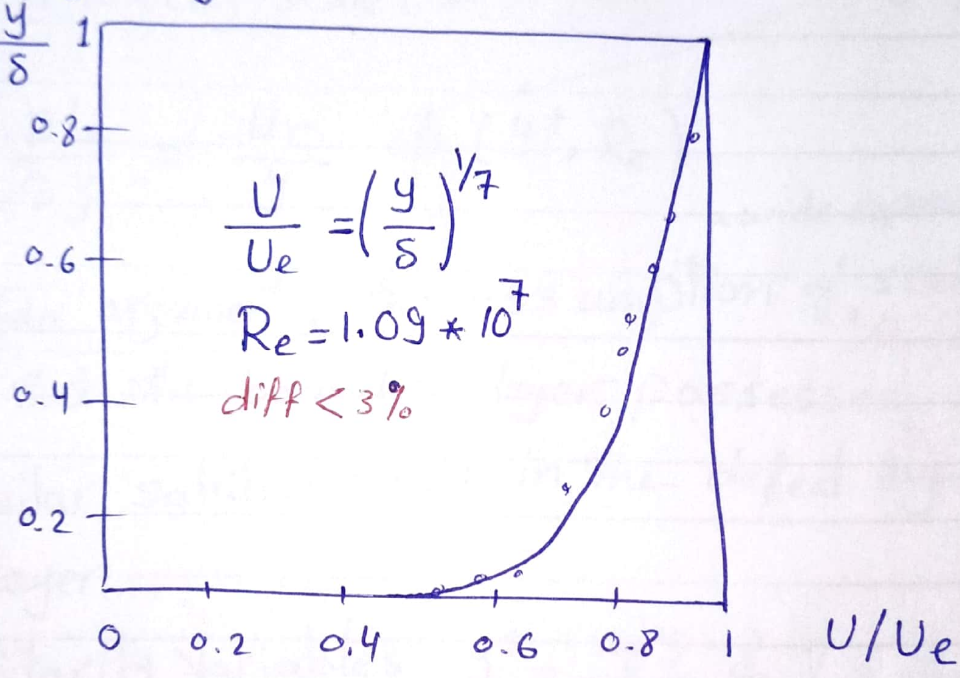
power laws

$$\frac{U}{U_e} = \left(\frac{y}{\delta}\right)^{1/n}$$ → integer between 6 and 8

* Power-law relationship is an approximation for turbulent boundary layer profiles.

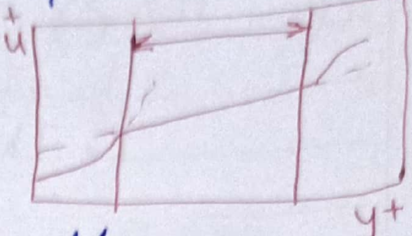
* Prandtl suggested $n = 7$. this yields a good approximation at high Re for flat-plate boundary layer.



* Barenblatt and others have challenged the validity of the law of the wall
  - a power-law variation of the velocity in the inner layer better correlates pipe-flow measurements and represents a more realistic description in a boundary layer.
  - Their critical assumption is the existence of a wide separation of scales (large $\frac{\delta}{(\nu/u_\tau)}$)

- They maintain that the turbulence in the overlap region is Re dependent.

- If this is true



the law of the wall will be replaced by

universal function

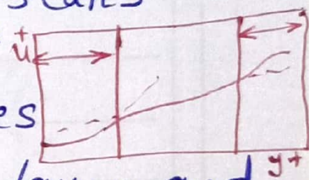
$$U = u_\tau \tilde{f}(y^+, Re)$$

defect law will be replaced by $U = U_e - u_o \tilde{g}(\eta, Re)$

u_o is a velocity scale. It is not necessarily equal to u_τ

$$\frac{\partial U}{\partial y} = \frac{u_\tau}{y} \Phi(y^+, Re)$$

* In Millikan argument, the assumption of a wide separation of scales implies that the boundary layer possesses self-similar solutions both in the defect layer and the sublayer.



- similarity variables $y^+ = \frac{u_\tau y}{\nu}$ & $\eta = \frac{y}{\Delta}$ exists in each region.
- the assumption that y^+ & η are distinct variables (described as Complete similarity in the overlap region)

[by Contrast, Barenblatt hypothesis Corresponds to in Complete similarity]

* InComplete similarity

$y^+ \longrightarrow \infty$ $\quad \Phi(y^+, Re)$ not necessarily approach a Constant value even if $Re \to \infty$

$$\Phi(y^+, R_e) = A(y^+)^{\alpha} \quad \text{for large } y^+$$

Coefficient f^n of Re — exponent f^n of Re

* Barenblatt, chorin and ProstoKishin assumed [incomplete similarity in y^+ and no similarity in Re]

- Combine the previous two equations

$$\frac{\partial U^+}{\partial y^+} = A(y^+)^{\alpha - 1}$$

integrate

$$U^+ = \frac{A}{\alpha}(y^+)^{\alpha}$$

- from experimental data for pipe flow

$$A = 0.577 \ln Re + 2.5 \quad \& \quad \alpha = \frac{1.5}{\ln Re}$$

Re based on average vel. and pipe diameter

* Zagarola, Perry and smits Considered more recent experiment with a much wider range of Re
- They Concluded that classical law of the wall provides closer correlation with measurements than the power law & they recommend larger value of K of 0.44
- or use $A = 0.7053 \ln Re + 0.3055$ &

$$\alpha = \frac{1.085}{\ln Re} + \frac{6.532}{(\ln Re)^2}$$

& still law of the wall is better