1) Input :- The Independent Variable of the differential equations, The excitation or forcing function To The System

2) output :- dependent Variables of The differential equation that represent the response of the System.

3) Modeling :- Process of Identifying the physical dynamic effects to be considered in analyzing System with differential equations

4) response :- The behavior of a System is characterized by its response to external Inputs, disturbances and initial conditions.

5) Major disciplines :- engineering Systems are Mechanics, electricity and electronics, fluid mechanics and fluid control.

6) System :- Set of Interacting components Connected together

7) Disturbance :- mean those External environmental effects

Mixed
Thermo - Mechanical.

Example. "Combustion engine"
I/P "Combustion of air &fuel"
O/P "Movement of engin"
disturbance. "Impurites in Gasoline"
response. "Movement of Piston inside Engin"

Mixed.
Electro Thermal

Ex "electric Water heater
I/P "electric Current or volt"
O/P "boiling water"
disturbance. "changing of electric Current"
response "rising of water Temprature"

Thermal System

Ex heating System.
I/P cold Temprature.
O/P heating "heatedroom"
Disturbance Any other source cold or hot
Response. changing of Temprature.

Fluid - Mechanical

Exp hydrulic PumP
I/P Pressurized fluid.
O/P Work " Action"
Disturbance Air Contaminent "Containing air" or Air in oil
Response Movement of Actuator "Motor"

Mechanical System :-

Example " Autombile"
I/P " fuel "
o/P " Speed or Motion "
disturbance. " bump"
response "decreasing or Increasing of Speed "

Electrical System

Example "Television receiver"
I/P " frequency of old channel.
o/P " frequency of New channel "
disturbance. rain & air & noise"
response. "changing channel"

Fluid System

Example "Water Tower"
I/P " water level (hight of water high) "
o/P " height of water low "
disturbance. "distroy of Pump" or "cut of water"
response. "changing of water level"

Mixed : electro- Mechanical

Example. "Loud Speaker" in stero system"
I/P "electrical current or volt"
o/P " Sound "
disturbance. " Noise "
response. " Vibration "

free response $\quad u(t) = 0$

$$[\tau D + 1]\, x = 0$$

$$\tau D x + x = 0$$

$$\tau x' + x = 0 \longrightarrow (1)$$

$$X(t) = \underbrace{X_h}_{\text{homogenous solution.}} + \cancel{X_p} \rightarrow 0$$

$$X_h = A e^{\lambda t}$$

$$X' = A \lambda e^{\lambda t}$$

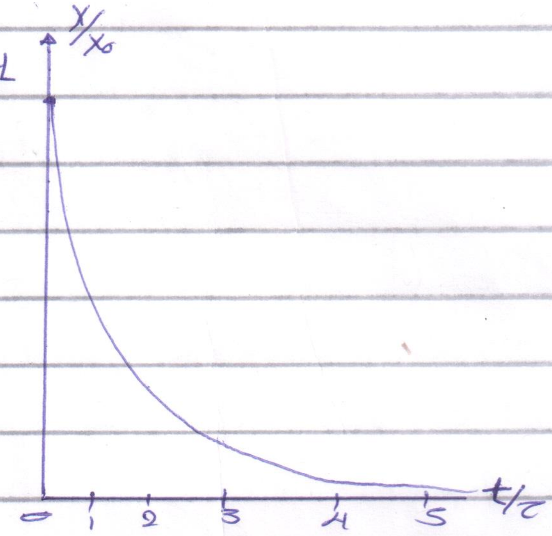
Sub in (1)

$$\tau A \lambda e^{\lambda t} + A e^{\lambda t}$$

$$\tau \cancel{A} \lambda \cancel{e^{\lambda t}} = -\cancel{A} \cancel{e^{\lambda t}}$$

$$\tau \lambda = -1$$

$$\lambda = \frac{-1}{\tau}$$



$$X(t) = A e^{-t/\tau}$$

To find A Apply The Initial Condition $(t=0)$

$$X(0) = X_0 = A e^{-t/\tau \to 0} \rightarrow 1$$

$$X_0 = A$$

$$X(t) = X_0 e^{-t/\tau}$$

Prove that Sol of 1st order Differential Equation using Step Responce. With I/P $U(t) = U_0$ will be in The form

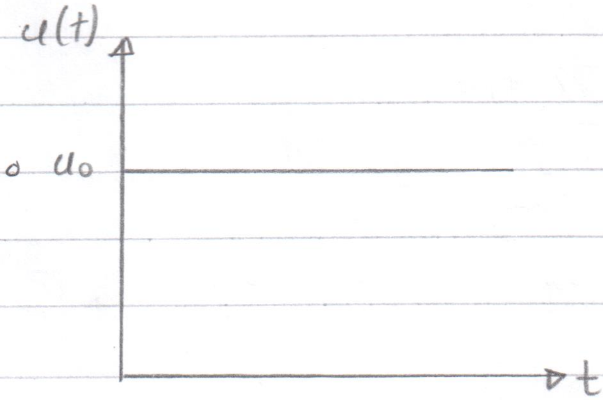
$$x(t) = X_0 e^{-t/\tau} + GU_0 \left[1 - e^{-t/\tau}\right]$$

$u(t) = u_0$

$\therefore [\tau D + 1] X = GU$

$\therefore \tau D X + X = GU_0 \rightarrow \because u = u_0$

$\therefore \tau \dot{X} + X = GU_0 \longrightarrow (1)$



$\therefore X = X_h + X_p$

$X_h = A e^{-t/\tau}$

$X_p = B$, where B is Constant

$\dot{X}_p = 0$

Sub in (1)

$\therefore \tau \dot{X}_p + X_p = GU_0$

$0 + B = GU_0$

$$\boxed{\therefore B = GU_0}$$

$X = A e^{-t/\tau} + B$

$X = A e^{-t/\tau} + GU_0$

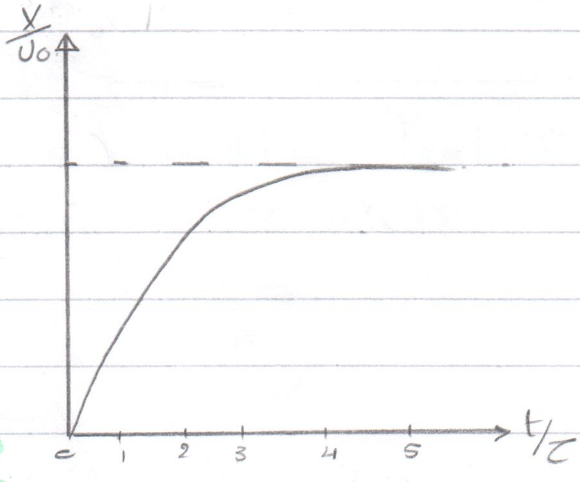
Apply Initial Condition To find A $(t=0)$

$\therefore X(0) = X_0 = A \underbrace{e^{-0/\tau}}_{1} + GU_0$

$$\boxed{\therefore A = X_0 - GU_0}$$

$x(t) = [X_0 - GU_0] e^{-t/\tau} + GU_0$

$$X(t) = X_0 e^{-t/\tau} + GU_0 \left[1 - e^{-t/\tau}\right]$$



c) Prove That The Solution of The first order differential equation using Step Input $u(t) = u_0$ will be In This form.

$$X(t) = X_o e^{-t/\tau} + GU_o [1 - e^{-t/\tau}]$$

$u(t) = u_0$

$[\tau D + 1] x = GU_0$

$\tau D x + x = GU_0$

$\tau x^{\circ} + x = GU_0 \longrightarrow 1$

$x(t) = Ae^{-t/\tau} + xp$

$x(t) = Ae^{-t/\tau} + B$

$x'(t) = -\frac{A}{\tau} e^{-t/\tau}$

Substitute In 1

$\tau\left[-\frac{A}{\tau} e^{-t/\tau}\right] + Ae^{-t/\tau} + B = GU_0$

$-\cancel{Ae^{-t/\tau}} + \cancel{Ae^{-t/\tau}} + B = GU_0$

$B = GU_0$

Initial condition $(t=0)$

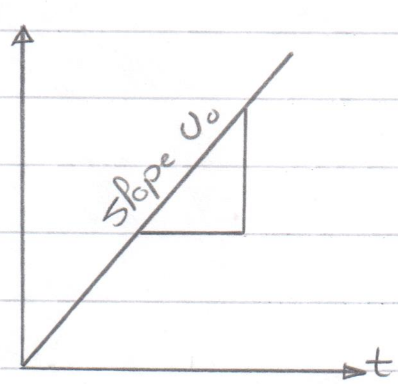
$x(0) = x_0 = A\cancel{e^{-t/\tau}}^{1} + GU_0$

$A = x_0 - GU_0$

$x(t) = [x_0 - GU_0] e^{-t/\tau} + GU_0$

$x(t) = x_0 e^{-t/\tau} + GU_0 [1 - e^{-t/\tau}]$

A) Prove. The Equation of 1st order classical Differinital. Equation of Ramp Input response?



$$[\tau D + 1]x = G\,u(t) \rightsquigarrow u(t) = u_o t$$

$$\tau D x + x = G\,U_o t$$

$$\tau x^{\cdot} + x = G\,U_o t \longrightarrow (1)$$

$$x(t) = X_h + X_P$$

$$X_h = A e^{-t/\tau} \qquad x_P = Rt + Q$$

where. A, R, Q are Constant

$$\left.\begin{aligned} x &= Ae^{-t/\tau} + Rt + Q \\ x^{\cdot} &= -\frac{A}{\tau} e^{-t/\tau} + R \end{aligned}\right\} \longrightarrow (2)$$

Substitute from (2) in (1)

$$\cancel{\tau} * \frac{-A}{\cancel{\tau}} e^{-t/\tau} + R\tau + Ae^{-t/\tau} + Rt + Q = GU_o t$$

$$Rt + R\tau = GU_o t - Q$$

$$R = GU_o$$

$$R\tau = -Q$$

$$Q = -R\tau$$

$$Q = -GU_o\tau$$

Sub in equ, 2

$$x(t) = Ae^{-t/\tau} + GU_o t - GU_o\tau$$

To find A apply Initial Condition at Time t(o)

$$x(0) = X_o = A \underbrace{e^{-0/\tau}}_{1} + G\underbrace{U_o 0}_{0} - GU_o\tau$$

$$A = X_o + GU_o\tau$$

$$X(t) = (X_o + GU_o\tau)e^{-t/\tau} + GU_o t - GU_o\tau$$

Ramp Response $u(t) = u_0 t$

$$[\tau D + 1]x = G u_0 t$$
$$\tau D x + x = G u_0 t$$
$$\tau x' + x = G u_0 t \longrightarrow (1)$$

$$x(t) = x_h + x_p = A e^{-t/\tau} + Rt + Q$$
$$x_h = A e^{-t/\tau} \qquad x_p = Rt + Q$$

$$x_p = Rt + Q$$
$$x_p' = R$$

Sub in (1)

$$\tau x_p' + x_p = G u_0 t$$
$$\tau R + Rt + Q = G u_0 t$$
$$\tau R + Rt = G u_0 t - Q$$

$$R = G u_0 \qquad \tau R = -Q$$
$$\tau G u_0 = -Q$$
$$Q = -\tau G u_0$$

$\therefore x(t) = A e^{-t/\tau} + G u_0 t - G u_0 \tau$

Initial Condition Apply ~~At~~ Time $t = 0$

$$x(t) = A e^{-t/\tau} + G u_0 t - \tau G u_0$$
$$x(0) = x_0 = A \underbrace{e^{-t/\tau}}_{1} - \tau G u_0$$
$$A = x_0 + \tau G u_0$$

$$x(t) = (x_0 + \tau G u_0) e^{-t/\tau} + G u_0 t - \tau G u_0$$

D)

i) Determine The Time Constant, response Variable External Input and Gain For Differential equation.

$$5\dot{x} + 7\dot{x} + 2x + 10 = 5t^2 + 2$$

i) $\therefore [\tau D + 1]x = GU$

$$5\dot{x} + 7\dot{x} + 2x + 10 = 5t^2 + 2.$$

$$12\dot{x} + 2x + 10 = 5t^2 + 2$$

$$12\dot{x} + 2x = 5t^2 - 8$$

$$12Dx + 2x = 5t^2 - 8$$

$$[12D + 2]x = 5t^2 - 8 \div 2$$

$$[6D + 1]x = 0.5[5t^2 - 8]$$

$\therefore$ τ = Time constant = 6 sec.

$\therefore$ response Variable = x

$\therefore$ Gain = G = 0.5

$\therefore$ Settling Time = 4τ = 4×6 = 24 sec

$\therefore$ external Input = $u(t) = 5t^2 - 8$

Find All Constant (Natural frequency - Gain - Input function damping ratio

$$25\ddot{x} + 4\dot{x} + 16x = 10 \sin 5t$$

$$\left[\frac{D^2}{\omega_n^2} + 2\eta \frac{D}{\omega_n} + 1\right]x = GU$$

$$25D^2x + 4Dx + 16x = 10 \sin 5t$$

$$[25D^2 + 4D + 16]x = 10 \sin 5t \div 16$$

$$\left[\frac{25}{16}D^2 + \frac{4}{16}D + 1\right]x = \frac{10}{16} \sin 5t$$

$$\frac{1}{\omega_n^2} = \frac{25}{16} \longrightarrow \omega_n = \sqrt{\frac{16}{25}} = \frac{4}{5} \text{ (Natural frequency)}$$

$\frac{2\eta}{\omega n} = \frac{4}{16} \longrightarrow \eta = 0.1$ (Damping ratio)

$\omega n = 4/5$

$Gain = \frac{10}{16} = \frac{5}{8}$

Input function $= \sin 5t$

iii) Determine response Variable, External input, Natural frequency damping Ratio and Gain for Differential equation
$LCD^2e_2 + RCDe_2 + e_2 = e_0$, if $l = 1mh$, $C = 10^{-4}F$ & $R = 14\Omega$.

$$\left[\frac{D^2}{\omega n^2} + 2\eta\frac{D}{\omega n} + 1\right]x = GU$$

$$LCD^2e_2 + RCDe_2 + e_2 = e_0$$

$l = 1*10^{-3}h \qquad C = 10*10^{-6} \qquad R = 14$

$$(1*10^{-3})(10*10^{-6})D^2e_2 + (14)(10*10^{-6})De_2 + e_2 = e_0$$

$$[(1*10^{-8})D^2 + (14*10^{-5})D + 1]e_2 = e_0$$

$\frac{1}{\omega n^2} = 1*10^{-8} \longrightarrow \omega n = 10^4$ Hz (Natural frequency)

$\frac{2\varepsilon}{\omega n} = 14*10^{-5} \longrightarrow \varepsilon = 0.7$ (Damping ratio)

$\omega n = 10^4$

$0.7 < 1 \rightarrow$ under Damping

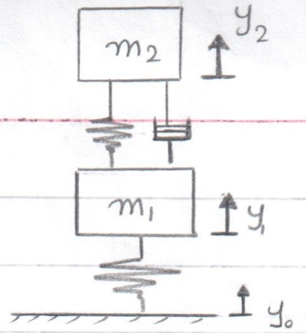
$\therefore$ Gain $= G = 1$

$\therefore$ External input $= e_0 = U(t)$

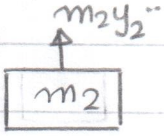
$\therefore$ response Variable $= e_2 = x$

Q-3 : A

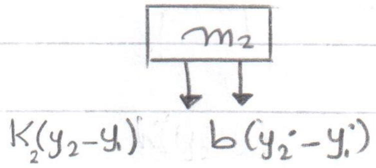
1)



Effective Force

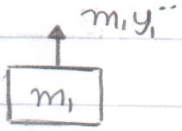


External Force

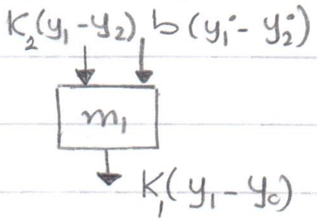


$$m_2\ddot{y}_2 - K(y_2-y_1) - b(\dot{y}_2-\dot{y}_1) = 0$$

Effective Force



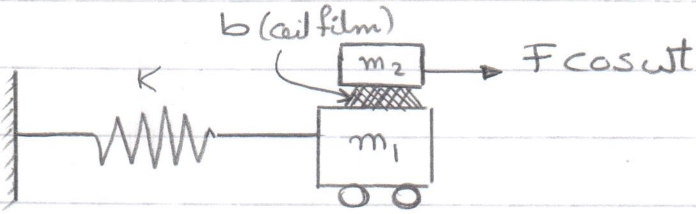
External force.



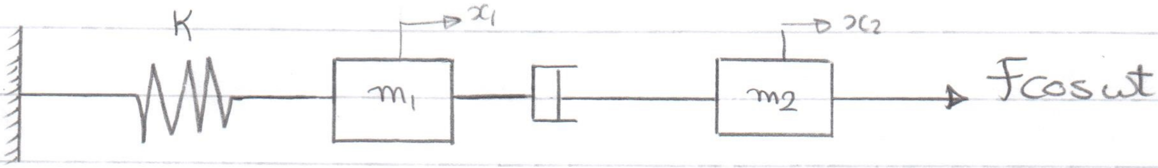
$$m_1\ddot{y}_1 = -K_2(y_1-y_2) - b(\dot{y}_1-\dot{y}_2) - K_1(y_1-y_0)$$

~~*~*~*~*

2)

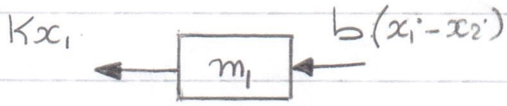


Ans



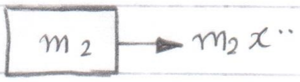
Effective force

External force.

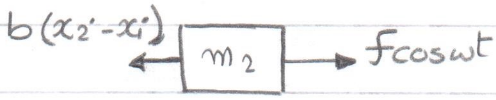


$$m_1\ddot{x} = -Kx_1 - b(\dot{x}_1-\dot{x}_2)$$

Effective force

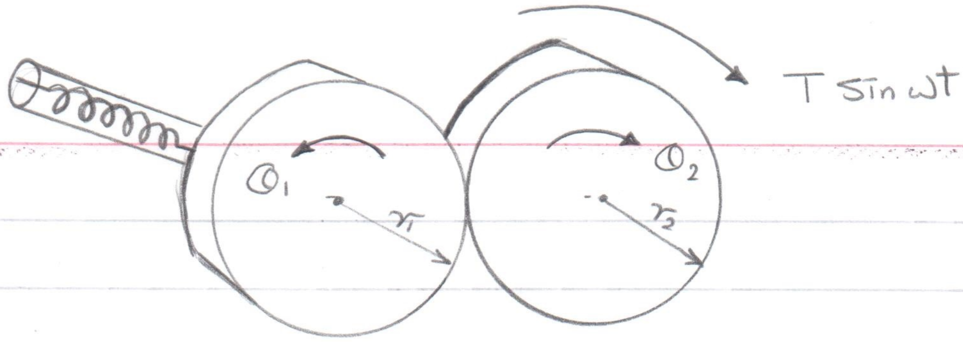


External force

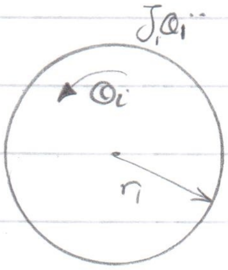


$$m_2\ddot{x} = -b(\dot{x}_2-\dot{x}_1) + F\cos\omega t$$

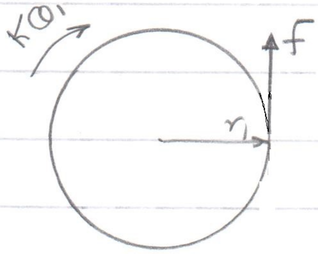
3)



Effective force

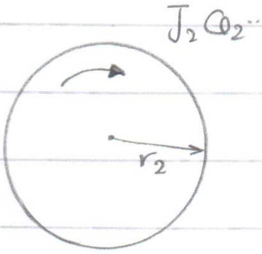


External force.

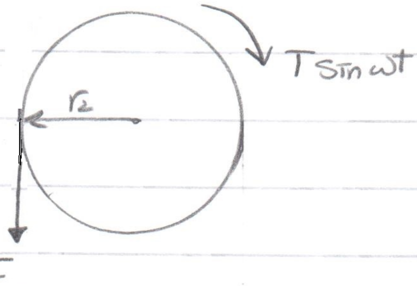


$$J_1\ddot{\theta}_1 = -K\theta_1 + \overbrace{F * r_1}^{M}$$

Effective force



External force



$$J_2\ddot{\theta}_2 = -F * r_2 + T\sin\omega t$$