Midterm 2014-2015

EME 403

Q1

A)

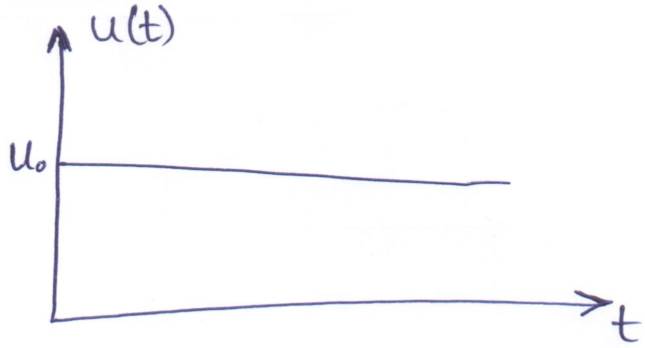
* Modeling: It is the process of identifying the principal physical dynamic effects to be considered in analyzing a system, writing the differential and algebraic eqns from the conservation laws of the relevant discipline and reducing the equation to be convenient differential equation form.

* Response: The behavior of a system is characterized by its response to external inputs, disturbances, and initial conditions.

* External disturbances: They are the external environmental effects that may occur randomly or unexpectedly.

* Initial Conditions: They are the initial values of the dynamic variables of the systems.

1.2. step input response $u(t) = U_o$



$$\boxed{\tau \dot{X} + X = G U_o}$$

as

$X(t) = X_h(t) + X_p(t)$

$X_h = A e^{-t/\tau}$ & $X_p = B$

$\dot{X}_p = 0$ as B is Constants

,* to find B, solve 1st order differential eqn for X_p only.

$$\tau \dot{X}_p + X_p = G U_o$$

$$0 + B = G U_o \qquad \therefore \boxed{B = G U_o}$$

,* $X(t) = A e^{-t/\tau} + B$

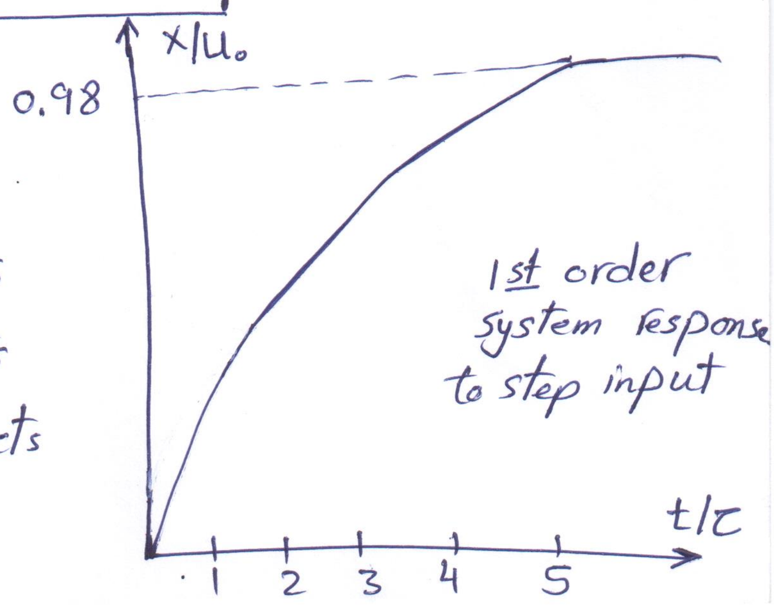
to find the Constant A apply the initial Conditions.

$$X_o = A \cancel{e^0}^{1} + G U_o \qquad \boxed{\therefore A = X_o - G U_o}$$

$$\therefore X(t) = [X_o - G U_o] e^{-t/\tau} + G U_o$$

$$\boxed{X(t) = X_o e^{-t/\tau} + G U_o [1 - e^{-t/\tau}]}$$

* As time increases (after a time equal four times from τ), $X(t)$ approaches τU_o and all dynamic effects died away (all derivative effects are negligible).



Q2

A) $L D i_L + R i_L = e_o$

$$\left[\frac{L}{R} D + 1\right] i_L = \frac{e_o}{R}$$

$$\left[\frac{6 * 10^{-3}}{100} D + 1\right] i_L = \frac{1}{100} e_o$$

$$\boxed{[\tau D + 1] X = G U}$$

time Constant $\tau = 6 * 10^{-5}$ sec

response variable i_L

external input e_o

Gain $G = \frac{1}{100}$

B) $LC D^2 e_2 + RCD e_2 + e_2 = e_o$

$$[LCD^2 + RCD + 1] e_2 = e_o$$

$$[1 \times 10^{-3} * 10 * 10^{-6} D^2 + 14 * 10 * 10^{-6} D + 1] e_2 = e_o$$

$$\boxed{\left[\frac{1}{\omega_n^2} D^2 + \frac{2\zeta}{\omega_n} D + 1\right] X = G U}$$

$$\frac{1}{\omega_n^2} = 1 * 10^{-3} * 10 * 10^{-6} = 1 * 10^{-8}$$

$\therefore \omega_n = 10^4$ Hz. natural frequency

$$\frac{2\zeta}{\omega_n} = 14 * 10 * 10^{-6}$$

$\therefore \zeta = 0.7$ damping ratio

$(\zeta < 1$ under damping$)$

response variable e_2

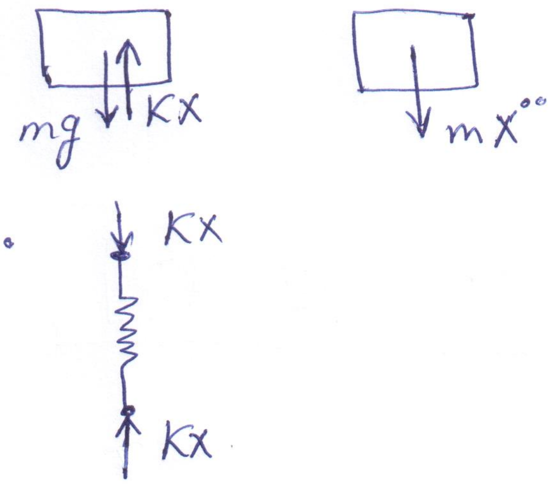
external input e_o

Gain $G = 1$

Q3

A)

$$+\downarrow \quad \Sigma F_y = m\ddot{x}$$



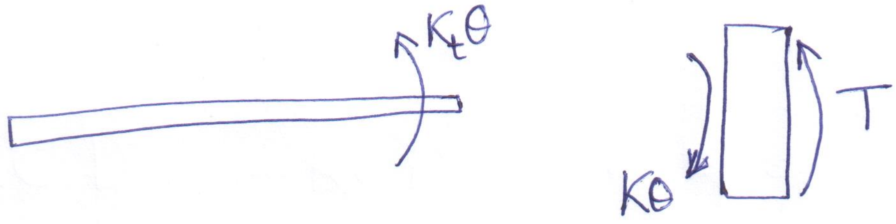
$$mg - Kx = m\ddot{x}$$

$$500 * 9.81 - 2.5 * 10^3 x = 500\ddot{x}$$

$$500\ddot{x} + 2.5 * 10^3 x = 4905$$

$$\boxed{\left[0.2 D^2 + 1\right] x = 1.96}$$

B)



$$\Sigma M = J\ddot{\theta}$$

$$T - K_t\theta = J\ddot{\theta}$$

$$J\ddot{\theta} + K_t\theta = T$$

$$0.5\ddot{\theta} + 20\theta = 2$$

$$\boxed{\left[0.025 D^2 + 1\right] \theta = 0.1}$$