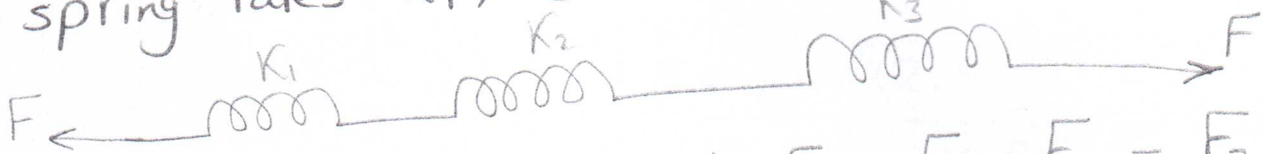
sheet

No 1

Expression for the resultant spring rate K

(a) spring rates K_1, K_2 and K_3 Connected in series

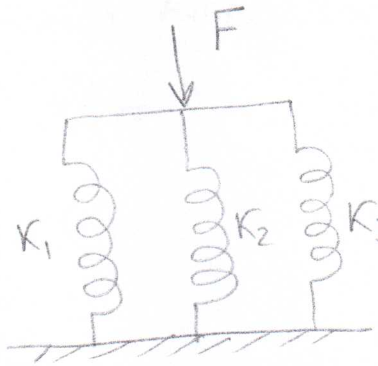


$\delta = \delta_1 + \delta_2 + \delta_3$ & $F = F_1 = F_2 = F_3$ & $F = K$

$$\frac{F}{K_{eq}} = \frac{F_1}{K_1} + \frac{F_2}{K_2} + \frac{F_3}{K_3}$$

$$\boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}}$$

(b) Connected in parallel



$\delta = \delta_1 = \delta_2 = \delta_3$

$F = F_1 + F_2 + F_3$

$K_{eq}\,\delta = K_1\delta_1 + K_2\delta_2 + K_3\delta_3$

$$\boxed{K_{eq} = K_1 + K_2 + K_3}$$

No 2

Compression spring $d = 3\,mm$ $N = 10$ Coils

$D = 25\,mm$

d 2.95 $\longrightarrow$ 3.05 mm

D 24.5 $\longrightarrow$ 25.5 mm

N 9.75 $\longrightarrow$ 10.25 Coil

G 78 $\longrightarrow$ 82 GPa

Find the spring rate when

(a) all the Variables tend to give the weakest spring

$$K = \frac{F}{\delta} = F * \frac{G d^4}{8 F D^3 N} = \frac{G d^4}{8 D^3 N}$$

$$K_{min} = \frac{G_{min}\, d^4_{min}}{8 D^3_{max}\, N_{max}} = \frac{78 * 10^3 * (2.95)^4}{8 * (25.5)^3 * 10.25} = 4.34 \frac{N}{mm}$$

(b) the stiffest spring

$$K_{max} = \frac{G_{max}\, d^4_{max}}{8 D^3_{min}\, N_{min}} = \frac{82 * 10^3 * (3.05)^4}{8 * (24.5)^3 * 9.75} = 6.186 \frac{N}{mm}$$

No (3)

helical spring $\quad F = 665\,N \quad \delta = (\delta_{max} - 10)\,mm$

$d = 6\,mm \quad c = 6 \quad \delta_{max}$ highly Compressed

static load $\quad \tau_{max} = 410\,MPa \quad N = ?? \quad G = 80\,GPa$

$$c = \frac{D}{d} \quad \therefore D = c\,d = 6 * 6 = 36\,mm$$

$$\tau_{max} = K \frac{8 F_{max} D}{\pi d^3}$$

$$K = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4*6-1}{4*6-4} + \frac{0.615}{6} = 1.2525$$

$$410 = 1.2525 * \frac{8 F_{max} * 36}{\pi * 6^3}$$

$$\therefore F_{max} = 771.3 \text{ Newton}$$

$$\delta_{max} = \frac{8 F_{max} D^3 N}{G d^4} = \frac{8 * 771.3 * (36)^3 N}{80 * 10^3 * 6^4} = 2.77 N$$

$$\delta = \delta_{max} - 10 = \frac{8 F D^3 N}{G d^4}$$

$$\frac{8 * 665 * 36^3 N}{80 * 10^3 * 6^4} = 2.77 N - 10$$

$$10 = 2.77 N - 2.394 N = 0.376 N$$

$$\boxed{N = 26.6 \simeq 27 \text{ Coils}}$$

NO (4)

two Concentric helical Compression springs $F = 45$

outer spring (1)

$D_o = 200\,mm$

$d = 40\ mm$

$N = 5.5$ Coils $- 2$
$= 3.5$ Coils

$l_{free} = 200\ mm$

$\delta = 30\,mm$

innerspring (2)

$D_o = 115\,mm$

$d = 25\,mm$

$N = 7$ Coils $- 2$
$= 5$ Coils

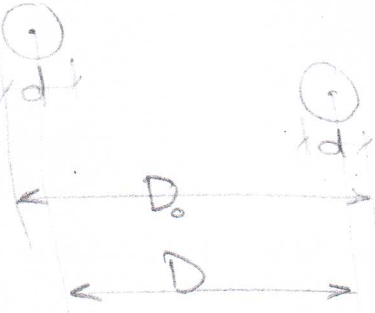
$l_{free} = 190\ mm$

$\delta = 20\ mm$

$l = 170m$

$\delta = l_{free} - l$

(a) Determine F_1 & F_2



outer

$$\delta_1 = 30 = \frac{8 F_1 D^3 N}{G d^4} \longrightarrow (1)$$

$D = D_o - d = 200 - 40 = 160\,mm$

sub. in (1)

$$30 = \frac{8 F_1 (160)^3 * 3.5}{G\ (40)^4} \qquad \therefore F_1 = 0.67\,G \longrightarrow (2)$$

inner

assume same material springs $G_1 = G_2 = G$

$D = D_o - d = 115 - 25 = 90\ mm$

$$\delta_2 = 20 = \frac{8 F_2 D^3 N}{G d^4} = \frac{8 F_2 * (90)^3 * 5}{G\,(25)^4}$$

$F_2 = 0.268\ G \longrightarrow (3)$

$F_1 + F_2 = F = 450 * 10^3$

$0.67\,G + 0.268\,G = 450 * 10^3 \qquad \therefore G = 479744$

from (2) $F_1 = 0.67\,G = 321428.48$ Newton $= 321.4\ KN$

from (3) $F_2 = 0.268\,G = 128571.4$ Newton $= 128.6\ KN$

(b) Determine τ_1 & τ_2

outer

$$C = \frac{D}{d} = \frac{160}{40} = 4$$

$$K_1 = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4*4-1}{4*4-4} + \frac{0.615}{4} = 1.4$$

$$\tau_1 = K_1 \frac{8 F_1 D_1}{\pi d_1^3} = 1.4 * \frac{8 * 321428.48 * 160}{\pi (40)^3} = 2872.5 MPa$$

inner

$$C = \frac{D}{d} = \frac{90}{25} = 3.6$$

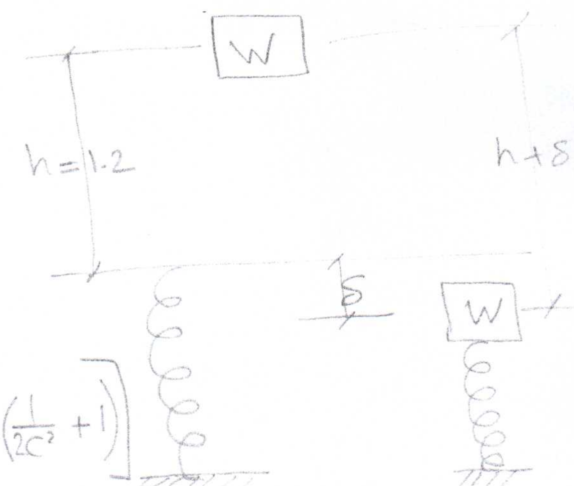
$$K_2 = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4*3.6-1}{4*3.6-4} + \frac{0.615}{3.6} = 1.46$$

$$\tau_2 = K_2 \frac{8 F_2 D_2}{\pi d_2^3} = 1.46 * \frac{8 * 128571.4 * 90}{\pi (25)^3} = 2753.3 MPa$$

No 5 8 identical springs in parallel $G = 80 GPa$

$W = 30 KN$ $h = 1.2 m$ from rest $\tau_{max\ each} = ??$

$d = 25 mm$ $C = 6$ $N = 15$ turns



Potential energy of elevator = total strain energy of 8 springs

$$W*(h+\delta) = 8U$$

$$30*10^3*\left(1.2*10^3 + \frac{8FD^3N}{Gd^4}\right) = 8\left[\frac{4F^2D^3N}{Gd^4}\left(\frac{1}{2C^2}+1\right)\right] \longrightarrow (1)$$

$$C = \frac{D}{d} \quad \therefore D = Cd = 6*25 = 150 mm$$

$$\frac{8D^3N}{Gd^4} = \frac{8(150)^3*15}{80*10^3*(25)^4} = 0.01296$$

sub. in (1)

$$3.6*10^7 + 30*10^3*0.01296F = 4*0.01296F^2\left(\frac{1}{2*36}+1\right)$$

$$0.05184F^2 - 388.8F - 3.6*10^7 = 0$$

$$F = \frac{+388.8 \pm \sqrt{(-388.8)^2 + 4*0.05184*3.6*10^7}}{2*0.05184}$$

$$F = \frac{388.8}{2*0.05184} \pm 26617.8$$

✓ $F = 30367.8$ Newton

✗ $F = -22867.8$ Newton

$\therefore F_{\substack{each\\max}} = 30367.8$ Newton

$$\tau_{\substack{max\\each}} = K\frac{8FD}{\pi d^3}$$

$$= 1.2525*\frac{8*30367.8*150}{\pi(25)^3}$$

$$= 929.83\,MPa$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4*6-1}{4*6-4} + \frac{0.615}{6}$$

$$= 1.2525$$