Chapter 1

Introduction

This book has been described by many writers as the "how-to guide for engineers interested in computing turbulent flows." This description is consistent with the contents of the book in the following sense. While the text provides some discussion of the physics of turbulent flows, it is by no means a thorough treatise on the complexities of the phenomenon. Rather, the discussion focuses on the most significant aspects of turbulence that underlie the engineering approximations introduced over the decades to facilitate affordable numerical computations.

In other words, the book presents as much of the physics of turbulence as necessary to understand why existing modeling approximations have been made—but no more. This is true because the theme of the book is the modeling of turbulence, which begins with understanding the physics involved. However, it also involves correlation of measurements, engineering judgment, a healthy dose of mathematics and a lot of trial and error.

The field is, to some extent, a throwback to the days of Prandtl, Taylor, von Kármán and all the many other clever engineers who spent a good portion of their time devising engineering approximations and models describing complicated

Figure 1.1: *Pioneers of turbulence modeling; from left Ludwig Prandtl (1875-1953), Geoffrey Taylor (1886-1975) and Theodore von Kármán (1881-1963).*

physical flows. The best efforts in turbulence modeling have been an attempt to develop a set of constitutive equations suitable for application to general turbulent flows, and to do it in as elegant and physically sound a manner as possible. These three fluid mechanics pioneers helped establish a solid framework for several generations of engineers to work in.

Turbulence modeling is one of three key elements in Computational Fluid Dynamics (CFD). Very precise mathematical theories have evolved for the other two key elements, viz., grid generation and algorithm development. By its nature — in creating a mathematical model that approximates the physical behavior of turbulent flows — far less precision has been achieved in turbulence modeling. This is not really a surprising event since our objective has been to approximate an extremely complicated phenomenon. Two key questions we must ask at the outset are the following. What constitutes the ideal turbulence model and how complex must it be?

1.1 Definition of an Ideal Turbulence Model

Simplicity combined with physical insight seems to have been a common denominator of the work of great men like Prandti, Taylor and von Kármán. Using their work as a gauge, **an ideal model should introduce the minimum amount of complexity while capturing the essence of the relevant physics**. This description of an ideal model serves as the keystone of this text.

1.2 How Complex Must a Turbulence Model Be?

Aside from any physical considerations, turbulence is inherently three dimensional and time dependent. Thus, an enormous amount of information is required to completely describe a turbulent flow. Fortunately, we usually require something less than a complete time history over all spatial coordinates for every flow property. Thus, for a given turbulent-flow application, we must pose the following question. Given a set of initial and/or boundary conditions, how do we predict the relevant properties of the flow? What properties of a given flow are relevant is generally dictated by the application. For the simplest applications, we may require only the skin-friction and heat-transfer coefficients. More esoteric applications may require detailed knowledge of energy spectra, turbulence fluctuation magnitudes and scales.

Certainly, we should expect the complexity of the mathematics required for a given application to increase as the amount of required flowfield detail increases. On the one hand, if all we require is skin friction for an attached flow, a simple mixing-length model (Chapter 3) may suffice. Such models are well developed and can be implemented with very little specialized knowledge. On the other hand, if we desire a complete time history of every aspect of a turbulent flow, only a solution to the complete Navier-Stokes equation will suffice. Such a solution requires an extremely accurate numerical solver and may require use of subtle transform techniques, not to mention vast computer resources. Most engineering problems fall somewhere between these two extremes.

Thus, once the question of how much detail we need is answered, the level of complexity of the model follows, qualitatively speaking.[1] In the spirit of Prandtl, Taylor and von Kármán, the conscientious engineer will strive to use as conceptually simple an approach as possible to achieve his ends. Overkill is often accompanied by unexpected difficulties that, in CFD applications, almost always manifest themselves as numerical difficulties!

1.3 Comments on the Physics of Turbulence

Before plunging into the mathematics of turbulence, it is worthwhile to first discuss physical aspects of the phenomenon. The following discussion is not intended as a complete description of this complex topic. Rather, we focus upon a few features of interest in engineering applications, and in construction of a mathematical model. For a more-complete introduction, refer to basic texts on the physics of turbulence such as those by Hinze (1975), Tennekes and Lumley (1983), Landahl and Mollo-Christensen (1992), Libby (1996) or Durbin and Pettersson Reif (2001).

1.3.1 Importance of Turbulence in Practical Situations

For "small enough" scales and "low enough" velocities, in the sense that the Reynolds number is not too large, the equations of motion for a viscous fluid have well-behaved, steady solutions. Such flows are controlled by viscous diffusion of vorticity and momentum. The motion is termed laminar and can be observed experimentally and in nature.

At larger Reynolds numbers, the fluid's inertia overcomes the viscous stresses, and the laminar motion becomes unstable. Rapid velocity and pressure fluctuations appear and the motion becomes inherently three dimensional and unsteady. When this occurs, we describe the motion as being turbulent. In the cases of fully-developed Couette flow and pipe flow, for example, laminar flow is assured only if the Reynolds number based on maximum velocity and channel height or pipe radius is less than 1500 and 2300, respectively.

[1] This is not a foolproof criterion, however. For example, a complicated model may be required to predict even the simplest properties of a very complex flow.

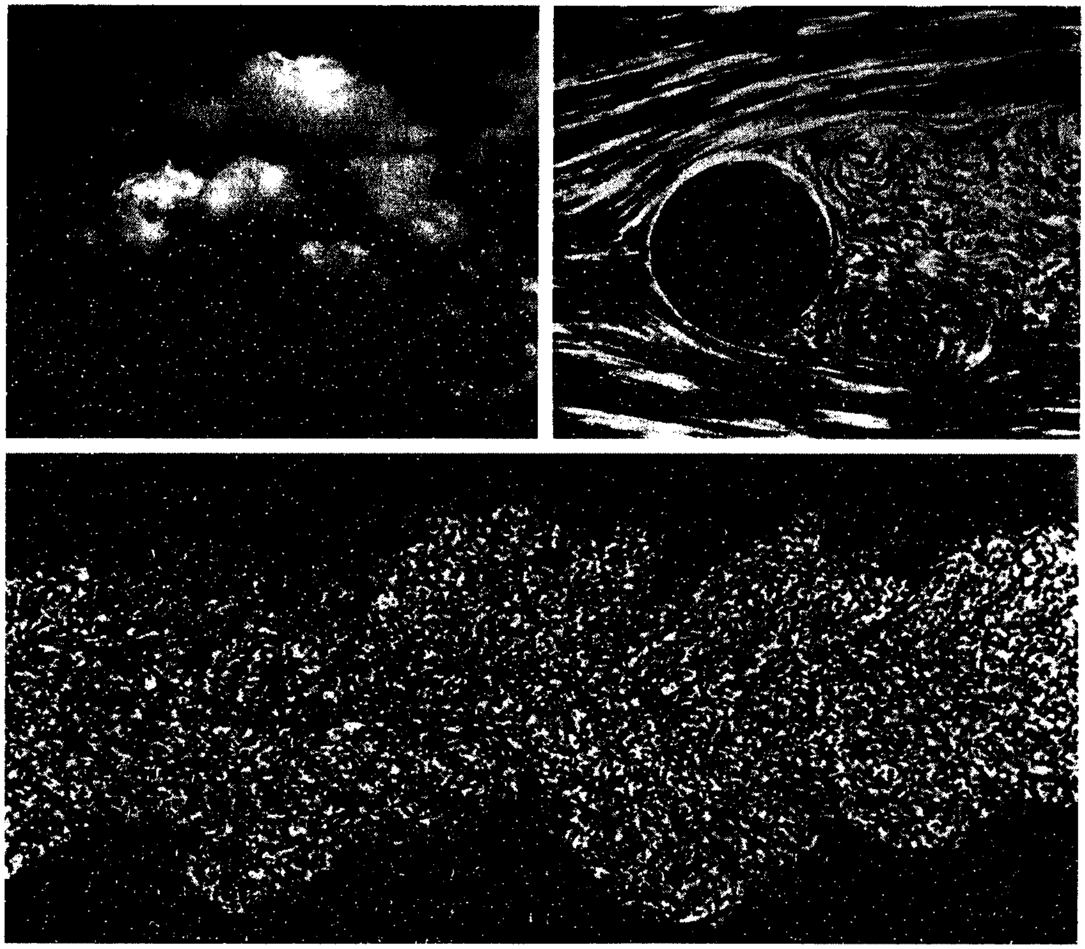

Figure 1.2: *Examples of turbulent motion. Upper left: a cumulus cloud; Upper right: flow in the wake of a cylinder; Bottom: flow in the wake of a bullet [Bottom photograph courtesy of Corrsin and Kistler (1954)].*

Virtually all flows of practical engineering interest are turbulent. Flow past vehicles such as rockets, airplanes, ships and automobiles, for example, is always turbulent. Turbulence dominates in geophysical applications such as river currents, the planetary boundary layer and the motion of clouds (Figure 1.2). Turbulence even plays a role at the breakfast table, greatly enhancing the rate at which sugar and cream mix in a cup of coffee!

Turbulence matters even in applications that normally involve purely laminar flow. For example, blood flow is laminar in the arteries and veins of a healthy human. However, the presence of turbulence generally corresponds to a health problem such as a defective heart valve.

Turbulent flow always occurs when the Reynolds number is large. For slightly viscous fluids such as water and air, "large" Reynolds number corresponds to anything stronger than a tiny swirl, a small breeze or a puff of wind. Thus, to analyze fluid motion for general applications, we must deal with turbulence. Although vigorous research has been conducted to help discover the mysteries of turbulence, it has been called the major unsolved problem of classical physics! In the following subsections, we will explore some of the most important aspects of turbulence.

1.3.2 General Properties of Turbulence

- **Basic Definition.** In 1937, von Kármán defined turbulence in a presentation at the Twenty-Fifth Wilbur Wright Memorial Lecture entitled "Turbulence." He quoted G. I. Taylor as follows [see von Kármán (1937)]:

  > *"Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighboring streams of the same fluid flow past or over one another."*

  As the understanding of turbulence has progressed, researchers have found the term "irregular motion" to be too imprecise. Simply stated, an irregular motion is one that is typically aperiodic and that cannot be described as a straightforward function of time and space coordinates. An irregular motion might also depend strongly and sensitively upon initial conditions. The problem with the Taylor-von Kármán definition of turbulence lies in the fact that there are nonturbulent flows that can be described as irregular.

  Turbulent motion is indeed irregular in the sense that it can be described by the laws of probability. Even though instantaneous properties in a turbulent flow are extremely sensitive to initial conditions, statistical averages of the instantaneous properties are not. To provide a sharper definition of turbulence, Hinze (1975) offers the following revised definition:

  > *"Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned."*

  To complete the definition of turbulence, Bradshaw [cf. Cebeci and Smith (1974)] adds the statement that *turbulence has a wide range of scales*. Time and length scales of turbulence are represented by frequencies and wavelengths that are revealed by a Fourier analysis of a turbulent-flow time history.

  The irregular nature of turbulence stands in contrast to laminar motion, so called historically because the fluid was imagined to flow in smooth laminae, or layers. In describing turbulence, many researchers refer to **eddying motion**, which is a local swirling motion where the vorticity can often be very intense. **Turbulent eddies** of a wide range of sizes appear and give rise to vigorous mixing and effective turbulent stresses (a consequence of the "mixing" of momentum) that can be enormous compared to laminar values.

- **Instability and Nonlinearity.** Analysis of solutions to the Navier-Stokes equation, or more typically to its boundary-layer form, shows that turbulence develops as an instability of laminar flow. To analyze the stability of laminar flows, classical methods begin by linearizing the equations of motion. Although linear theories achieve some degree of success in predicting the onset of instabilities that ultimately lead to turbulence, the inherent nonlinearity of the Navier-Stokes equation precludes a complete analytical description of the actual transition process, let alone the fully-turbulent state. For a real (i.e., viscous) fluid, mathematically speaking, the instabilities result mainly[2] from interaction between the Navier-Stokes equation's nonlinear inertial terms and viscous terms. The interaction is very complex because it is rotational, fully three dimensional and time dependent.

  As an overview, the nonlinearity of the Navier-Stokes equation leads to interactions between fluctuations of differing wavelengths and directions. As discussed below, the wavelengths of the motion usually extend all the way from a maximum comparable to the width of the flow to a minimum fixed by viscous dissipation of energy. The main physical process that spreads the motion over a wide range of wavelengths is vortex stretching. The turbulence gains energy if the vortex elements are primarily oriented in a direction in which the mean velocity gradients can stretch them. Most importantly, wavelengths that are not too small compared to the mean-flow width interact most strongly with the mean flow. **Consequently, the larger-scale turbulent motion carries most of the energy and is mainly responsible for the enhanced diffusivity and attending stresses.** In turn, the larger eddies randomly stretch the vortex elements that comprise the smaller eddies, cascading energy to them. Energy is dissipated by viscosity in the shortest wavelengths, although the *rate* of dissipation of energy is set by the long-wavelength motion at the start of the cascade. The shortest wavelengths simply adjust accordingly.

- **Statistical Aspects.** The time-dependent nature of turbulence also contributes to its intractability. The additional complexity goes beyond the introduction of an additional dimension. Turbulence is characterized by

[2]Inviscid instabilities, such as the Kelvin-Helmholtz instability, also play a role.

random fluctuations thus mandating the use of statistical methods to analyze it. On the one hand, this aspect is not really a problem from the engineer's viewpoint. Even if we had a complete time history of a turbulent flow, we would usually integrate the flow properties of interest over time to extract **time averages**, or **mean values**. On the other hand, as we will see in Chapter 2, time-averaging operations lead to terms in the equations of motion that cannot be determined a priori.

- **Turbulence is a Continuum Phenomenon.** In principle, we know that the time-dependent, three-dimensional continuity and Navier-Stokes equations contain all of the physics of a given turbulent flow. That this is true follows from the fact that turbulence is a continuum phenomenon. As noted by Tennekes and Lumley (1983),

  > *"Even the smallest scales occurring in a turbulent flow are ordinarily far larger than any molecular length scale."*

  Nevertheless, the smallest scales of turbulence are still extremely small (we will see just how small in the next subsection). They are generally many orders of magnitude smaller than the largest scales of turbulence, the latter often being of the same order of magnitude as the dimension of the object about which the fluid is flowing. Furthermore, the ratio of smallest to largest scales decreases rapidly as the Reynolds number increases. To make an accurate numerical simulation (i.e., a fully time-dependent three-dimensional solution) of a turbulent flow, all physically relevant scales must be resolved.

  While more and more progress is being made with such simulations, computers of the early twenty-first century have insufficient memory and speed to solve any turbulent-flow problem of practical interest. To underscore the magnitude of the problem, Speziale (1985) notes that a numerical simulation of turbulent pipe flow at a Reynolds number of 500,000 would require a computer 10 million times faster than a Cray Y/MP. While standard personal computers are comparable in speed to a vintage 1985 Cray Y/MP, modern mainframe computers are still confined to simple geometries at low Reynolds numbers. This is true because, as discussed in Chapter 8, the number of numerical operations in such a computation is proportional to $Re^{9/4}$, where Re is a characteristic Reynolds number. However, the results are very useful in developing and testing approximate methods.

- **Vortex Stretching.** The strongly rotational nature of turbulence goes hand-in-hand with its three dimensionality. The vorticity in a turbulent flow is itself three dimensional so that vortex lines in the flow are non-parallel. The resulting vigorous stretching of vortex lines maintains the

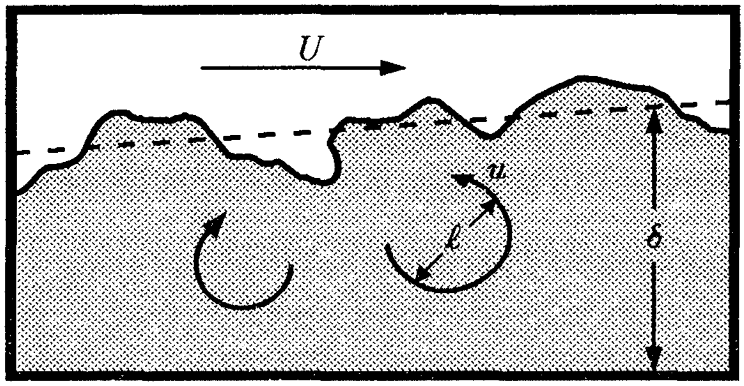

Figure 1.3: *Schematic of large eddies in a turbulent boundary layer. The flow above the boundary layer has a steady velocity U; the eddies move at randomly-fluctuating velocities of the order of a tenth of U. The largest eddy size (ℓ) is comparable to the boundary-layer thickness (δ). The interface and the flow above the boundary is quite sharp [Corrsin and Kistler (1954)].*

ever-present fluctuating vorticity in a turbulent flow. Vortex stretching is absent in two-dimensional flows so that turbulence must be three dimensional. This inherent three dimensionality means there are no satisfactory two-dimensional approximations for determining fine details of turbulent flows. This is true even when the average motion is two dimensional. The induced velocity field attending these skewed vortex lines further increases three dimensionality and, at all but very low Reynolds numbers, the vorticity is drawn out into a tangle of thin tubes or sheets. Therefore, **most of the vorticity in a turbulent flow resides in the smallest eddies**.

- **Turbulence Scales and the Cascade.** Turbulence consists of a continuous spectrum of scales ranging from largest to smallest, as opposed to a discrete set of scales. In order to visualize a turbulent flow with a spectrum of scales we often cast the discussion in terms of eddies. As noted above, a turbulent eddy can be thought of as a local swirling motion whose characteristic dimension is the local turbulence scale (Figure 1.3). Alternatively, from a more mathematical point of view, we sometimes speak in terms of wavelengths. When we think in terms of wavelength, we imagine we have done a Fourier analysis of the fluctuating flow properties.

  We observe that eddies overlap in space, large ones carrying smaller ones. Turbulence features a **cascade process** whereby, as the turbulence decays, its kinetic energy transfers from larger eddies to smaller eddies. Ultimately, the smallest eddies dissipate into heat through the action of molecular viscosity. Thus, we observe that, like any viscous flow, **turbulent flows are always dissipative**.

Figure 1.4: *Laser-induced fluorescence image of an incompressible turbulent boundary layer. Flow is from left to right and has been visualized with disodium fluorescein dye in water. Reynolds number based on momentum thickness is 700. [From C. Delo—Used with permission.]*

- **Large Eddies and Turbulent Mixing.** An especially striking feature of a turbulent flow is the way large eddies migrate across the flow, carrying smaller-scale disturbances with them. The arrival of these large eddies near the interface between the turbulent region and nonturbulent fluid distorts the interface into a highly convoluted shape (Figures 1.3 and 1.4). In addition to migrating across the flow, they have a lifetime so long that they persist for distances as much as 30 times the width of the flow [Bradshaw (1972)]. **Hence, the state of a turbulent flow at a given position depends upon upstream history and cannot be uniquely specified in terms of the local strain-rate tensor as in laminar flow.**

- **Enhanced Diffusivity.** Perhaps the most important feature of turbulence from an engineering point of view is its enhanced diffusivity. Turbulent diffusion greatly enhances the transfer of mass, momentum and energy. Apparent stresses in turbulent flows are often several orders of magnitude larger than in corresponding laminar flows.

In summary, turbulence is dominated by the large, energy-bearing, eddies. The large eddies are primarily responsible for the enhanced diffusivity and stresses observed in turbulent flows. Because large eddies persist for long distances, the diffusivity and stresses are dependent upon flow history, and cannot necessarily be expressed as functions of local flow properties. Also, while the small eddies ultimately dissipate turbulence energy through viscous action, the rate at which they dissipate is controlled by the rate at which they receive energy from the largest eddies. These observations must play an important role in the formulation of any rational turbulence model. As we progress through the following chapters, we will introduce more specific details of turbulence properties for common flows on an as-needed basis.