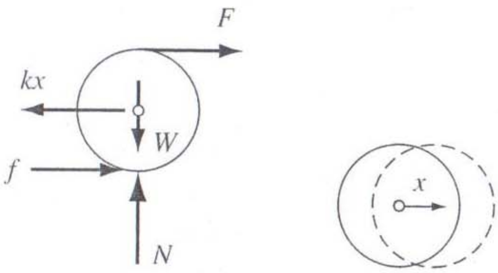


Figure 3.26 Free-body diagram of wheel.

$$\sum F_x = m\ddot{x}_{cg} \qquad F + f - kx = m\ddot{x} \tag{3.92}$$

$$\sum M_{cg} = J_{cg}\ddot{\theta} \qquad rF - rf = J_{cg}\ddot{\theta} \tag{3.93}$$

solve for f from the moment equation:

$$f = F - J_{cg}\frac{\ddot{\theta}}{r} \tag{3.94}$$

$$x = r\theta \quad \text{and} \quad \ddot{x} = r\ddot{\theta}, \quad \text{so} \quad \ddot{\theta} = \frac{\ddot{x}}{r} \tag{3.95}$$

$$f = F - J_{cg}\frac{\ddot{x}}{r^2} \tag{3.96}$$

Substitution gives

$$F + \left(F - J_{cg}\frac{\ddot{x}}{r^2}\right) - kx = m\ddot{x} \tag{3.97}$$

$$m\ddot{x} + J_{cg}\frac{\ddot{x}}{r^2} + kx = 2F \tag{3.98}$$

$$\left(m + \frac{J_{cg}}{r^2}\right)\ddot{x} + kx = 2F \tag{3.99}$$

For a uniform circular disk, the mass moment of inertia with respect to an axis through its center of gravity is $mr^2/2$. Substituting this for J_{cg} gives

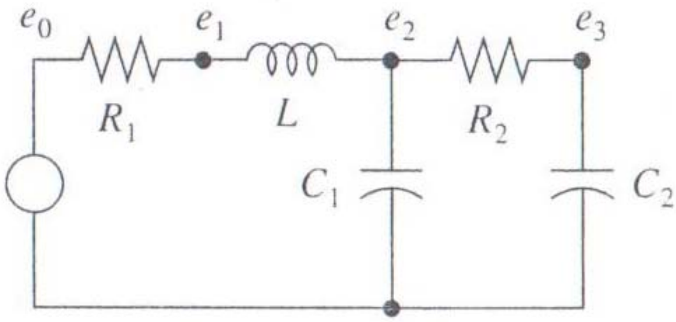
$$\frac{3}{2}m\ddot{x} + kx = 2F \tag{3.100}$$

If a rigid body rotates about a fixed axis O, the moment equation in Newton's second law can be written for that axis. Both the sum of the moments and the mass moment of inertia are then written for that axis:

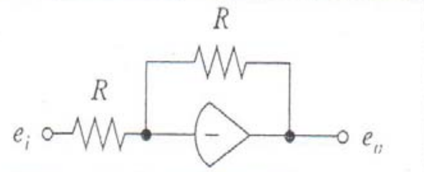
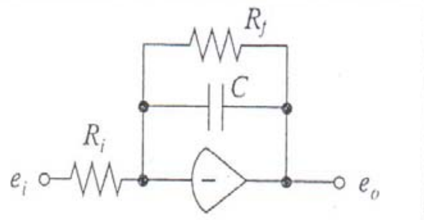
$$\sum M_O = J_O\ddot{\theta} \tag{3.101}$$

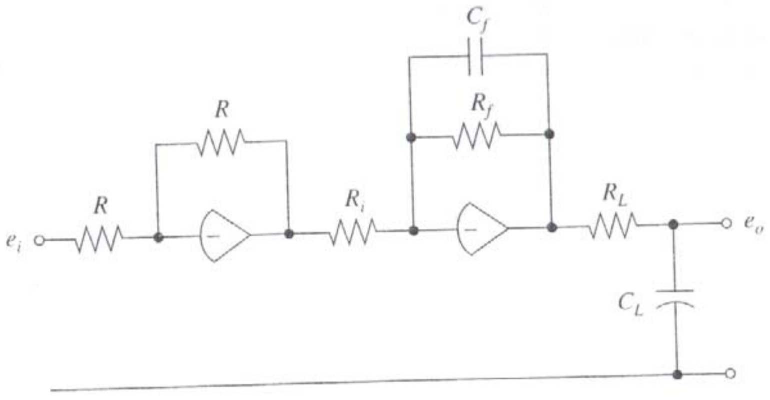
Question Two (15 marks)

A) Write the modelling equation for the circuit shown in figure. What is the gain?



B) An electronic circuit with an op-amp buffer is shown in figure. Derive the differential equation for e_o as a function of the input e_i. Calculate the static gain, natural frequency, and damping ration if R_f = 10 kΩ, R_i = 10 kΩ, C_f = 1 μf, R_L = 500 Ω, and C_L = 10 μf.

Transfer Function	Circuit
$e_o = -e_i$	
$e_o = \dfrac{-\dfrac{R_f}{R_i} e_i}{(\tau D + 1)}$ $\tau = R_f C$	



Question Three (15 marks)

A) Declare, with neat sketches, basic effects of thermal systems.

Basic Effects

Thermal Conduction

It is the ability of solid or continuous media to conduct heat.

heat transfer

$$\boxed{\frac{Q_h}{A} = -K_t \frac{dT}{dx}}$$

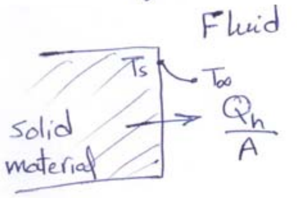
Fourier's law of heat conduction

thermal conductivity

temp. gradient in the direction of heat flow

Thermal Convection

It's the process of heat transfer between a surface of a solid material and a fluid that is exposed to the solid surface.



$$\boxed{\frac{Q_h}{A} = h\,(T_s - T_\infty)}$$

Newton's law of Cooling

h: Convection coefficient

* Convection → free convection, forced convection

Thermal Radiation

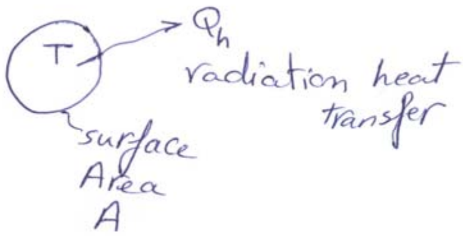
It's the process of heat transfer in which the energy is high enough to transfer heat without the presence of a surrounding medium, such as fluid or solid.

Thermal radiation can take place in vacuum.

$$\boxed{\frac{Q_h}{A} = \sigma T^4}$$

as $\sigma = 56.68 * 10^{-9}\ \frac{\text{Watt}}{m^2 K^4}$

stefan-Boltzmann Constant



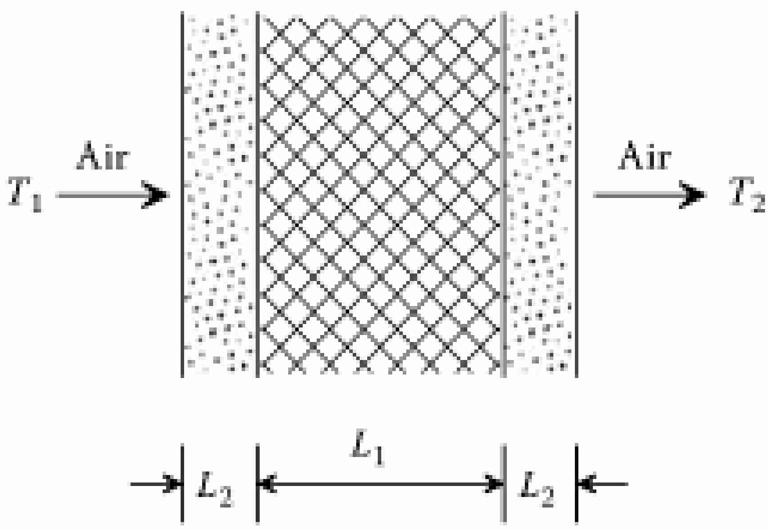
⊛ Thermal Capacitance

It's the behavior of the material when it holds or store ↑ heat.

$$\boxed{Q_h = \underbrace{C_p\, m}_{\text{total heat storage (heat capacity)}} \frac{dT}{dt}}$$

dynamic eqn
(diff. eqn)
flow ∝ $\frac{d\text{effort}}{d\text{time}}$

B) Consider heat transfer through an insulated wall as shown in figure. The wall is made of a layer of brick with thermal conductivity k_1 = 0.5 W/(m.°C) and two layers of foam with thermal conductivity k_2 = 0.17 W/(m.°C) for insulation. The left surface of the wall is at temperature T_1 = 38°C and exposed to air with coefficient h_1 = 10 W/(m^2.°C). The right surface of the wall is at temperature T_2 = 20°C and exposed to air with coeffiecient h_2 = 10 W/(m^2.°C). The thickness of the brick layer is 0.1 m, the thickness of each foam layer is 0.03 m, and the cross-sectional area of the wall is 16 m^2. Write the modelling equation and determine the heat flow rate through the wall.



a. The heat transfer through the insulated wall can be represented using a thermal circuit with five thermal resistances connected in series as shown in Figure 7.30. Two modes of heat transfer, conduction and convection, are involved. The corresponding thermal resistances are

$$R_1 = \frac{1}{h_1 A} = \frac{1}{10 \times 16} = 6.25 \times 10^{-3}\,{}^\circ\text{C·s/J},$$

$$R_2 = R_4 = \frac{L_2}{k_2 A} = \frac{0.03}{0.17 \times 16} = 1.10 \times 10^{-2}\,{}^\circ\text{C·s/J},$$

$$R_3 = \frac{L_1}{k_1 A} = \frac{0.1}{0.5 \times 16} = 1.25 \times 10^{-2}\,{}^\circ\text{C·s/J},$$

$$R_5 = \frac{1}{h_2 A} = \frac{1}{10 \times 16} = 6.25 \times 10^{-3}\,{}^\circ\text{C·s/J}.$$

The total thermal resistance is

$$R_{eq} = \sum_{i=1}^{5} R_i = 4.70 \times 10^{-2}\,{}^\circ\text{C·s/J}$$

Thus, the heat flow rate through the insulated wall is

$$q_h = \frac{\Delta T}{R_{eq}} = \frac{38 - 20}{4.70 \times 10^{-2}} = 382.98\,\text{W}$$

Question Four (15 marks)

A) Compare between viscosity of liquids and gases.

Viscosity $\mu = \frac{\text{shear stress}}{\text{shear rate}} = \frac{\tau}{\partial u/\partial y}$

* Newtonian fluid: a fluid in which the absolute viscosity is independent of the shear rate.
* Non-Newtonian fluid has a variable viscosity, depending upon the shear rate.
* Kinematic viscosity $\nu = \frac{\mu}{\rho}$

Liquids $\mu \downarrow$ as Temp $\uparrow$ exponential decay

$$\mu = \mu_o e^{-\lambda_L (T - T_o)}$$

as: μ_o, T_o values at reference conditions

λ_L Const. depends on the liquid

Gas $\mu \uparrow$ as temp $\uparrow$ straight line relation

$$\mu = \mu_o + \underbrace{\lambda_G}_{\text{Const. depends on the gas}} (T - T_o)$$

B) For Fluid systems, define: capacitance, inductance, and resistance.

* Fluid Capacitance:

It relates how fluid energy can be stored by virtue of pressure

$$Q = \frac{V}{\beta} \dot{P}_{cv}$$

$$\boxed{i = c \frac{de}{dt}}$$

$$\therefore C_f = \frac{V}{\beta}$$

Inductance $\boxed{e = L \frac{di}{dt}}$

$$L = \frac{\rho l}{A}$$

as $\delta P = \frac{\rho l}{A} \dot{Q}$

Fluid Inductance is often called fluid "inertance", since its effect is due to the inertia of a moving incomp. fluid in a fluid line of constant area.

$$\delta P = P_1 - P_2$$

⊛ Resistance

A fluid resistor dissipates power and can have a large variety of forms: laminar flow resistance, orifice type or head loss resistance and compressible flow resistance.

*** laminar flow resistance**

$\boxed{e = iR}$

$$\delta P = RQ \qquad \therefore R = \frac{32\,\mu L}{A\,d_h^2}$$

d_h = hydraulic diameter $= \dfrac{4\,\text{area}}{\text{perimeter}}$

$R = \dfrac{128\,\mu L}{\pi d^4}$ for a circular section with dia. d

$= \dfrac{32\,\mu l}{w^4}$ for a square section with width w