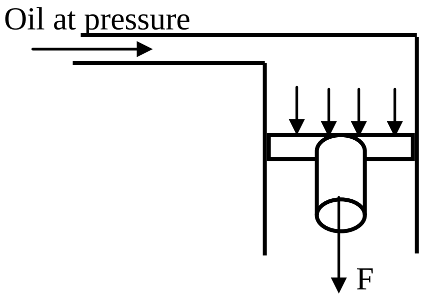
Fluid Statics

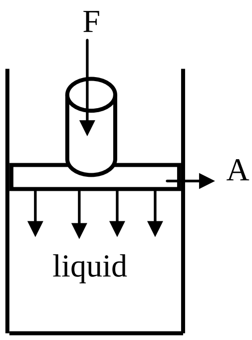
- Fluid Statics deals with problems associated with fluids at rest.
- In fluid statics, there is no relative motion between adjacent fluid layers.
- Therefore, there is no shear stress in the fluid trying to deform it.
- The only stress in fluid statics is *normal stress*
  - ✓ Normal stress is due to pressure
  - ✓ Variation of pressure is due only to the weight of the fluid → fluid statics is only relevant in presence of gravity fields.
- Applications: Floating or submerged bodies, water dams and gates, liquid storage tanks, etc.

- Pressure : is the Normal force per unit area

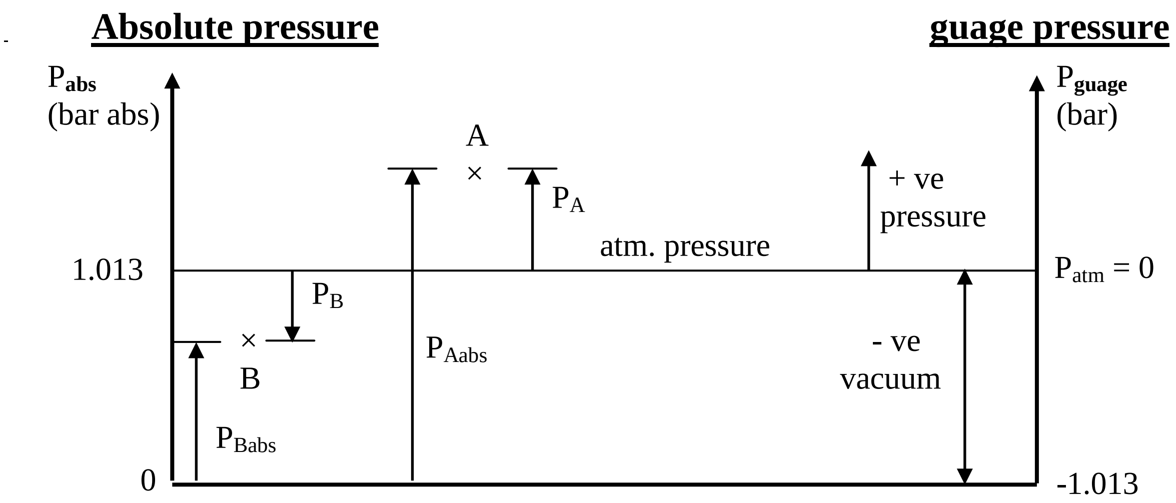
$$p = \frac{F}{A}$$

$$Pa = \frac{N}{m^2}$$





* Absolute, atmospheric and guage pressure



Absolute pressure = true pressure

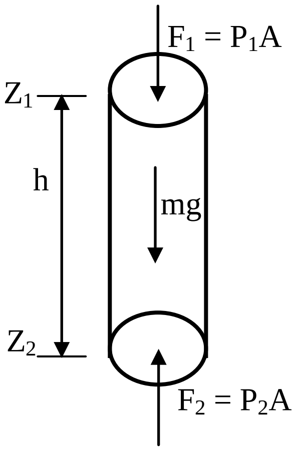
$$\boxed{P_{abs} = P_{guage} + P_{atm}}$$

* All given values for pressure are guage except if :
1. (abs) is mentioned beside the unit.
2. Dealing with atmospheric pressure.
3. Dealing with vapour pressure.

* No pressure guage value less than -1.013 bar
* 1 bar = 10^5 pascal
* -ve pressure is called vacuum

* In a static liquid :

1. The pressure, at a certain point, is the same in all directions.
2. The pressure is constant in the same horizontal plane.
3. The pressure changes in the vertical direction.



$$\rho = \frac{m}{V} \qquad \therefore m = \rho V = \rho A h$$

$$\sum F_y = 0 \quad \downarrow +$$

$$F_1 + mg - F_2 = o$$

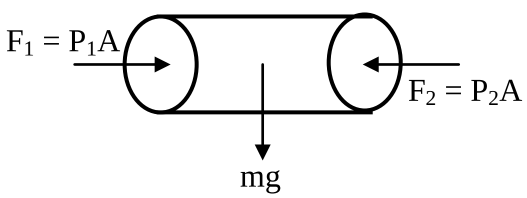
$$P_1 A + \rho A (Z_2 - Z_1) g - P_2 A = o \qquad \div A$$

$$P_1 + \rho h g - P_2 = o$$

$$\boxed{P_2 - P_1 = \rho\ g\ h} \quad \text{or} \quad \boxed{P_2 - P_1 = \gamma\ h}$$

$$\rightarrow + \sum F_x = o$$

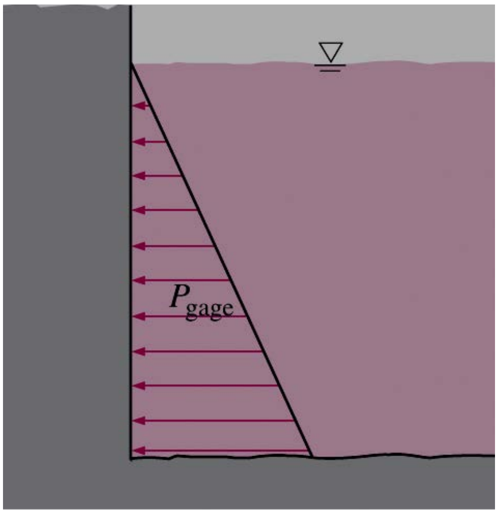
$$F_1 - F_2 = o$$



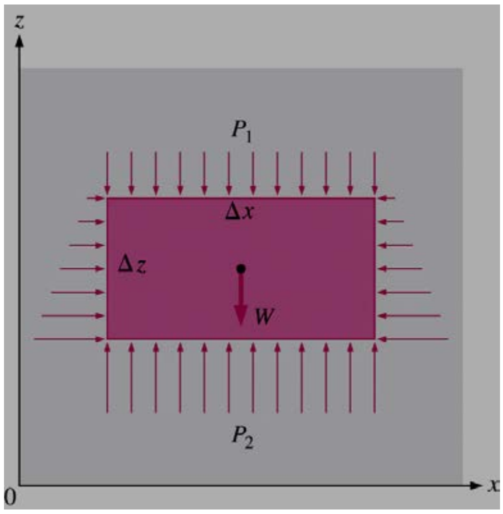
$$P_1 A - P_2 A = o \qquad \div A$$

$$P_1 - P_2 = o$$

$$\boxed{P_1 = P_2}$$



The pressure of a fluid at rest increases with depth (as a result of added weight).



Free-body diagram of a rectangular fluid element in equilibrium.

* Pressure and head

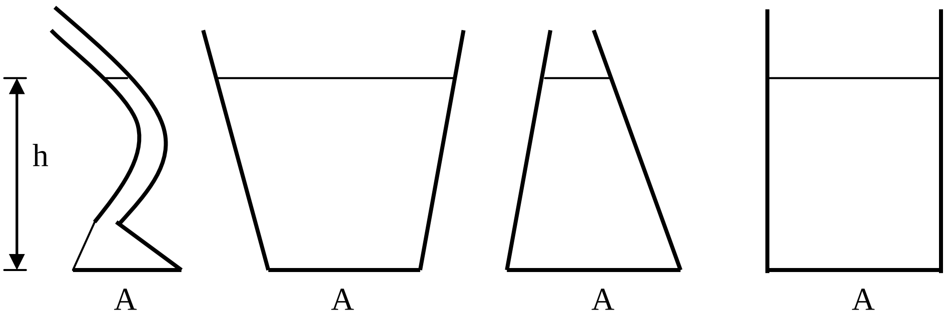
$$P = \gamma h$$

Pressure — head

N/m^2 — m of liquid

* head : is the vertical length that can define the pressure.

*The hydrostatic paradox :



$P_{\mathbf{bottom}} = \gamma h$ then $F_{\mathbf{bottom}} = PA = \gamma hA$

Although the weight of fluid is different, the force in the base of the four vessels is the same. This force depends on the depth (h) and the base area A.

Example :

A cylinder contains a fluid at pressure of 350 KN/m^2

- Express the pressure in terms of a head of :-

a) Water $\rho_\omega = 1000\,kg/m^3$ b) Mercury $SG_m = 13.6$

- Determine the absolute pressure if $P_{atm} = 101.3$ KN/m^2 ?

Solution

$P = \gamma h = \rho g h$

a) $350 * 10^3 = 1000 * 9.81 * h_w$ $\therefore h_w = 35.68$ m of water

b) $P = SG_m \rho_\omega g h$ $SG_m = \frac{\rho_m}{\rho_\omega}$ $\rho_m = SG_m \rho_\omega$

$350 * 10^3 = 13.6 * 1000 * 9.81 * h_m$ $\therefore h_m = 2.62$ m of mercury

$P_{abs} = P_{gage} + P_{atm}$

$= 350 * 10^3 + 101.3 * 10^3$

$= 451300\ N/m^2 * 10^{-3}$

$= 451.3\ KN/m^2$

Example :

If $h_{atm} = 76$ cm Hg, determine P_{atm}?

Solution

$P_{atm} = \gamma_m h$ $SG_m = \frac{\gamma_m}{\gamma_w}$

$= SG_m \gamma_w h$ $\gamma_m = SG_m \gamma_w$

$= 13.6 * 9800 * (76 * 10^{-2})$

$= 1.013 * 10^5\ N/m^2$

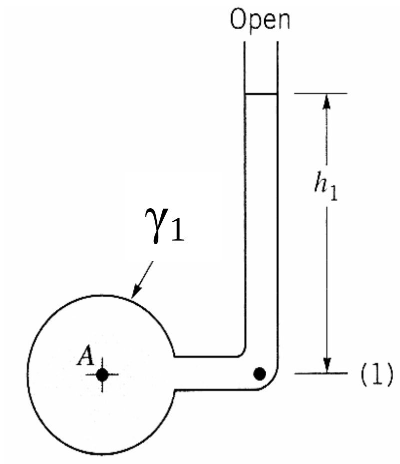
* pressure measurements by manometers

* Piezometer

Pressure tube or piezometer

Consists of a single vertical tube

$$P_A = \gamma_1 h_1$$



* U- tube manometer

Statics

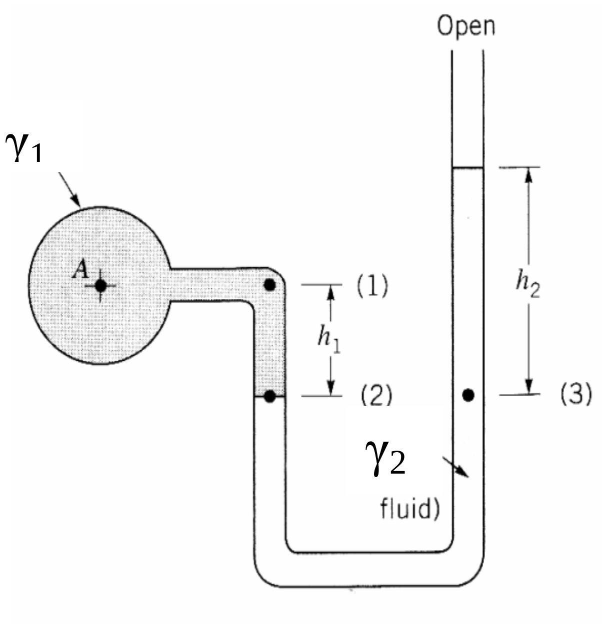
Same horizontal plane

* to make pressure equivalence

1 – Still liquid

2 – Continues liquid

3 – Same liquid



U-tube manometer

$P_I = P_{II}$

$$P_A + \rho_1 g h_1 = \rho_2 g h_2$$

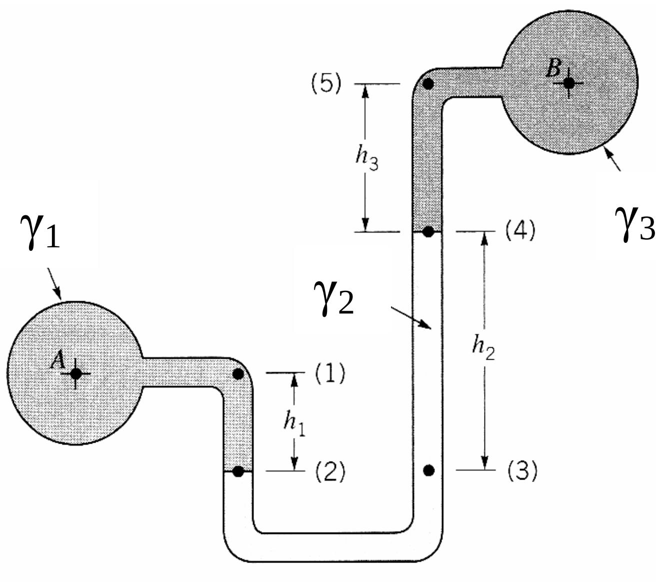
$$P_A + \gamma_1 h_1 = \gamma_2 h_2$$

$P_I = P_{II}$

$$P_A + \rho_1 g h_1 = P_B + \rho_2 g h_2 + \rho_3 g h_3$$

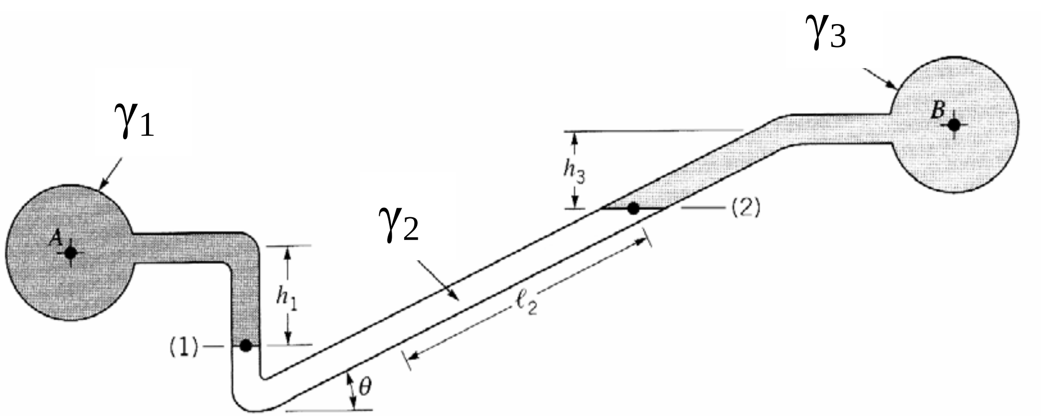
$$P_A - P_B = \rho_2 g h_2 + \rho_3 g h_3 - \rho_1 g h_1$$

$$P_A - P_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$



Differential U-tube manometer.

* Inclined-Tube Manometer



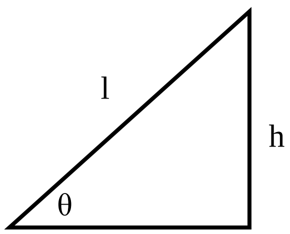
Inclined-tube Manometer.

$P_I = P_{II}$

$$P_A + \rho_1 g h_1 = P_B + \rho_2 g l_2 \sin\theta + \rho_3 g h_3$$

$$P_A - P_B = \rho_2 g l_2 \sin\theta + \rho_3 g h_3 - \rho_1 g h_1$$

$$P_A - P_B = \gamma_2 l_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$



$$\sin\theta = \frac{h}{l}$$

$$h = l \sin\theta$$

* U-tube with one enlarged

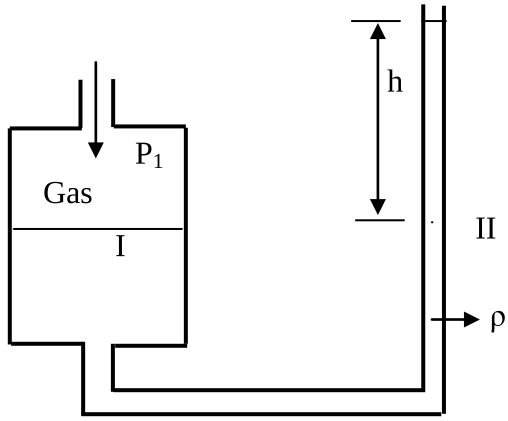
volume = volume

$A * \ell\ell = a * h$

$$\ell\ell = \frac{a}{A} * h$$

$$= \frac{\frac{\pi}{4} d^2}{\frac{\pi}{4} D^2} * h$$

$$\ell\ell = \frac{d^2}{D^2} * h$$

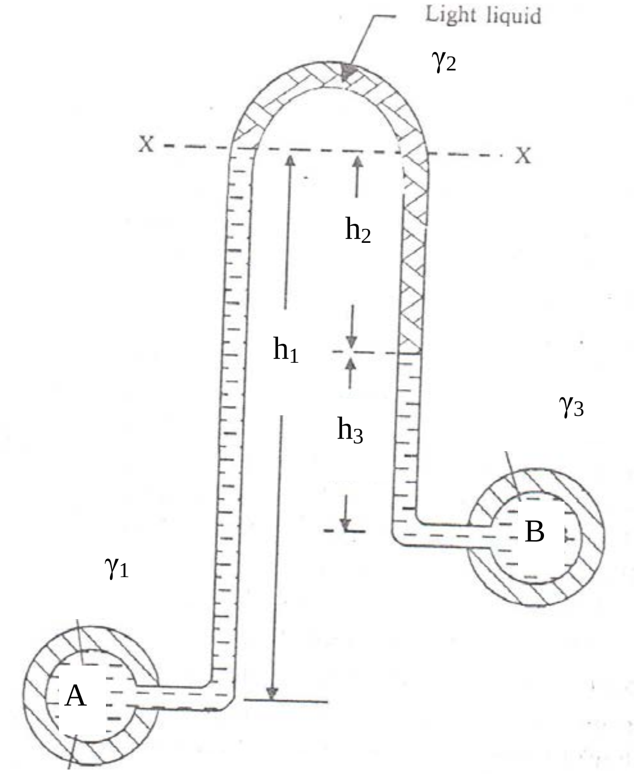


$P_I = P_{II}$

$$P_1 = \rho g \ell\ell + \rho g h$$

$$= \rho g * \frac{d^2}{D^2} h + \rho g h$$

$$= \rho g h \left(\frac{d^2}{D^2} + 1 \right)$$



* Inverted U-tube

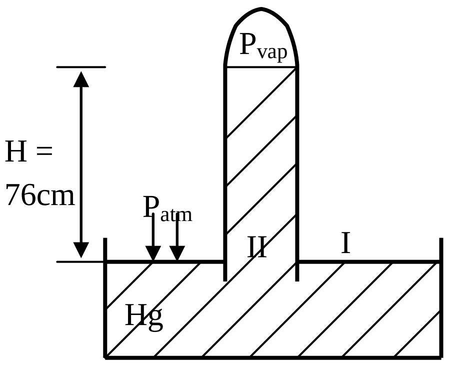
$P_I = P_{II}$

$$P_A - \rho_1 g h_1 = P_B - \rho_3 g h_3 - \rho_2 g h_2$$

$$P_A - P_B = \rho_1 g h_1 - \rho_3 g h_3 - \rho_2 g h_2$$

$\Delta P =$

*Atmospheric pressure (Barometric pressure)



$$P_{\underset{Hg}{vap}} = 1.7 * 10^{-5} \quad \text{bar}$$

$$= 1.7 \frac{N}{m^2} \approx 0 \qquad \text{neglected}$$

$$P_{\mathbf{I}} = P_{\mathbf{II}}$$

$$P_{atm} = P_{\underset{hg}{vap}} + \rho_m g H$$

$$= 13600 * 9.8 * 0.76$$

$$= 1.013 * 10^{\mathbf{5}} \quad \text{N/m}^{\mathbf{2}}$$

$$= 1.013 \quad \text{bar}$$

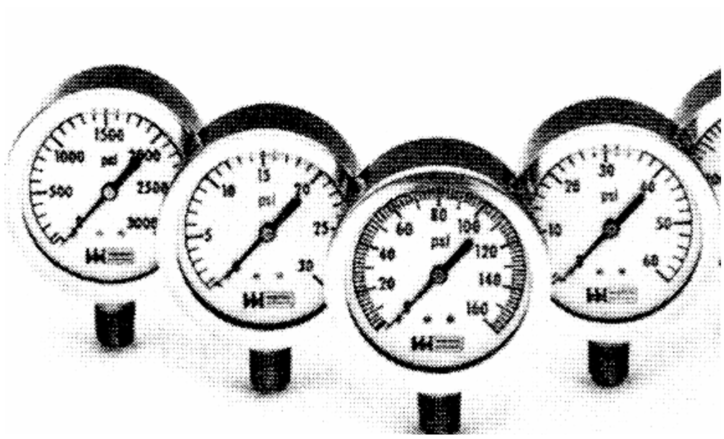
* Bourdon tube gauge

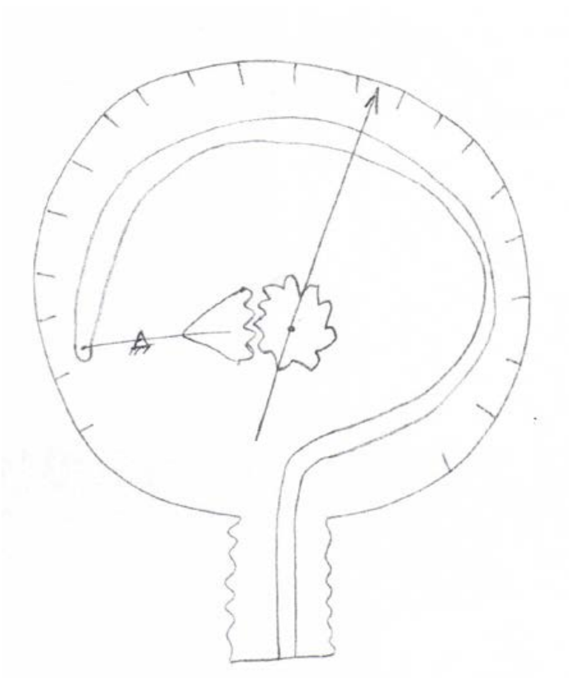
It is used for measuring pressure in almost all ranges except minutely small pressure.

Disadvantages:

1 – Needs calibration on dead weight tester.

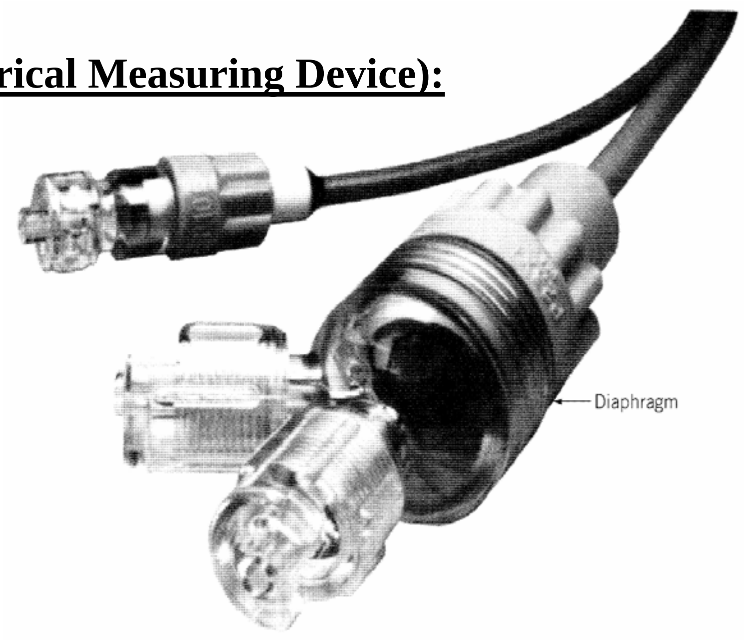
2 – Accuracy is less than liquid Columns.





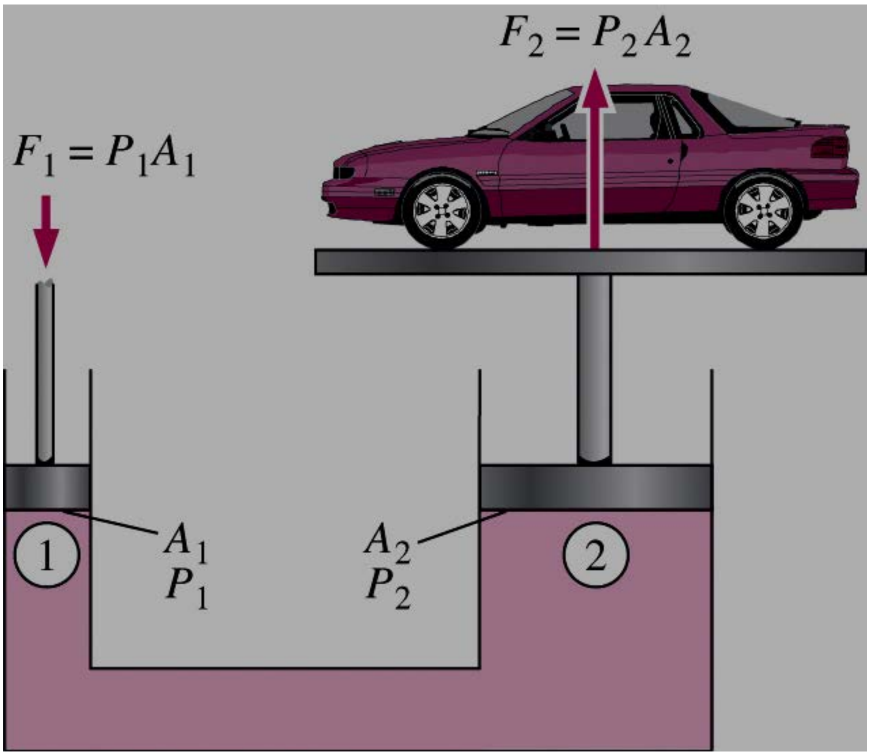
* Pressure Transducer (Electrical Measuring Device):

A pressure transducer converts pressure into an electrical output.



Applications

Lifting of a large weight by a small force by the application of Pascal's law.



Pascal's law: The pressure applied to a confined fluid increases the pressure throughout by the same amount.

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

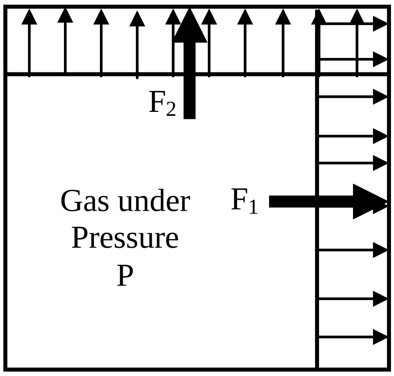
Hydrostatic Forces on Plane Surfaces

* Forces due to fluid pressure on Flat surface

* For gases

F_1 & F_2 perpendicular on the surface & acts at the center of area subjected to pressure

$$F_1 = PA_1 \quad \& \quad F_2 = PA_2$$



* for liquids

* F_1= volume of pressure prism

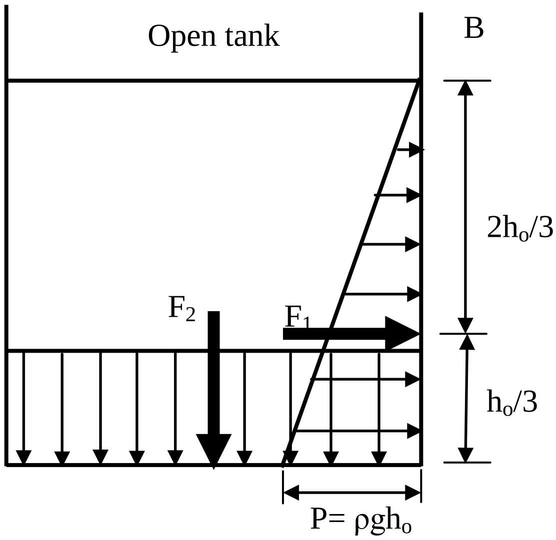
$$= \gamma\, h_0 * \frac{h_0}{2} * B$$

$$= \frac{\gamma h_o^2 B}{2}$$

* F_1 acts at the center of volume of the prisme $\perp$ to the surface

* F_2 (on bottom) = PA = $\gamma\, h_o * A$

* F_2 acts $\perp$ on bottom and at center of area

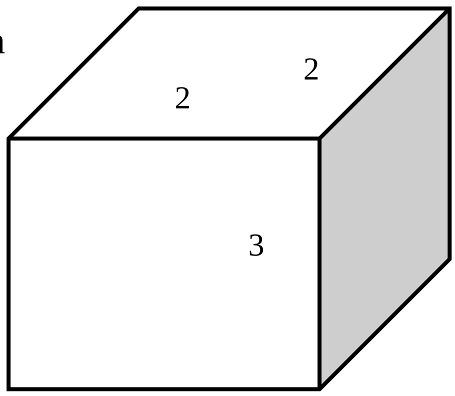


Example :

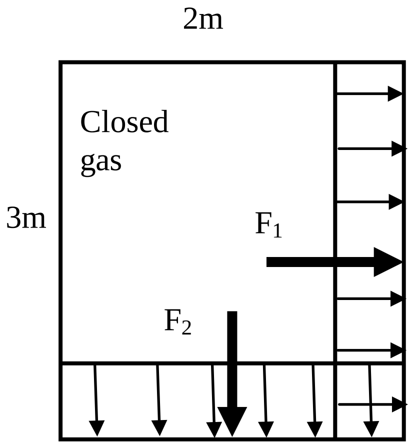
A square tank (2 × 2) × 3 m high. Calculate the force on one of the vertical sides of the tank and in its bottom on the following cases :-

1 – Tank is closed containing gas of pressure 5 bar

2 – Tank is opened containing water height of 2.5 m



1 – gas at 5 bar



a) side $\quad F_1 = PA_1$

$= 5 * 10^5 * 2 * 3$

$= 3 * 10^6$ N

$= 3$ MN $\quad \perp$ side at center

b) bottom $\quad F_2 = PA_2$

$= 5 * 10^5 * 2 * 2$

$= 2 * 10^6$ N

$= 2$ MN $\quad \perp$ bottom at center

2 – Water with 2.5 height

a) side $\quad F_1 = \gamma_\omega \, h_0 * \frac{h_0}{2} * B$

$= 9800 * \frac{(2.5)^2}{2} * 2$

$=$ N $\quad \perp$ side at $\frac{2.5}{3}$ from bottom

b) bottom $\quad F_2 = \gamma_\omega \, h_0 * A$

$= 9800 * 2.5 * (2*2)$

$=$ N $\quad \perp$ bottom at center