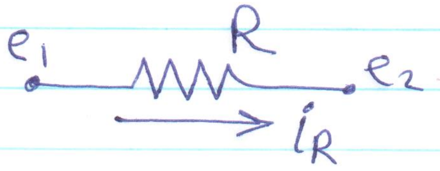
Electrical system

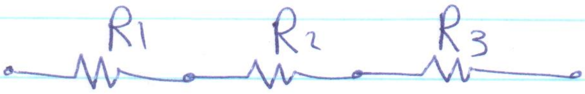
(1) Passive circuit :- (R - L - c).

(a) Resistors :-



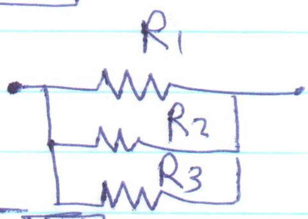
$$\boxed{i_R = \frac{e_1 - e_2}{R}}$$

* series Resistors:



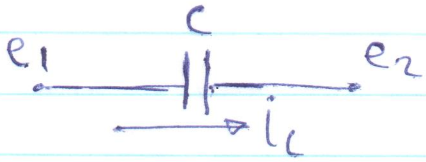
$$\boxed{R_T = R_1 + R_2 + R_3}$$

* Parallel Resistors :



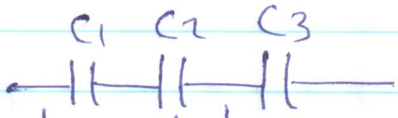
$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

(b) Capacitors :-



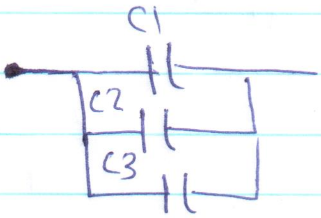
$$\boxed{i_c = c D (e_1 - e_2)}$$

* Series capacitors:-



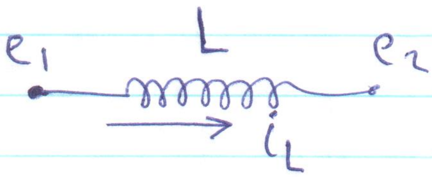
$$\boxed{\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

* Parallel capacitors:-



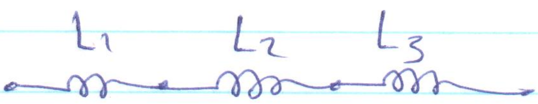
$$\boxed{C_T = C_1 + C_2 + C_3}$$

(c) inductors:-



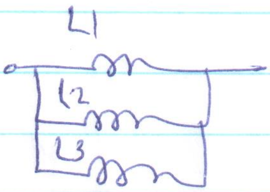
$$\boxed{i_L = \frac{1}{LD}(e_1 - e_2)}$$

* series inductors



$$\boxed{L_T = L_1 + L_2 + L_3}$$

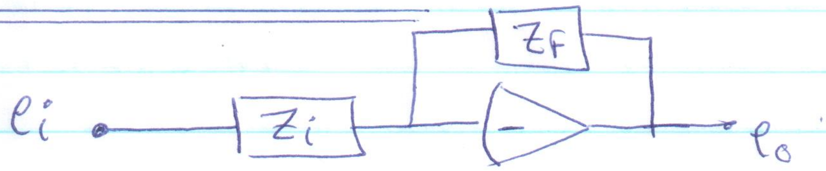
* Parallel inductors



$$\boxed{\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

2 Active circuits:

*** operational Amplifier: (op-amp)**

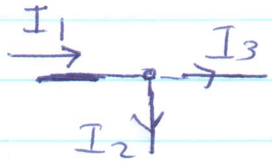


$$e_o = -\frac{Z_f}{Z_i} e_i$$

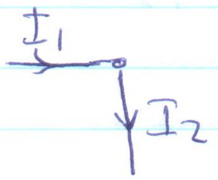
$$e_o = -G\, e_i \quad \text{"Given in exam".}$$

{Notes} 1 kirchhoff's current law

$$\boxed{I_1 = I_2 + I_3}$$



$$\boxed{I_1 = I_2}.$$



2 ohm's law

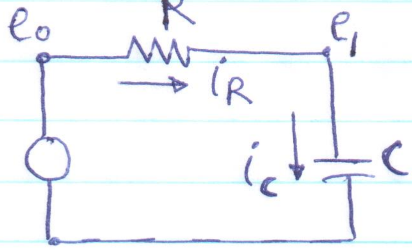
$$V = I \; Z$$

(a) Resistors : $\boxed{Z = R}$.

(b) Capacitors : $\boxed{Z = 1/CD}$.

(c) Inductors $\boxed{Z = LD}$.

Example (1) :-



The circuit shown in figure has a resistence 8Ω, Find the capacitance necessary to give a Settling time 2.5 m sec.

— Sol —

K.C.L $\quad i_R = i_c$

$$\because \frac{e_0 - e_1}{R} = C D e_1$$

$$e_0 - e_1 = CRDe_1$$

$$CRDe_1 + e_1 = e_0$$

$$\therefore (CRD + 1)\, e_1 = e_0$$

$\because \tau = CR \qquad \because$ Settling time $= 4\tau$

$4\tau = 2.5 * 10^{-3} \qquad \therefore \tau = \frac{2.5}{4} * 10^{-3}$

$C * 8 = \frac{2.5}{4} * 10^{-3} \qquad \therefore C = 78.125\ \mu F$

1st order D.E $\quad (\tau D + 1)\, X(t) = G\, U(t).$

$$\therefore \tau = CR$$

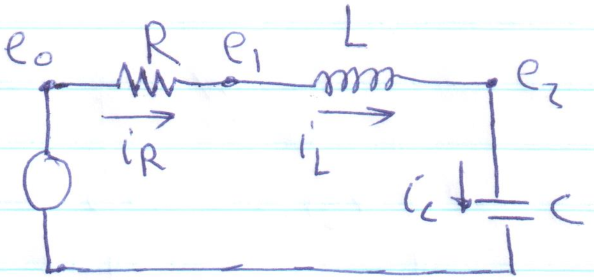
$$G = 1.$$

Example (2) :-

$L = 1\,mh$

$c = 10\,\mu F$

$R = 14\,\Omega$.



Find the damping ratio and natural freq.

— sol —

1st Node, $\quad i_R = i_L$

$$\frac{e_o - e_1}{R} = \frac{1}{LD}(e_1 - e_2)$$

$$e_o - e_1 = \frac{R}{LD} e_1 - \frac{R}{LD} e_2$$

$$e_o + \frac{R}{LD} e_2 = \left(\frac{R}{LD} + 1\right) e_1 \longrightarrow (1)$$

2nd Node $\quad i_L = i_c$

$$\frac{1}{LD}(e_1 - e_2) = cD\, e_2$$

$$e_1 - e_2 = LCD^2 e_2$$

$$e_1 = (LCD^2 + 1)\, e_2 \longrightarrow (2)$$

by Sub. from (2) in (1).

$$e_o + \frac{R}{LD} e_2 = \left(\frac{R}{LD} + 1\right)(LCD^2 + 1)\, e_2$$

$$e_o + \frac{R}{LD} e_2 = \left(RCD + \frac{R}{LD} + LCD^2 + 1\right) e_2$$

$$e_o = \left(RCD + \cancel{\frac{R}{LD}} + LCD^2 + 1 - \cancel{\frac{R}{LD}}\right) e_2$$

$$e_o = (LCD^2 + RCD + 1) e_2$$

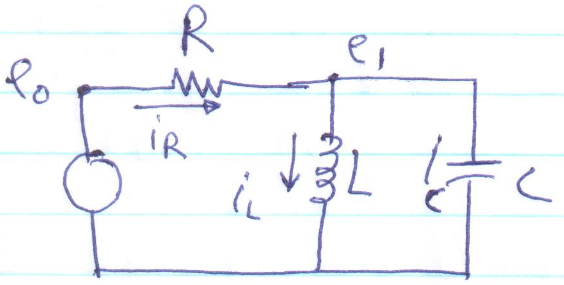
$$\therefore \quad G\xi u(t) = \left(\frac{D^2}{\omega_n^2} + \frac{2\xi}{\omega_n} D + 1\right) x(t).$$

$$\therefore \quad \omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1*10^{-3}*10*10^{-6}}} = 10000 \text{ rad/s}$$

$$\frac{2\xi}{\omega_n} = RC \qquad \therefore \xi = \frac{RC\omega_n}{2}$$

$$\xi = \frac{14*10*10^{-6}*10000}{2} = 0.7 \quad \text{"under damped".}$$

Example (3):-



1st Node

$$i_R = i_L + i_C$$

$$\frac{e_o - e_1}{R} = \frac{1}{LD} e_1 + CDe_1$$

$$e_o - e_1 = \frac{R}{LD} e_1 + CRDe_1 \quad *LD$$

$$LDe_o - LDe_1 = Re_1 + CRLD^2 e_1$$

$$\therefore (CRLD^2 + LD + R) e_1 = LDe_o \quad \% R$$

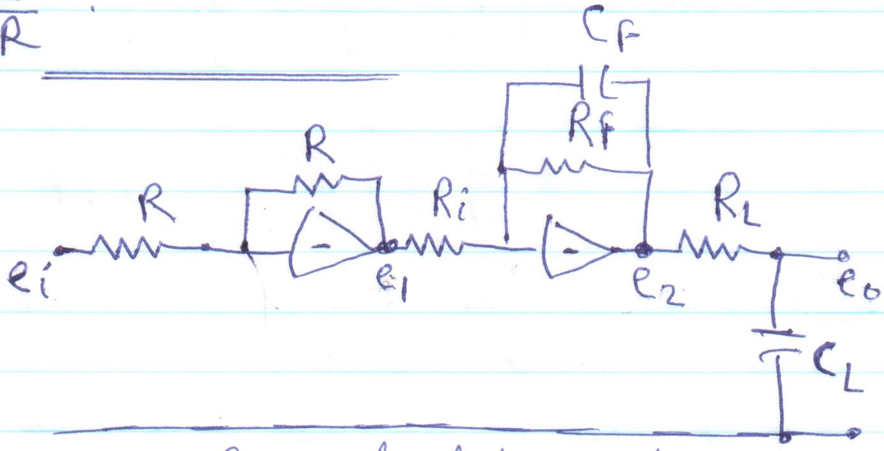
$$\therefore \left(CLD^2 + \frac{L}{R} D + 1\right) e_1 = \frac{L}{R} D e_o$$

$$\therefore \left(\frac{D^2}{\omega_n^2} + \frac{2\xi}{\omega_n} D + 1\right) X(t) = G\, u(t).$$

$$\therefore \omega_n = \frac{1}{\sqrt{LC}} \qquad \therefore \frac{2\xi}{\omega_n} = \frac{L}{R}.$$

$$G = \frac{L}{R}.$$

Example (4):



$R_f = 10\,k\Omega$
$R_i = 10\,k\Omega$
$C_f = 1\,\mu F$
$R_L = 500\,\Omega$ $\quad C_L = 10\,\mu F$. calculate static gain, natural frequency, and damping ratio

— Sol —

[1] From 1st op-amp.

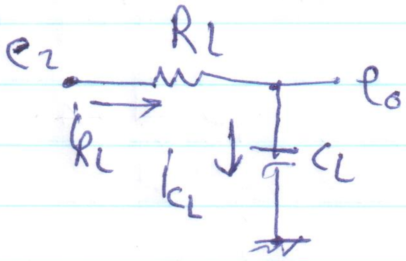
$$e_1 = -e_i \quad \text{from table} \longrightarrow (1)$$

[2] from 2nd op-amp.

$$e_2 = \frac{-\frac{R_f}{R_i}\, e_1}{(R_f C_f D + 1)} \quad \text{from } (1)$$

$$e_2 = \frac{\frac{R_f}{R_i}}{(R_f C_f D + 1)}\, e_i \longrightarrow (2)$$

[3] From RC circuit



$$i_{R_L} = i_{C_L}$$

$$\frac{e_2 - e_0}{R_L} = C_L D e_0$$

$$e_2 - e_0 = C_L R_L D e_0$$

$$e_2 = (C_L R_L D + 1) e_0 \quad \text{from (2)}$$

$$\frac{\frac{R_f}{R_i}}{(R_f C_f D + 1)} e_i = (C_L R_L D + 1) e_0$$

$$\cancel{\frac{R_f}{R_i}} e_i = (R_f C_f D + 1)(C_L R_L D + 1) e_0$$

$$R_i = R_f$$

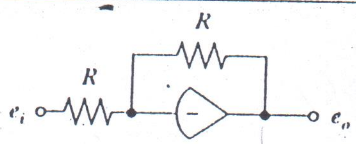
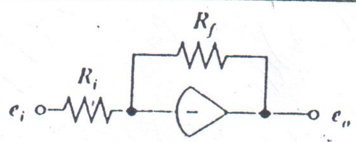
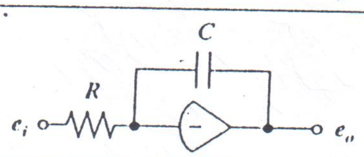
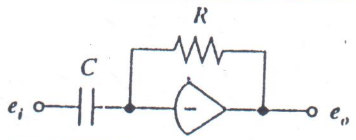
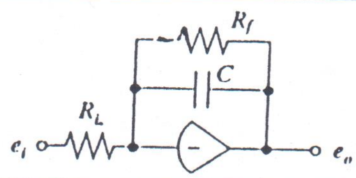
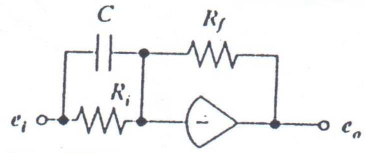
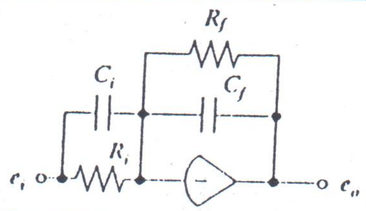
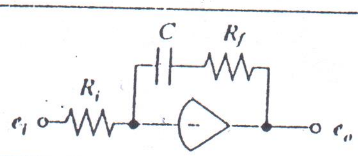
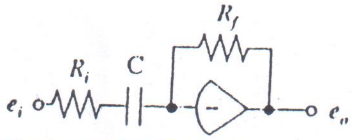
$$e_i = (R_f C_f R_L C_L D^2 + R_f C_f D + C_L R_L D + 1) e_0$$

$$e_i = (R_f C_f R_L C_L D^2 + (R_f C_f + C_L R_L) D + 1) e_0$$

$$\therefore \omega_n = \frac{1}{\sqrt{R_f C_f R_L C_L}}$$

$$\frac{2\xi}{\omega_n} = R_f C_f + R_L C_L$$

$$G = 1.$$

	Description	Transfer Function	Circuit
EX.4	Sign Changer	$e_o = -e_i$	
	Amplifier	$e_o = -\frac{R_f}{R_i} e_i$	
	Integrator	$e_o = \frac{-e_i}{\tau D}$ $\tau = RC$	
	Differentiator	$e_o = -\tau D e_i$ $\tau = RC$	
EX.4	Lag	$e_o = \frac{-\frac{R_f}{R_i} e_i}{(\tau D + 1)}$ $\tau = R_f C$	
	Lead	$e_o = -\frac{R_f}{R_i}(\tau D + 1)$ $\tau = R_i C$	
	Lead-Lag or Lag-Lead	$e_o = -\frac{R_f}{R_i}\frac{(\tau_i D + 1)e_i}{(\tau_f D + 1)}$ $\tau_i = R_i C_i$ $\tau_f = R_f C_f$	
	Bandwidth-Limited Integrator	$e_o = \frac{-(\tau_f D + 1)e_i}{\tau_i D}$ $\tau_f = R_f C$ $\tau_i = R_i C$	
	Bandwidth-Limited Differentiator	$e_o = \frac{-\tau_f D e_i}{(\tau_i D + 1)}$ $\tau_f = R_f C$ $\tau_i = R_i C$	

Thermal systems

[1] Thermal conduction.

it is the ability of solid or continuous media to conduct heat

$$\boxed{q_h / A = -k_t \frac{dT}{dx}}$$

q_h : heat transfer
k_t : thermal conductivity
$\frac{dT}{dx}$: temp. gradient

[2] Thermal convection

it is the process of heat transfer between a surface of a solid material and a fluid that is exposed to the solid surface

$$\boxed{\frac{q_h}{A} = h (T_s - T_\infty)}$$

h : convection coefficient.

[3] Thermal Radiation

it's the process of heat transfer in which the energy is high enough to transfer heat without medium such as fluid or solid

$$\boxed{\frac{q_h}{A} = \sigma T^4}$$

σ : stefan - Boltzman constant

4 Thermal Capacitance:-

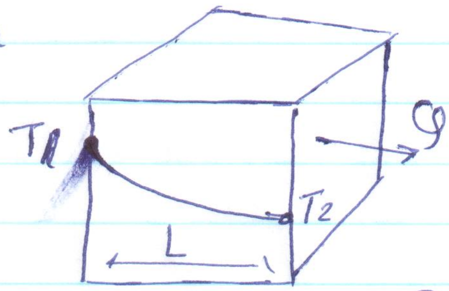
it's the behavior of the material when it holds, or store heat

$$\boxed{q_h = c_p \, m \, \frac{dT}{dt}}$$

$m c_p$:- total heat storage "heat capacity".

* Find the thermal resistance in conduction, convection, and radiation. illustrate by drawing

(1) Conduction



$$q_h = -k_t A \frac{\partial T}{\partial x}$$

$$= -k_t A \frac{\Delta T}{\Delta x}$$

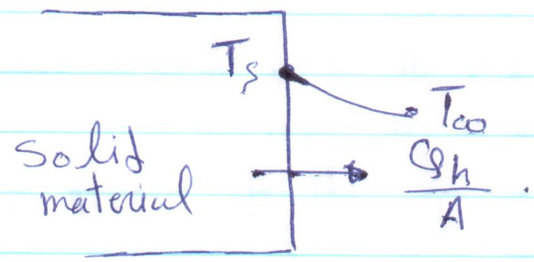
$$= -\frac{k_t A}{L} \Delta T$$

$$q_h = \frac{-k_t A}{L} (T_2 - T_1)$$

$$q = \frac{\Delta T}{R} \equiv I = \frac{V}{R}$$

$$\therefore \boxed{R = \frac{L}{k_t A}}$$

[2] Convection.



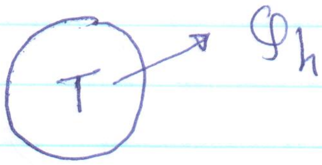
$$\frac{q_h}{A} = h(T_s - T_\infty)$$

$$q_h = Ah(T_s - T_\infty).$$

$$q_h = Ah\,\Delta T \qquad \because q_h = \frac{\Delta T}{R}.$$

$$\therefore \boxed{R = \frac{1}{Ah}}$$

[3] Thermal Radiation.



$$\frac{q_h}{A} = \sigma T^4$$

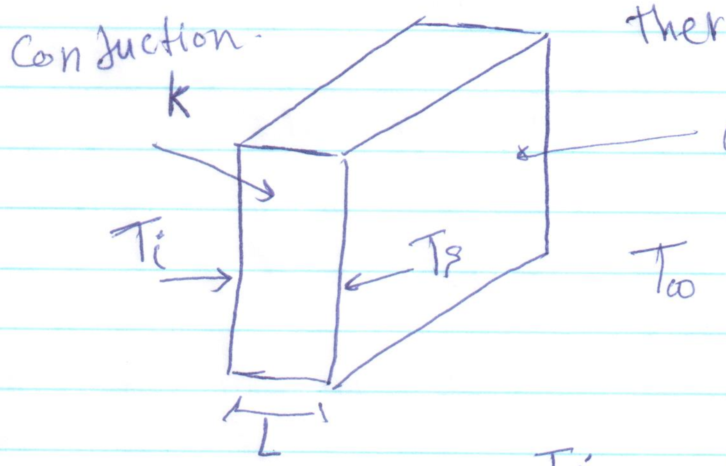
$$q_h = A\sigma T^4$$

$$\because q_h = \frac{\Delta T}{R}$$

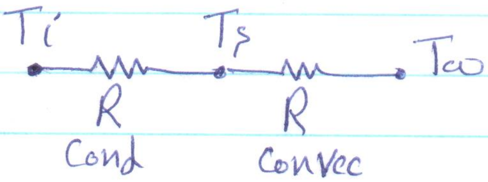
$$\therefore \boxed{R = \frac{1}{\sigma A}}$$

Example 6.2 :-

Consider the system shown in figure, in which a plate of Plexiglas that is wall of a container is exposed to an internal temp $T_i = 50°C$ on one side and is subjected to free convection to room temp. 25°C on the other side we want to know how much heat is lost and what, will be the outer surface temperature T_s. The plate is 100 mm by 100 mm and is 6 mm thick. the thermal conductivity is 0.195 W thermal convection is 20 W/m²k



$T_i = 50°C$, $T_\infty = 25°C$



$$R_{Cond} = \frac{L}{KA} = \frac{0.006}{0.195 * 0.1 * 0.1} = 3.08 \ k/W$$

$$R_{Convec} = \frac{1}{hA} = \frac{1}{20 * 0.1 * 0.1} = 5 \ ^\circ k/W$$

$$q_{h\,total} = \frac{\Delta T}{R_{Cond} + R_{Conv}} = \frac{50-25}{3.08+5} \approx 3.1 \ \text{Watts}$$

$$T_s = \frac{R_{conv} T_i + R_{cond} T_\infty}{R_{cond} + R_{conv}}$$

$$= \frac{5(50) + 3.08(25)}{3.08 + 5} = 40.4°C$$

Fluid systems

1) Drive the equation of state of liquid and gases?

a) Liquids

$$\rho = \rho_o + \left.\frac{\partial \rho}{\partial P}\right|_{P_o,T_o} (P - P_o) + \left.\frac{\partial \rho}{\partial T}\right|_{P_o,T_o} (T - T_o).$$

$$\rho = \rho_o \left[1 + \frac{1}{\beta}(P - P_o) - \alpha (T - T_o) \right]$$

$\beta = \rho_o \left.\frac{\partial P}{\partial \rho}\right|_{P_o,T_o}$ Bulk modulus.

$\beta = \frac{-\partial P}{\partial V / V_o}$ $\longrightarrow$ for fixed mass of fluid

$\beta_a = \frac{c_p}{c_v} \beta$ Adiabatic bulk modulus

$\alpha = \frac{-1}{\rho_o} \left.\frac{\partial \rho}{\partial T}\right|_{P_o,T_o}$ thermal expansion coefficient

$\alpha = \left.\frac{\partial V / V_o}{\partial T}\right|_{P_o,T_o}$ $\longrightarrow$ for a fixed mass

b) Gases

$\rho = \frac{P}{RT}$

P, T are absolute

R is gas constant

$\frac{P}{\rho^n} = \text{Constant} = c$

$P = c\rho^n$

$n = 1$ isothermal

$n = k$ adiabatic

$n = 0$ isobaric

$n = \infty$ isovolumetric

[2] Compare between liquid and gases for viscosity?

[a] Liquid $\mu = \mu_o \, e^{-\lambda_L (T - T_o)}$ $\mu \downarrow$ as $T \uparrow$

μ_o, T_o : values at reference conditions
λ_L : constant depend on the liquid.

[b] gases $\mu = \mu_o + \lambda_G (T - T_o)$.
λ_G constant deped on the gas

$\mu \uparrow$ as $T \uparrow$

[3] Compare between fluid capacitance, inductance and resistance?

[a] fluid capacitance:. It relates how fluid energy can be stored by virtue of pressure.

$$Q = \frac{V}{\beta} \dot{P}_{CV} \qquad \therefore \boxed{C_f = \frac{V}{\beta}}.$$

[b] fluid inductance:- it's the effect due to the inertia of a moving fluid.

$$\boxed{L = \frac{\rho L}{A}} \qquad \Delta P = \frac{\rho L}{A} \dot{Q}$$

[c] fluid Resistance:. fluid resistor dissipates power and can have a large variety of forms laminar flow resistance, orifice type or head loss.

$$\boxed{R = \frac{32 \mu L}{A \, d_h^2}}$$

d_h: hydrolic diameter $= \dfrac{4 \text{ Area}}{\text{Perimeter}}$

[4] define :- (a) the Reynolds number
(b) Propagation Speed.

[a] the Reynolds number :-

$$\boxed{N_r = \frac{\text{inertial flow forces}}{\text{viscous flow forces}} = \frac{Vd}{\nu}}$$

[b] Propagation speed :-

$$C_o = \sqrt{\frac{\beta}{\rho}}$$

β :- bulk modulus
ρ :- density

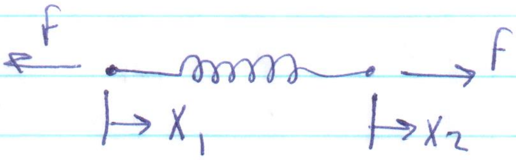
$$\beta = kP$$

$$\therefore \boxed{C_o = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{kP}{\rho}} = \sqrt{kRT}}$$

Mechanical Systems

[1] Compare between spring, damper, and mass ?

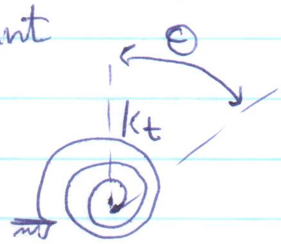
[a] Springs :



$$\boxed{F = k\,(x_2 - x_1)}$$

"linear springs".

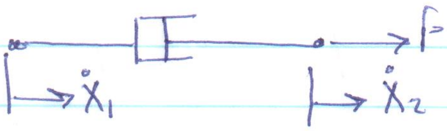
k :- spring constant



$$\boxed{T = k_t\,\theta}$$

"Torsional spring".

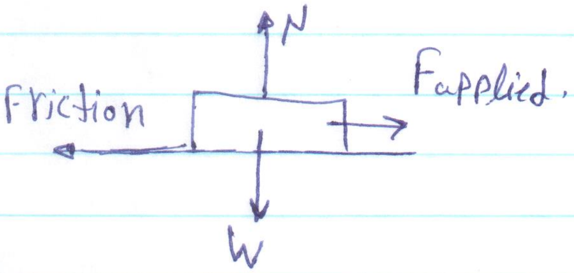
[b] Dampers :-

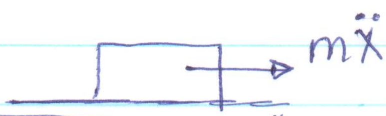


$$\boxed{F = b\,(\dot{x}_2 - \dot{x}_1)}$$

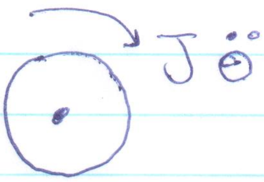
b :- damping constant

[c] Discrete Mass :-



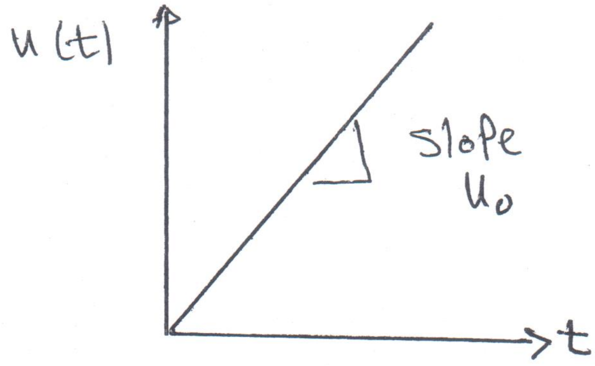


$$\boxed{F = m\,\ddot{x}}$$ "linear".



$$\boxed{\Sigma M = J\,\ddot{\theta}}$$ "Rotation".

3 Ramp input response :- $u(t) = u_0 t$



$$\tau \dot{X} + X = G u_0 t$$

$$X(t) = X_h(t) + X_p(t).$$

$$X_h(t) = A e^{-t/\tau}$$

$$X_p(t) = Rt + \varphi \qquad R \,\&\, \varphi \text{ :- are constants}$$

$$\dot{X}_p(t) = R$$

* to find $R \,\&\, \varphi$ solve 1st order differential eqn for X_p only

$$\tau \dot{X}_p + X_p = G u_0 t$$

$$\tau R + Rt + \varphi = G u_0 t$$

$$\tau R + \varphi + Rt = G u_0 t$$

نساوي t بـ t والرقم بالرقم

$$\tau R + \varphi = 0 \qquad \& \qquad \boxed{R = G u_0}$$

$$\because \tau \overbrace{G u_0}^{R} + \varphi = 0 \qquad \therefore \boxed{\varphi = -\tau G u_0}.$$

* to find the constant A apply the initial conditions

$$X_0 = A \cancel{e^0} + \underbrace{G u_0}_{R} \cancel{t}^{\,0} - \underbrace{\tau G u_0}_{\varphi}$$

$$X_0 = A - \tau G u_0 \qquad \therefore \boxed{A = X_0 + \tau G u_0}$$

$$\therefore X(t) = X_0 e^{-t/\tau} + \tau G u_0 e^{-t/\tau} + G u_0 t - \tau G u_0$$

$$\boxed{X(t) = X_0 e^{-t/\tau} + G u_0 \left[t - \tau(1 - e^{-t/\tau})\right]}$$

1 Free response :- $u = 0$

$$\tau \dot{X} + X = 0$$

$$\boxed{\dot{X} = \frac{-1}{\tau} X} \longrightarrow (1) \qquad \because X = X_h + \cancel{X_p} \ (\to 0)$$

$$X_h = A e^{\lambda t} \qquad \therefore \dot{X}_h = A \lambda e^{\lambda t}$$

by Sub in (1)

$$\cancel{A} \lambda \cancel{e^{\lambda t}} = \frac{-1}{\tau} \cancel{A} \cancel{e^{\lambda t}}$$

$$\therefore \boxed{\lambda = \frac{-1}{\tau}}$$

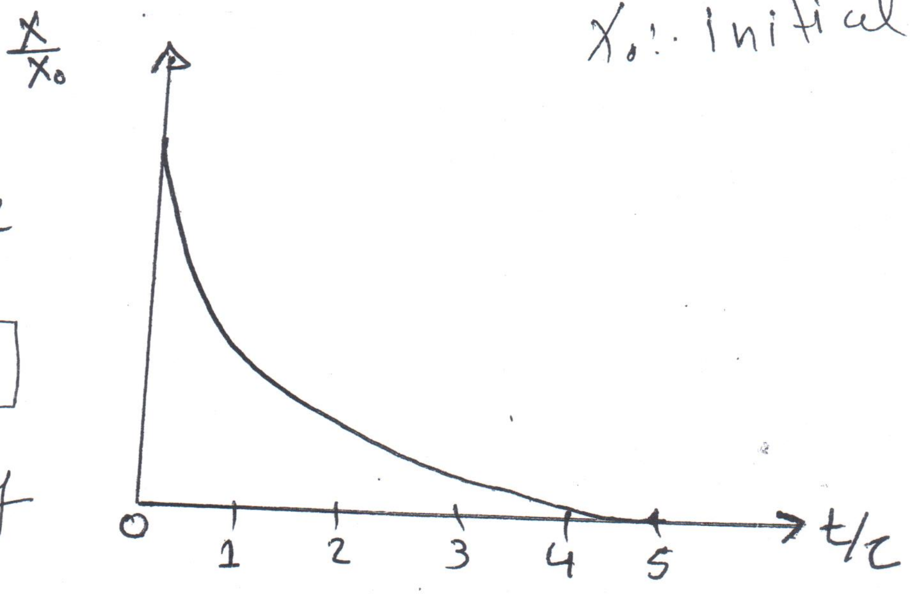
to find the constant A applying the initial condition.

$$X(0) = X_o = A \underbrace{e^{0}}_{1} \qquad \therefore \boxed{A = X_o}$$

the complete solution will be :-

$$\boxed{X(t) = X_o e^{\frac{-t}{\tau}}}$$

X_o :- initial value.



Final response after a time equal to $\boxed{4\tau}$

Settling time.

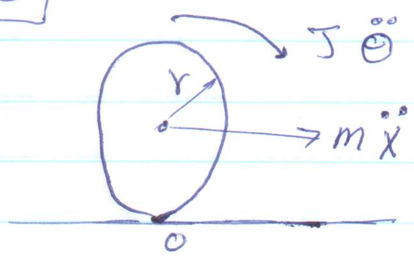
Example 3.12:-

Find the equation of motion for the shown figure.

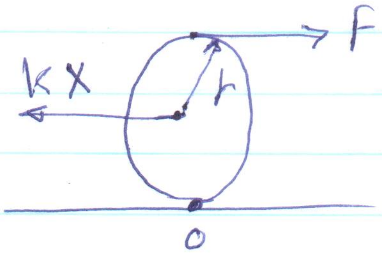
Sol-

$\boxed{x = r\theta}$

effective



external.



$$\underset{(+)}{\Sigma M_o} = m\ddot{x}(r) + J\ddot{\theta}$$

$$= mr^2\ddot{\theta} + J\ddot{\theta}$$

$$= [mr^2 + J]\ddot{\theta}$$

$$\underset{(+)}{\Sigma M_o} = -kx(r) + f(2r)$$

$$= -kr^2\theta + 2rF$$

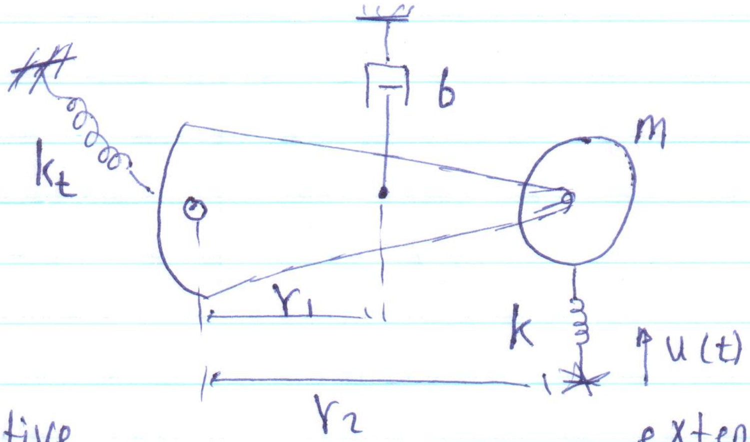
$$\Sigma M_{o\,eff} = \Sigma M_{o\,ext}.$$

$$[mr^2 + J]\ddot{\theta} = -kr^2\theta + 2rf$$

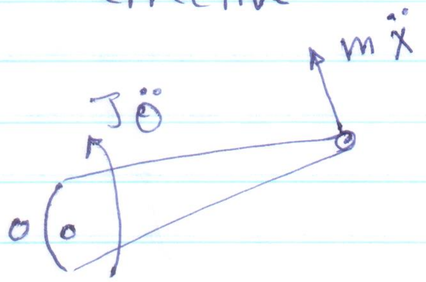
$$\boxed{[mr^2 + J]\ddot{\theta} + kr^2\theta = 2rF}.$$

Example 3.13 :-

Develop the equations governing the angular motion.



effective



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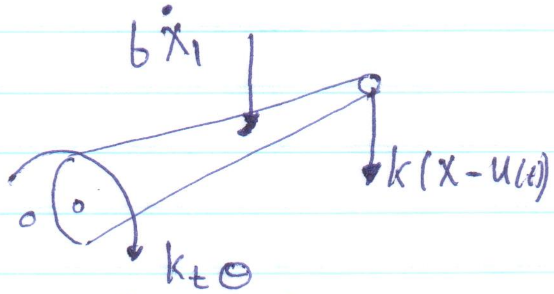

$x = r_2\theta.$

$\dot{x}_1 = r_1\dot{\theta}.$

$$\underset{(+)}{\Sigma M_{o\,eff}} = J\ddot{\theta} + m\ddot{x}(r_2) = J\ddot{\theta} + m r_2^2\ddot{\theta}$$

$$\underset{(+)}{\Sigma M_{o\,ext}} = -k_t\theta - b\dot{x}_1(r_1) - k r_2(x - u(t)). = -k_t\theta - b r_1^2\dot{\theta} - k r_2^2\theta + k r_2 u(t).$$

$$\Sigma M_{eff} = \Sigma M_{ext}.$$

$$J\ddot{\theta} + m r_2^2\ddot{\theta} = -k_t\theta - b r_1^2\dot{\theta} - k r_2^2\theta + k r_2 u(t)$$

$$\boxed{(J + m r_2^2)\ddot{\theta} + b r_1^2\dot{\theta} + [k_t + k r_2^2]\theta = k r_2 u(t)}$$