	Alexandria Higher Institute of Engineering & Technology (AIET)			
	Department of: Mechatronics	Fourth Year	4th Year	
	EME403	Dynamic System Analysis	Midterm-of-Semester-1 Exam, Nov., 23, 2015	
	Examiners:	Dr. Rola Afify and committee		Time: 1.5 hour

Answer the following questions:

Question one: (6 marks)

A) Define: **System**,

A **system** is a set of interacting components connected together in such a way that the variation or response in the state of one component affects the state of the others.

Inputs,

By **inputs,** we mean functions of the independent variable of the differential equation, the excitation, or the forcing function to the system.

and **Response**.

The behavior of a system is characterized by its **response** to external inputs, disturbances, and initial conditions.

B) Give one example (declare input, output, and disturbances) for each system:

i) Electrical System

A television receiver has a dynamic response of the beam that traces the picture on the screen of the set. The TV tuning circuit, which allows you to select the desired channel, also has a dynamic response, and a simpler, though no less important, example is the dynamic voltage and current responses that occur when you switch a light on or off.

ii) Fluid System

A city water tower has a dynamic response of the height of the water as a function of the amount of water pumped into the tower and the amount being used by the citizens. If a garden hose is suddenly blocked at its end when water is flowing through it, the pressure in the hose will have a dynamic response.

iii) Thermo-mechanical systems

A combustion engine used in a car, truck, ship, or airplane is a thermo-fluid-mechanical (or simply, thermomechanical) device, since it converts thermal energy into fluid power and then into mechanical power.

Question Two: (6 marks)

A) Prove that the solution of the first order differential equation using free response input will be in this form $x(t) = x_o e^{-t/\tau}$ (use neat sketches).

1.1. Free response $u = 0$ $\qquad x = x_h + \cancel{x_p} \to 0$

$$\boxed{\tau \dot{x} + x = 0} \qquad \therefore \boxed{\dot{x} = \frac{-1}{\tau} x}$$

The homogeneous solution is of the form $x = A e^{\lambda t}$

$\therefore \dot{x} = A \lambda e^{\lambda t}$

sub. in the 1st order eq^n

$$\cancel{A}\lambda \cancel{e^{\lambda t}} = \frac{-1}{\tau} \cancel{A}\cancel{e^{\lambda t}}$$

$$\therefore \boxed{\lambda = \frac{-1}{\tau}} \qquad \therefore X(t) = A e^{\frac{-t}{\tau}}$$

Applying the initial Condition to find the Constant A

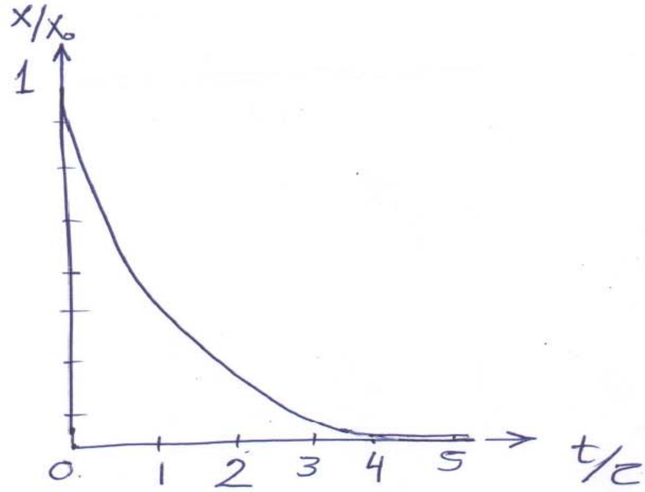
$$X(0) = X_o = A \cancel{e^{0}}$$

$$\therefore A = X_o$$

the Complete solution will be :-

$$\boxed{X(t) = X_o \, e^{\frac{-t}{\tau}}}$$

1st order systems will be very near to their final response after a time equal to 4τ has Collapsed.



$$\boxed{\begin{aligned} X &= X_h + X_p \\ X &= A e^{\lambda t} + 0 \end{aligned}}$$

B) Determine the time constant, response variable, external input and Gain for this differential equation $0.006Di_L + 100i_L = e_o$.

$0.006Di_L + 100i_L = e_o$

$$\left[\frac{6*10^{-3}}{100} D + 1\right] i_L = \frac{1}{100} e_o \qquad \boxed{[\tau D + 1]X = G\,U}$$

time Constant $\tau = 6 * 10^{-5}$ sec

response Variable i_L

external input e_o

Gain $G = \frac{1}{100}$

C) Determine response variable, external input, natural frequency, damping ratio and Gain for this differential equation $10^{-8}D^2e_2 + 10^{-5}De_2 + e_2 = e_o$.

$$\boxed{\left[\frac{1}{\omega_n^2}D^2 + \frac{2\zeta}{\omega_n}D + 1\right]X = GU}$$

$\frac{1}{\omega_n^2} = 1 * 10^{-8}$

$\therefore \omega_n = 10^4$ Hz. natural frequency

$\frac{2\zeta}{\omega_n} = 10^{-5}$

$\therefore \zeta = 0.05$ damping ratio

$(\zeta < 1$ under damping)

response variable e_2

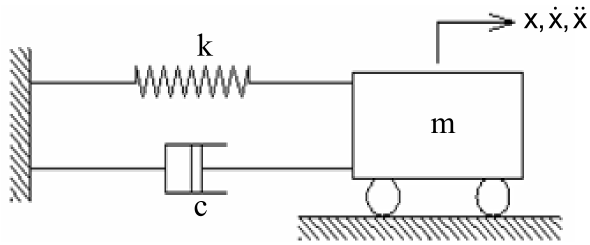
external input e_o

Gain $G = 1$

Question Three: (8 marks)

Develop, using neat sketches, an equation describing the motion of each of the following:-

a) A mass of 5 kg is attached to a spring and a damper shown in figure. The spring stiffens is 2000 N/m and damping coefficient is 12.5 N.s/m. Determine: The natural frequency, The damping factor.



$2000x = Kx$

$12.5x^{\cdot} = bx'$

N

$mg = 5*9.8$

$mx^{\cdot\cdot} = 5x^{\cdot\cdot}$

$\Sigma F_x = mx^{\cdot\cdot}$

$-Kx - bx^{\cdot} = mx^{\cdot\cdot}$

$mx^{\cdot\cdot} + Kx + bx' = 0$

$5x^{\cdot\cdot} + 2000x + 12.5x^{\circ} = 0$

$[5D^2 + 12.5D + 2000]X = 0$

$[2.5*10^{-3}D^2 + 6.25*10^{-3}D + 1]X = 0$

$\left[\frac{1}{\omega_n^2}D^2 + \frac{2\zeta}{\omega_n}D + 1\right]X = Gu(t)$

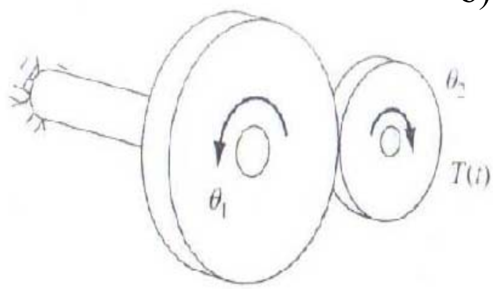
$\frac{1}{\omega_n^2} = 2.5*10^{-3}$

$\omega_n = 20$ Hz

$\frac{2\zeta}{\omega_n} = 6.25*10^{-3}$

$\zeta = 0.0625 < 1$

under damped



b) A shaft is fixed at one end and has the larger gear of a pair of gears at the other end. The pitch radii of the steel gears are r_1 = 8 inches (64 teeth) and r_2 = 4 inches (32 teeth); the tooth face widths are 0.5 inch. Find the equation of motion of this system if the smaller gear has a torque Tsinωt applied to it, where ω is the frequency, in rad/s, of the excitation torque.

The equations of motion for the two gears are:

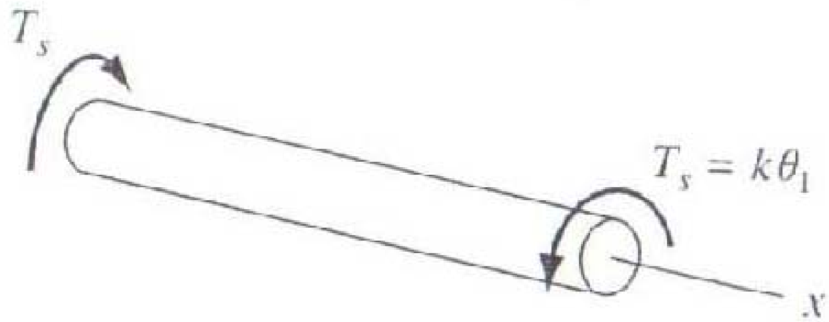
For gear 1:

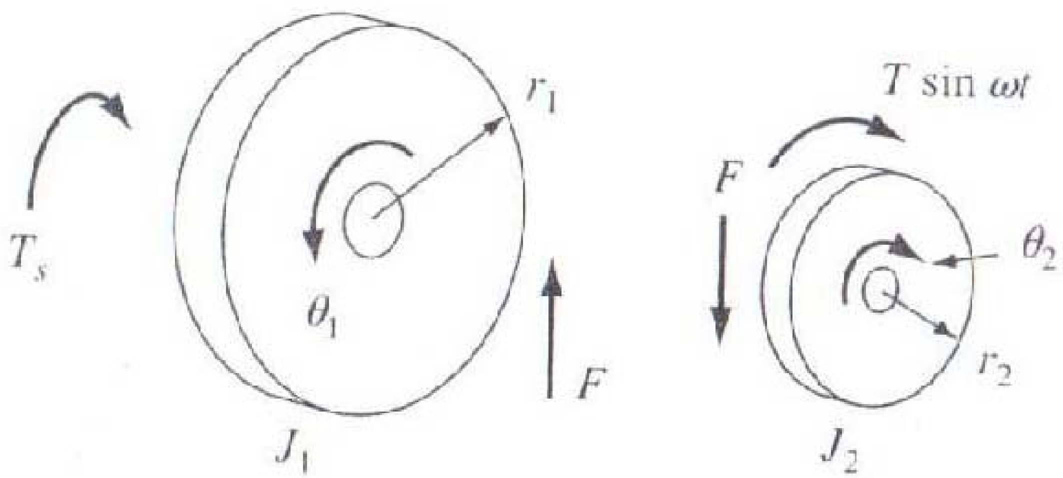
$$\sum M_x = J_1\ddot{\theta}_1$$

$$-T_s + Fr_1 = J_1\ddot{\theta}_1$$

For gear 2:

$$\sum M_x = J_2\ddot{\theta}_2$$





$$T \sin \omega t - F r_2 = J_2 \ddot{\theta}_2 \tag{3.81}$$

Solving for F from the last equation and substituting into the equation for gear 1, we obtain

$$F = -\frac{J_2 \ddot{\theta}_2 - T \sin \omega t}{r_2} \tag{3.82}$$

$$J_1 \ddot{\theta}_1 + k\theta_1 = F r_1 = -\frac{r_1}{r_2}(J_2 \ddot{\theta}_2 - T \sin \omega t) \tag{3.83}$$

$$J_1 \ddot{\theta}_1 + J_2 \ddot{\theta}_2 \frac{r_1}{r_2} + k\theta_1 = \frac{r_1}{r_2}(T \sin \omega t) \tag{3.84}$$

This problem requires only one variable to define the position of the gears. As the two gears turn in contact with each other, the arc lengths they traverse are equal. We have the following constraint equation to consider:

$$r_1 \theta_1 = r_2 \theta_2 \tag{3.85}$$

Thus, there is only one independent coordinate for the problem. We solve for θ_2 and differentiate the result to get $\ddot{\theta}_2$:

$$\theta_2 = \frac{r_1}{r_2}\theta_1 \quad \text{and} \quad \ddot{\theta}_2 = \frac{r_1}{r_2}\ddot{\theta}_1 \tag{3.86}$$

We substitute this result into Eq. (3.84) and obtain an equation in θ_1 only:

$$J_1 \ddot{\theta}_1 + J_2 \ddot{\theta}_1 \left(\frac{r_1}{r_2}\right)^2 + k\theta_1 = \frac{r_1}{r_2}(T \sin \omega t) \tag{3.87}$$

Dr. Rola Afify