Question (1) :-

(A) Define The Following :-

(1) System : It is a Collection From The Component of major engineering desciplines system.

(2) Inputs : The Independent variables of The differential equation.

(3) Response : refers to external input, Initial Condition and disturbances.

(B) Give one excample (declare input, output and disturbances) For each System :-

(I) Electrical System :

Excample → Television recieuier
Input → Frequency of old channel
output → frequency of New channel
disturbances → Noise
response → Changing channel.

(II) Fluid System :

Excample → City water Tower
Input → water pumped.
output → hight of water
disturbances → The amount being used In Citizens
response → Changing in water Level.

(III) Thermo-mechanical system

Example → Combustion engine
Input → Combustion of fuel & air
Output → movement of engine
disturbances → Impurties in Gasoline
response → movement of piston inside engine

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Question (2) :-

(A) prove the first order differential equation using free response will be in this form $x(t) = x_0 e^{-t/\tau}$ (use neat sketch).

* Solution *

* Free response $u(t) = 0$

$\tau$ : Time const
G : Gain

$$(\tau D + 1)x = G\,u(t)$$

$$\tau \dot{x} + x = 0 \longrightarrow (1)$$

$$x(t) = x_h + x_p$$

$x_h$ : homogenous solns
$x_p$ : particular soln

$$x_h = A e^{\lambda t} \qquad \Bigg\| \qquad x_p = \text{Zero}.$$

$$\left.\begin{aligned} \therefore x(t) &= A e^{\lambda t} \\ \dot{x} &= A \lambda e^{\lambda t} \end{aligned}\right\} \rightarrow ②$$

Substituting from equation (2) in equation (1).

$$\tau A \lambda e^{\lambda t} + A e^{\lambda t} = 0 \quad \div A e^{\lambda t}$$

$$\tau \lambda + 1 = 0$$

$$\boxed{\therefore \lambda = \frac{-1}{\tau}}$$

Substituting in equation (2)

$$x(t) = A e^{-t/\tau} \quad \rightarrow (3)$$

To find $A$, apply the intial condition of eqn (3) at time $(t=0)$

$$x(0) = x_0 = A e^0$$

$$\boxed{\therefore A = x_0}$$

$$\boxed{\therefore x(t) = x_0 e^{-t/\tau}}$$

(B) Determine 1 Time const ($\tau$)
2 response variable
3 external input
4 Gain (G)

$$0.006\, D\, i_L + 100\, i_L = e_o$$

$$[0.006\, D + 100]\, i_L = e_o \qquad \div 100$$

$$[6 * 10^{-5}\, D + 1]\, i_L = 0.01\, e_o.$$

$$\boxed{(\tau D + 1)\, x = G\, u(t)}$$ $1^{st}$ order differential equation

$\therefore$ Time const $= 6 * 10^{-5}$ sec.

response variable $\longrightarrow i_L$

external input $= e_o$

Gain $= 0.01$

(C) Determine
1. response variable
2. external input
3. natural frequency ($\omega_n$)
4. damping ratio ($\varepsilon_r$)
5. Gain (G)

$$10^{-8} D^2 e_2 + 10^{-5} D e_2 + e_2 = e_o$$

* Solution *

$$\left[10^{-8} D^2 + 10^{-5} D + 1\right] e_2 = e_o.$$

$2^{nd}$ order differential equation.

$$\boxed{\left[\frac{1}{\omega_n^2} D^2 + \frac{2\varepsilon_r}{\omega_n} D + 1\right] x = G\,U(t).}$$

→ $\therefore \frac{1}{\omega_n^2} = 10^{-8}$ → $\omega_n = \sqrt{10^8} = 10000$ HZ → Natural frequency

→ $\frac{2\varepsilon_r}{\omega_n} = 10^{-5}$ → $\varepsilon_r = \frac{10^{-5}}{2} * 10000 = 0.05 < 1$

(under damped)

damping ratio

→ response variable is $e_2$

→ Gain $(G) = 1$

→ external Input $= e_o$

QUESTION (3) :-

Develop, using neat sketch, an equation describing the motion of each of the following

(A) $m = 5\ Kg.$

$K = 2000\ N/m$

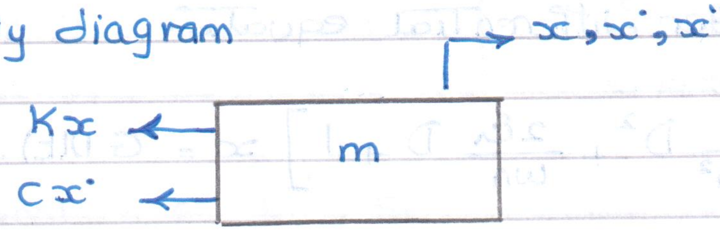
$C = 12.5\ N.S/m.$

Determine the Natural Frequency $\omega_n = ?$.

damping Factor $\varepsilon_n = ??$.

* Solution *

The Free body diagram




$$\Sigma \text{Forces} = m\ddot{x}$$

$$m\ddot{x} = -Kx - C\dot{x}$$

$$m\ddot{x} + C\dot{x} + Kx = 0$$

$$[mD^2 + CD + K]\,x = 0$$

$$[5D^2 + 12.5D + 2000]\,x = 0 \qquad \div 2000$$

$$[2.5 * 10^{-3} D^2 + 6.25 * 10^{-3} D + 1]\,x = 0$$

2nd order differential equation

$$\boxed{\left[\frac{1}{\omega_n^2} D^2 + \frac{2\varepsilon_n}{\omega_n} D + 1\right] x = G\,U(t).}$$

$$\rightarrow \quad \therefore \frac{1}{\omega_n^2} = 2.5 * 10^{-3} \longrightarrow \omega_n = \sqrt{\frac{1}{2.5 * 10^{-3}}} = 20 \text{ HZ}$$

Natural Frequency

$$\rightarrow \quad \frac{2\xi}{\omega_n} = 6.25 * 10^{-3}$$

$$\xi = \frac{6.5 * 10^{-3}}{2} * 20 = 0.0625 < 1 \quad \text{"underdamped"}$$

↳ damping ratio

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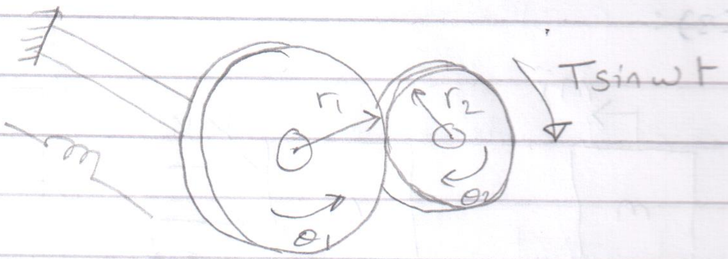
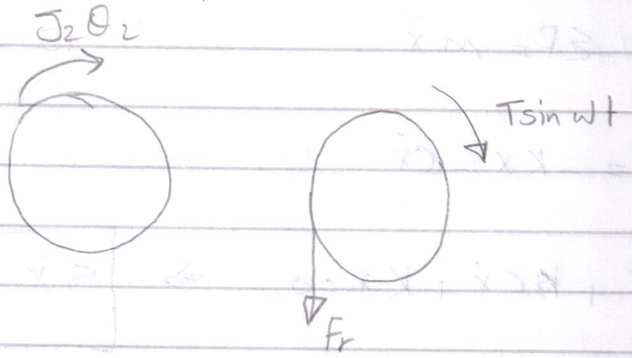
Question (3) → (b) في الخلف

b)



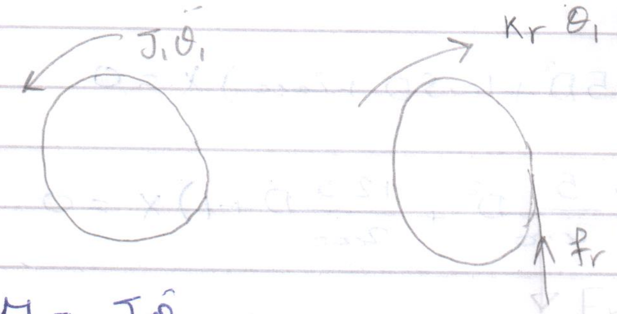

for $\theta_2$:




$$\Sigma M = J\ddot{\theta}$$

$$J_2\ddot{\theta}_2 = T\sin\omega t - F_r * r_2 \Rightarrow (1)$$

for $\theta_1$:




$$\Sigma M = J\ddot{\theta}$$

$$J_1\ddot{\theta}_1 = f_r * r_1 - k_r\theta_1$$

$$F_r = \frac{J_1\ddot{\theta}_1}{r_1} + \frac{k_r\theta_1}{r_1} = \frac{J_1\ddot{\theta}_1 + k_r\theta_1}{r_1} \Rightarrow (2)$$

Sub from ② in ①

$$J_2\ddot{\theta}_2 = T\sin\omega t - \left(\frac{J_1\ddot{\theta}_1 + K_r\theta_1}{r_1}\right)r_2$$

$$J_2\ddot{\theta}_2 = T\sin\omega t - \frac{J_1\ddot{\theta}_1 r_2}{r_1} - \frac{K_r r_2\theta_1}{r_1}$$

$$\therefore\; J_2\ddot{\theta}_2 + \frac{J_1\ddot{\theta}_1 r_2}{r_1} + \frac{K_r r_2\theta_1}{r_1} = T\sin\omega t$$

$$J_2\ddot{\theta}_2 + \frac{J_1\ddot{\theta}_1(4)}{8} + \frac{K_r(4)}{8}\theta_1 = T\sin\omega t$$