Fluid Kinematics

It deals with the motion of fluids without considering the forces and moments which create the motion.
We define field variables which are functions of space and time

Velocity field $\vec{V} = \vec{V}(x, y, z, t)$

$$\vec{V} = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k}$$

Acceleration field, $\vec{a} = \vec{a}(x, y, z, t)$

$$\vec{a} = a_x(x, y, z, t)\vec{i} + a_y(x, y, z, t)\vec{j} + a_z(x, y, z, t)\vec{k}$$

Streamline:

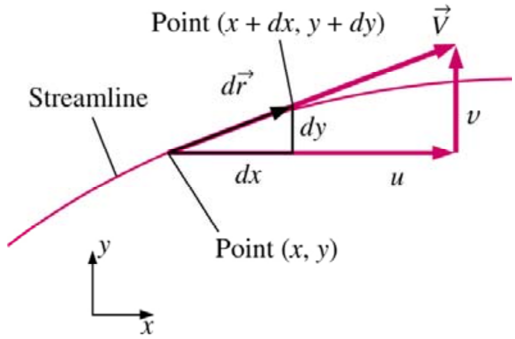
A Streamline is a curve that is everywhere tangent to it at any instant represents the instantaneous local velocity vector.

$$\tan\theta = \frac{dy}{dx} = \frac{v}{u}$$

$$\frac{u}{dx} = \frac{v}{dy}$$

in — general — for 3 − D

$$\frac{u}{dx} = \frac{v}{dy} = \frac{w}{dz}$$



Stream line equation

Where:
u velocity component in- X- direction
v velocity component in-Y- direction
w velocity component in -Z- direction

$V = \sqrt{u^2 + v^2 + w^2}$

velocity vector can written as:

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

Where:
i ,j, k are the unit vectors in+ ve x, y, z directions

Acceleration Field

- From Newton's second law, $\vec{F}_{particle} = m_{particle}\vec{a}_{particle}$
- The acceleration of the particle is the time derivative of the particle's velocity.

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt}$$

- However, particle velocity at a point is the same as the fluid velocity,

$\overline{V} = f^{n.}(x, y, z, t)$

Mathematically the total derivative equals the sum of the partial derivatives

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz + \frac{\partial u}{\partial t}dt$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t}$$

$$a_x = \frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w + \frac{\partial u}{\partial t}$$

$$a_x = \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{Convective component}} + \underbrace{\frac{\partial u}{\partial t}}_{\text{Local component}}$$

Convective component   Local component

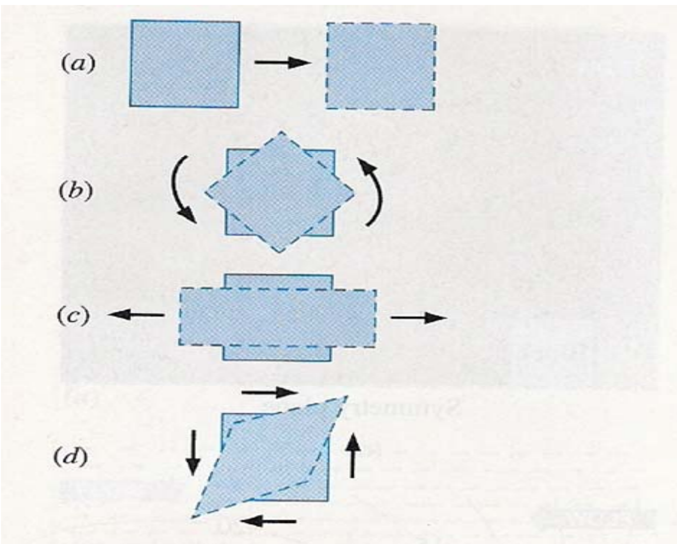
Similarly:

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$a = \sqrt{a^2{}_x + a^2{}_y + a^2{}_z}$$

Types of motion or deformation of fluid element



Linear translation

Rotational translation

Linear deformation

angular deformation

FIGURE 4–34
Fundamental types of fluid element motion or deformation: (a) translation, (b) rotation, (c) linear strain, and (d) shear strain.