Stress Analysis

ME276

Text Book

* Mechanics of Materials
  R.C. Hibbeler, 8th edition in SI units.

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* **Mechanics of materials :-**

It is a branch of mechanics that studies the internal effects of stress and strain in a solid body that is subjected to an external loading.

stress is associated with the strength of the material from which the body is made, while strain is a measure of deformation of the body.

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* **Equilibrium of a deformable body :-**

$$\sum \vec{F} = 0$$

$$\sum M_o = 0$$

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* **Load :-**

It is any external force acting upon a machine part.

It has four types :-

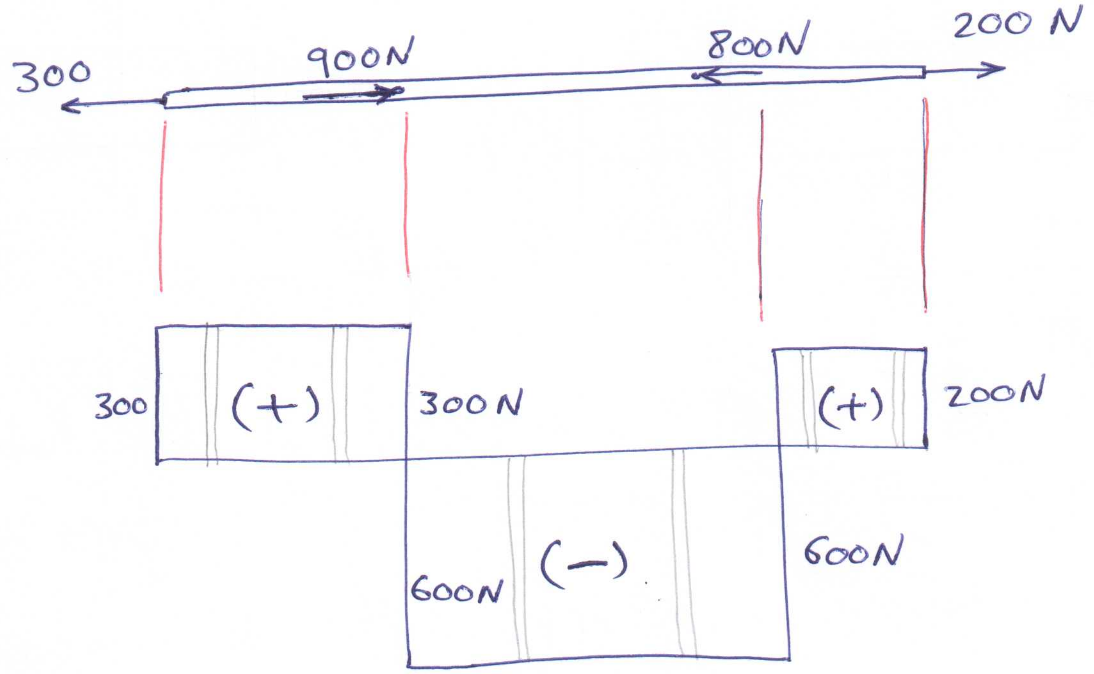
1- Dead or steady load: doesn't change in magnitude or direction.

2- Live or variable load: changes continuously.

3- suddenly applied or shock loads: It is suddenly applied or removed

4- Impact load: It is applied with some initial velocity

* Normal Force Diagram (NFD)

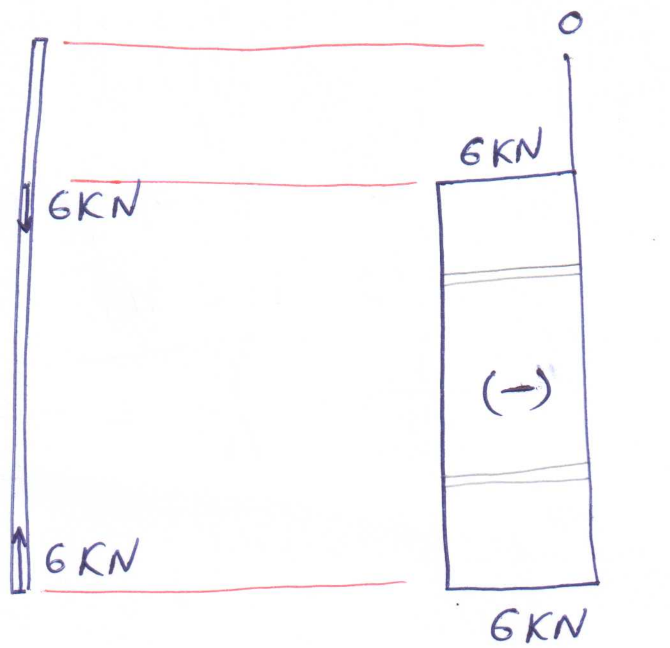


(N.F.D.)

من اليمين إلى الشمال ← −   + →

من فوق إلى تحت −↓   ↑+

(N.F.D)



Introduction to the Concept of stress and strain: Normal stress and strains

* **stress**

It is the force per unit area. $\sigma = \frac{F}{A}$ (Pa)

i.e. It is the effect of applied force on the design part.

* **strength**

It is a property of the material shows how strong it is.

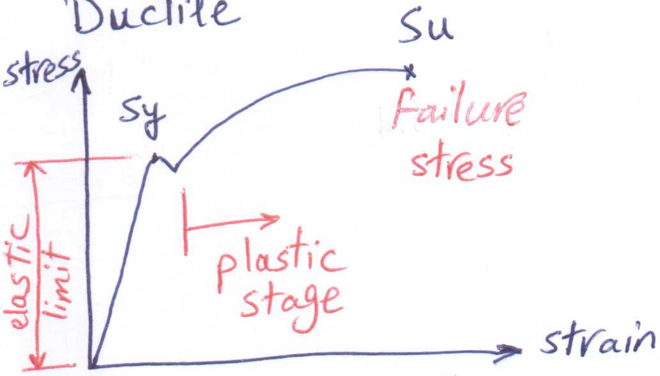
* To avoid failure in any part of the designed machine, the max. stress found in it must be less than its corresponding strength

$$\text{allowable max. stress (Design or working stress)} = \frac{\text{strength}}{\text{factor of safety}}$$

* **strain** $\varepsilon = \epsilon = \frac{\text{change in length}}{\text{original length}}$ (dimensionless)

* Materials

Ductile

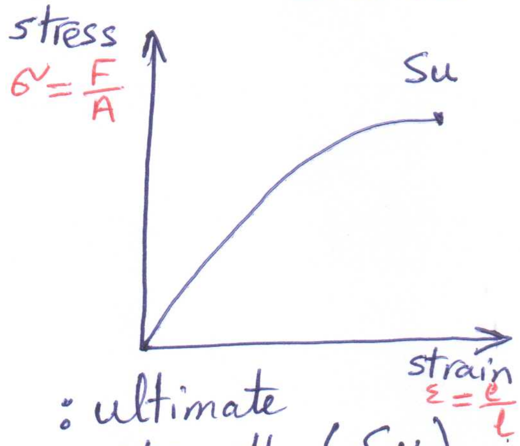


Design strength is: yield strength (S_y)

$$\sigma_{all} = \frac{S_y}{f.S.}$$

$$\tau_{all} = \frac{0.5 S_y}{f.S.}$$

Brittle



: ultimate strength (S_u)

$$\sigma_{all} = \frac{S_u}{F.S.}$$

$$\tau_{all} = \frac{0.5 S_u}{F.S.}$$

as:

S_y : yield strength (MPa) — إجهاد الخضوع (التحمل)

S_u : ultimate strength (MPa) — إجهاد الكسر (أقصى إجهاد يتحمله الجسم)

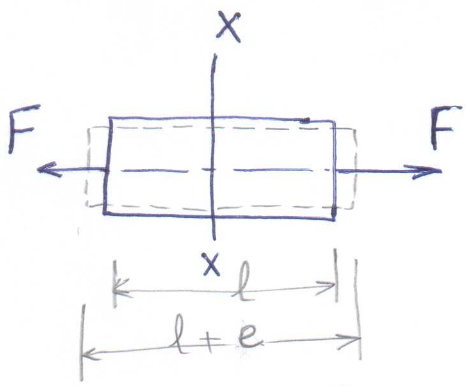
f.s. : factor of safety (2 ~ 3)

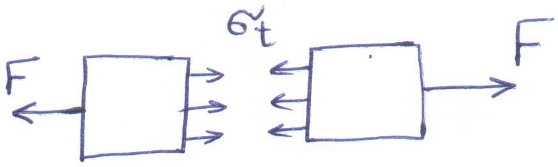
σ_{all} : maximum allowable stress

τ_{all} : maximum allowable shear stress

* stress Analysis
  - simple
    - Normal (σ)
      - tension stress
      - Compression stress
      - bending stress
    - Bearing (σ_{Br})
    - shear (τ)
      - direct shear
      - shear due to torsion
      - shear due to bending
  - Combined
    - from different Kinds
      - bending + torsion
      - $\sigma + \tau$
    - from same Kind
      - tension or Compression + bending
      - $\sigma + \sigma$

* Tensile stress and strain





$$\sigma_t = \frac{F}{A}$$

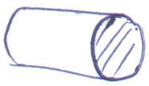
$$\text{Normal stress } (N/m^2)\ (Pa) = \frac{\text{Normal force } (N)}{\text{Area of failure } (m^2)}$$

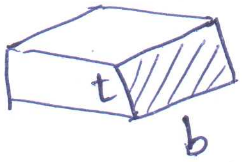
$$\varepsilon = \frac{e}{l}$$

$$\text{strain } (-) = \frac{\text{change of length } (m)}{\text{original length } (m)}$$

* load (F) must pass at the center
* Design eqn $\frac{F}{A} \leq \sigma_{all}$ as: $\sigma_{all} = \frac{S_y}{f.s.}$ or $\frac{S_u}{f.s}$
* Area of failure (cross-section area)

① circle $A = \frac{\pi}{4} d^2$



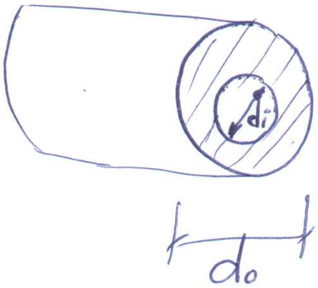


② rectangle $A =$ length * width

$= t * b$

③ hollow cylinder

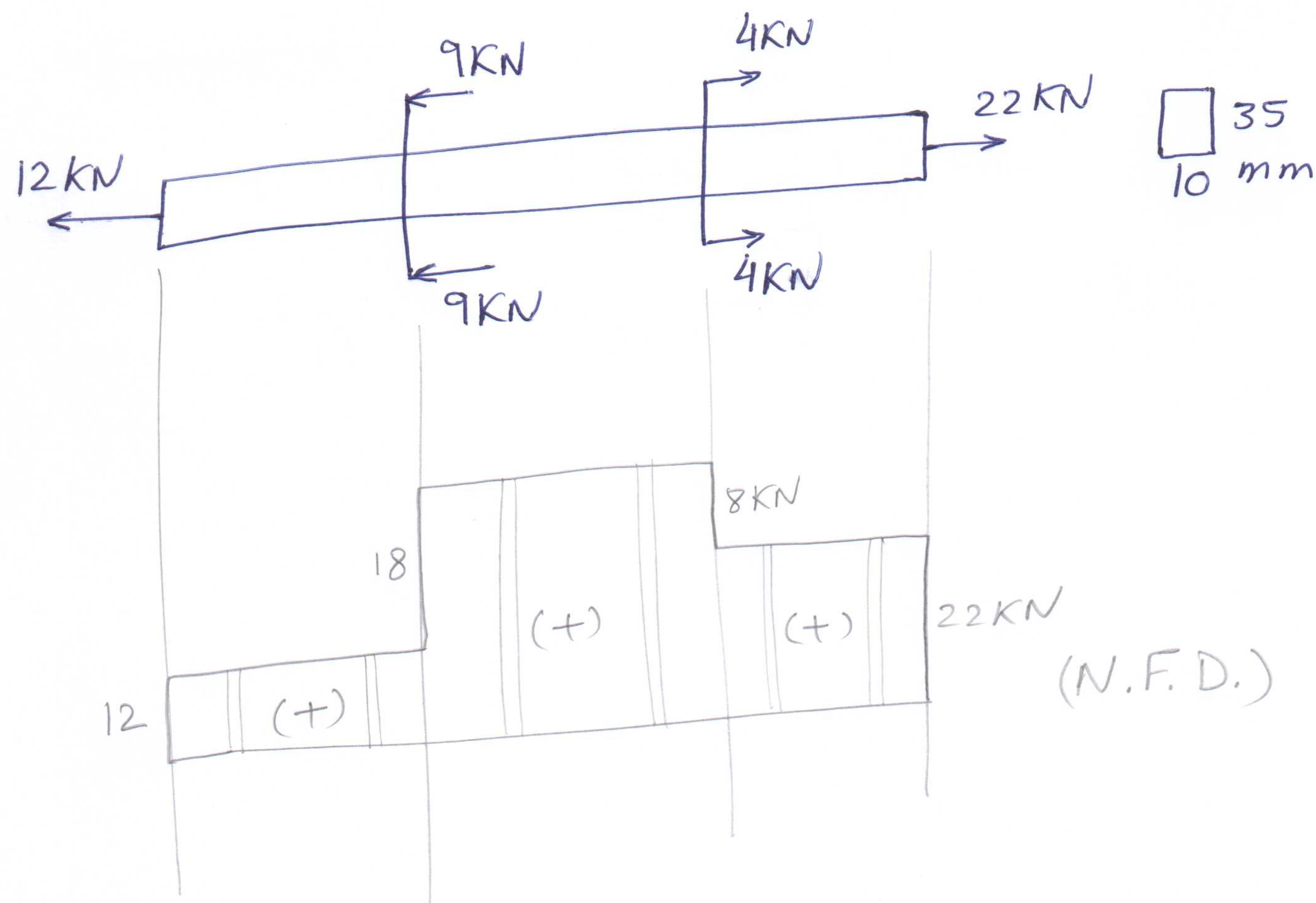
$$A = \frac{\pi}{4}(d_o^2 - d_i^2)$$



Ex. 1-6

Pg 28

or Q5 (sheet)



max. normal force = + 30 KN

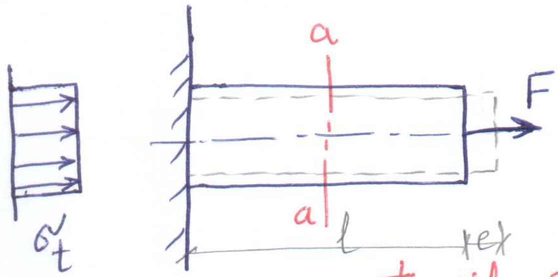
at sec BC

$$\sigma_{max.} = \frac{F_{max.}}{A}$$

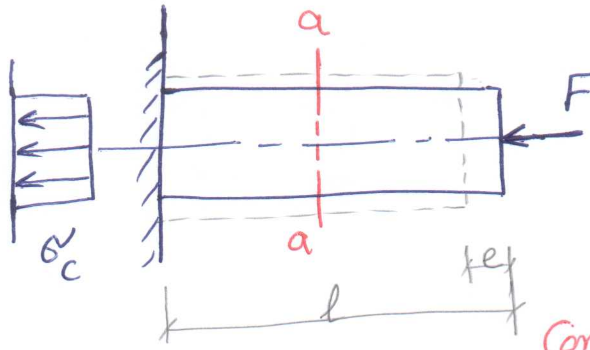
$$= \frac{30 * 10^3}{35 * 10}$$

$= 85.7$ MPa

* Tensile stress and Compressive stress



tensile stress



Compressive stress

at section a-a

$$\sigma_t = \frac{F}{A} \qquad \sigma_c = -\frac{F}{A}$$

$$\epsilon = \frac{e}{l} \qquad \epsilon = \frac{-e}{l}$$

In mechanical design, the +ve sign means tensile and the -ve sign means Compressive.

* Young's modulus (Modulus of elasticity) (E)

* Hooke's law

(after Robert Hooke)

when a material is loaded within elastic limit, the stress is directly proportional to strain.

$$\sigma \propto \varepsilon$$

$$\sigma = E\,\varepsilon$$

$$\therefore E = \frac{\sigma}{\varepsilon} = \frac{F}{A} * \frac{l}{e}$$

$$\therefore \boxed{E = \frac{Fl}{Ae}} \quad (MPa)$$

if e is required (elongation)

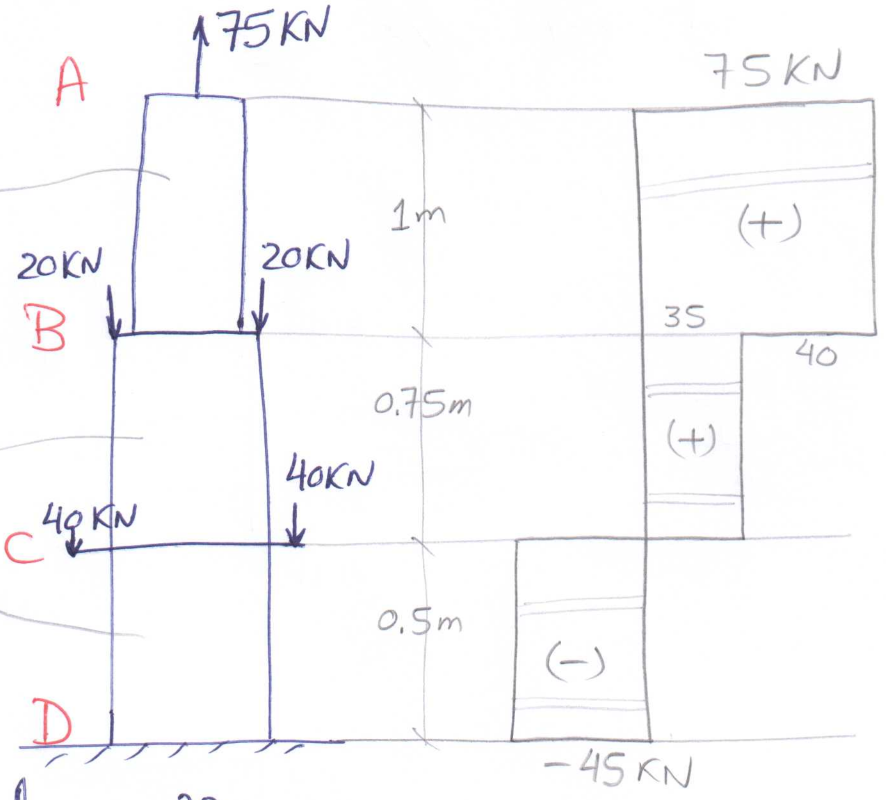
$$\boxed{e = \frac{Fl}{AE}} \quad mm$$

Ex. 4-1

Pg 126 or Q 6 (sheet)

Area$_{AB}$ = 600 mm^2

Area$_{BD}$ = 1200 mm^2



Reqd

$\Delta l_A = ??$ & $\Delta l_{B/C} = ??$

$$\Delta l_A = \frac{Fl}{AE}\Big|_{CD} + \frac{Fl}{AE}\Big|_{BC} + \frac{Fl}{AE}\Big|_{AB}$$

$$= \frac{-45*10^3*0.5*10^3}{1200*200*10^3} + \frac{35*10^3*0.75*10^3}{1200*200*10^3} +$$

$$\frac{75*10^3*1*10^3}{600*200*10^3} = +0.641 \text{ mm}$$

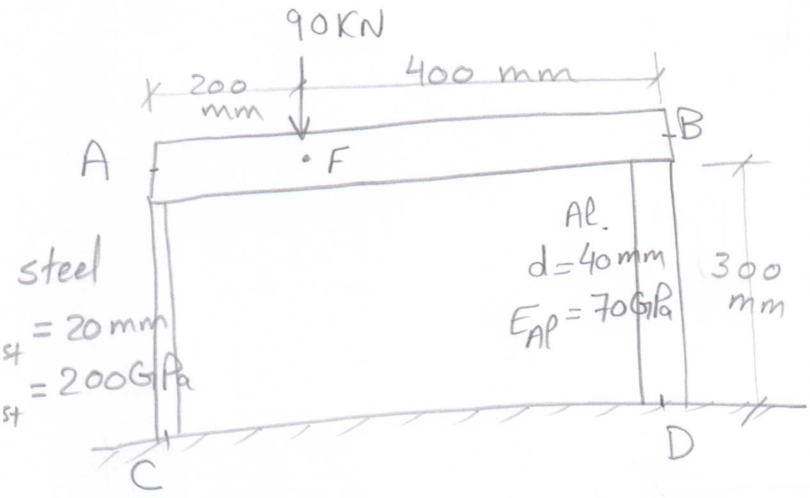
$$\Delta l_{B/C} = \frac{Fl}{AE}\Big|_{BC} = \frac{35*10^3*0.75*10^3}{1200*200*10^3}$$

$$= +0.109 \text{ mm}$$

Ex. 4-3

Pg 128

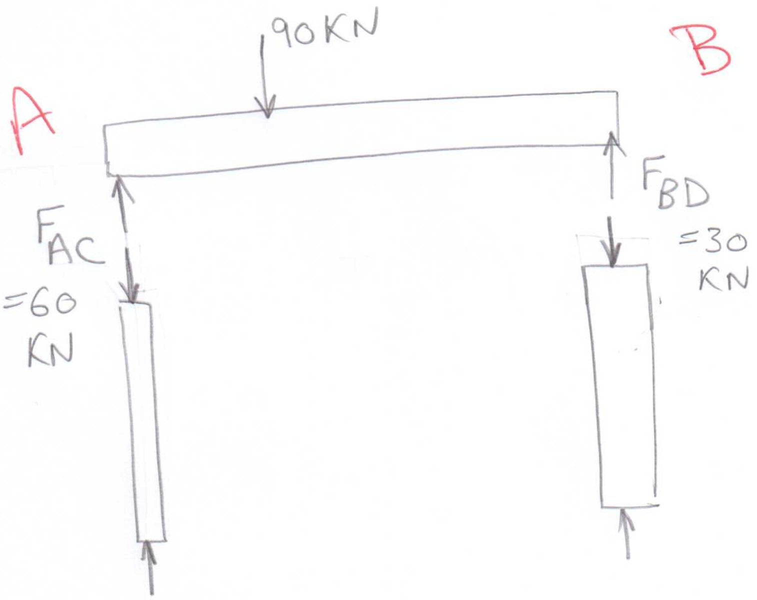
or Q 11 (sheet)



Reqd

$\Delta l_F = ??$

Soln



$\Sigma F_y = 0 \quad \uparrow +$

$F_{AC} + F_{BD} = 90 \text{ KN} \longrightarrow (1)$

$\Sigma M_A = 0 \quad (+\circlearrowleft)$

$F_{BD} * 600 - 90 * 200 = 0$

$F_{BD} = 30 \text{ KN}$

sub. in (1)

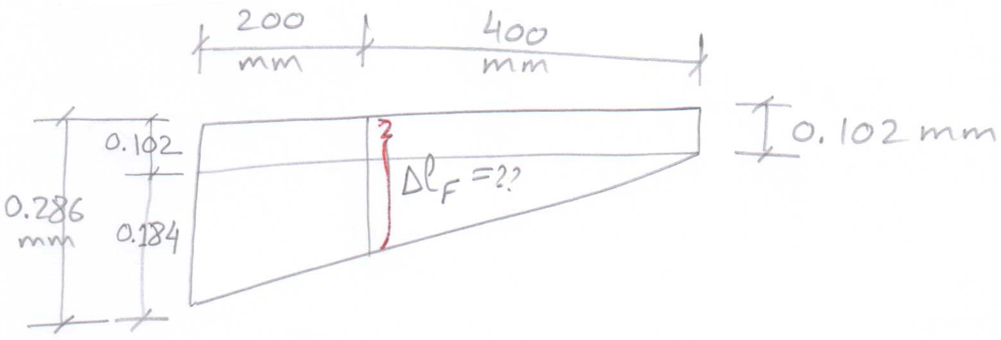
$\therefore F_{AC} = 60 \text{ KN}$

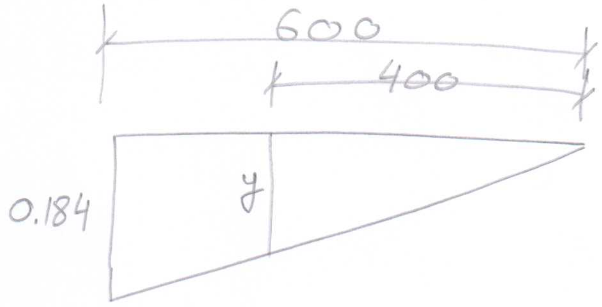
For member AC

$$\Delta l_{AC} = \frac{Fl}{AE} = \frac{-60 * 10^3 * 300}{\frac{\pi}{4}(20)^2 * 200 * 10^3} = -0.286 \text{ mm}$$

for member BD

$$\Delta l_{BD} = \frac{Fl}{AE} = \frac{-30 * 10^3 * 300}{\frac{\pi}{4}(40)^2 * 70 * 10^3} = -0.102 \text{ mm}$$





from triangle

$$\frac{y}{400} = \frac{0.184}{600}$$

$$y = 0.123 \text{ mm}$$

$$\Delta l_F = 0.102 + y$$

$$= 0.102 + 0.123$$

$$= 0.225 \text{ mm}$$