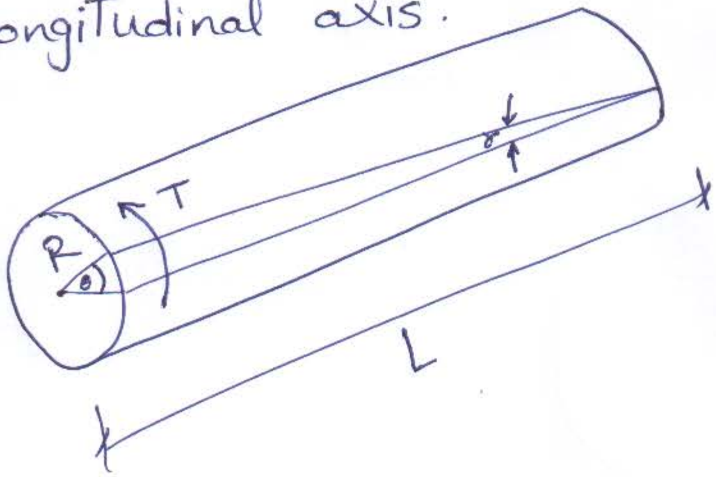
Torsion stress

Torque is a moment that tends to twist a member about its longitudinal axis.



$$\boxed{\tau = \frac{T * r}{J}}$$

as T: Torque (N. mm)

r: radius at which τ is calculated (mm)

J: 2nd polar moment of area (mm^4)

* τ_{max} will be found at the outer surface ($r_{max.}$)

$$r_{max.} = \frac{\text{outer diameter}}{2}$$

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* For the same arc

$$\theta R = \gamma L = \text{arc length}$$

$$\boxed{\theta = \frac{T * L}{G J}} \qquad \theta \text{ (degree)} = \theta(\text{rad}) * \frac{\pi}{180}$$

as θ: twist angle in rad

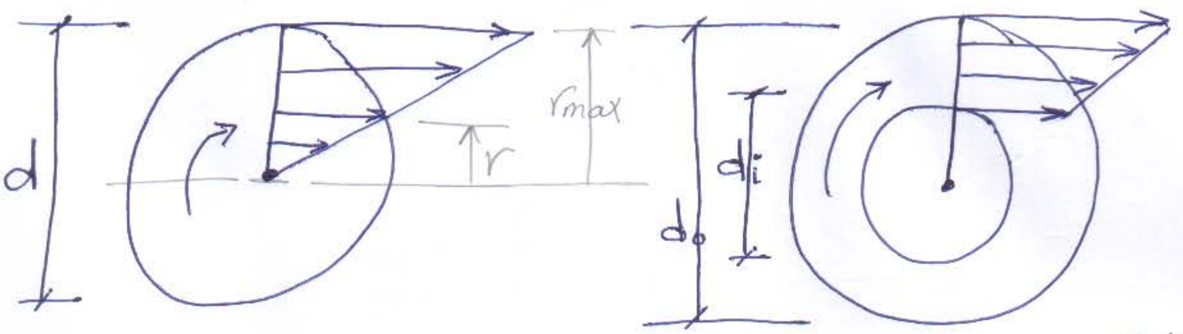
L: shaft Length (mm)

G: modulus of Rigidity (MPa)

from τ and θ eqns

$$\boxed{\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}} = \frac{\tau_{max}}{r_{max}}$$

stress distribution



$J = \frac{\pi}{32} d^4$

$J = \frac{\pi}{32}\left[d_o^4 - d_i^4\right]$

Design eqns

$$\tau_{max.} = \frac{T\, r_{max}}{J} \leq \tau_{all}$$

if θ_{all} is given $\qquad \theta = \frac{TL}{GJ} \leq \theta_{all}$

$$P = T * \omega$$

P: power in watt

T: Torque in N.m

ω: rotational speed in rad/sec

$$\omega = \frac{2\pi\,(rpm)}{60}$$

$$= 2\pi\,(rps)$$

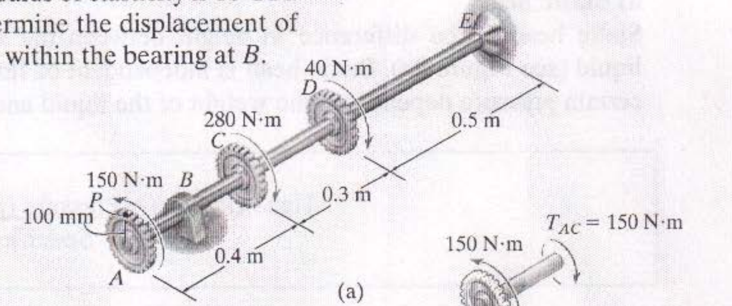
$$= 2\pi f$$

as f: frequency in (Hz)

power transmission

EXAMPLE 5.5

The gears attached to the fixed-end steel shaft are subjected to the torques shown in Fig. 5–19*a*. If the shear modulus of elasticity is 80 GPa and the shaft has a diameter of 14 mm, determine the displacement of the tooth P on gear A. The shaft turns freely within the bearing at B.



(a)

SOLUTION

Internal Torque. By inspection, the torques in segments AC, CD, and DE are different yet *constant* throughout each segment. Free-body diagrams of appropriate segments of the shaft along with the calculated internal torques are shown in Fig. 5–19*b*. Using the right-hand rule and the established sign convention that positive torque is directed away from the sectioned end of the shaft, we have

$$T_{AC} = +150\ \text{N}\cdot\text{m} \qquad T_{CD} = -130\ \text{N}\cdot\text{m} \qquad T_{DE} = -170\ \text{N}\cdot\text{m}$$

These results are also shown on the torque diagram, Fig. 5–19*c*.

Angle of Twist. The polar moment of inertia for the shaft is

$$J = \frac{\pi}{2}(0.007\ \text{m})^4 = 3.771(10^{-9})\ \text{m}^4$$

Applying Eq. 5–16 to each segment and adding the results algebraically, we have

$$\phi_A = \sum \frac{TL}{JG} = \frac{(+150\ \text{N}\cdot\text{m})(0.4\ \text{m})}{3.771(10^{-9})\ \text{m}^4\,[80(10^9)\ \text{N/m}^2]} + \frac{(-130\ \text{N}\cdot\text{m})(0.3\ \text{m})}{3.771(10^{-9})\ \text{m}^4\,[80(10^9)\ \text{N/m}^2)]} + \frac{(-170\ \text{N}\cdot\text{m})(0.5\ \text{m})}{3.771(10^{-9})\ \text{m}^4\,[80(10^9)\ \text{N/m}^2)]} = -0.2121\ \text{rad}$$

Since the answer is negative, by the right-hand rule the thumb is directed *toward* the end E of the shaft, and therefore gear A will rotate as shown in Fig. 5–19*d*.

The displacement of tooth P on gear A is

$$s_P = \phi_A r = (0.2121\ \text{rad})(100\ \text{mm}) = 21.2\ \text{mm} \qquad \textit{Ans.}$$

NOTE: Remember that this analysis is valid only if the shear stress does not exceed the proportional limit of the material.

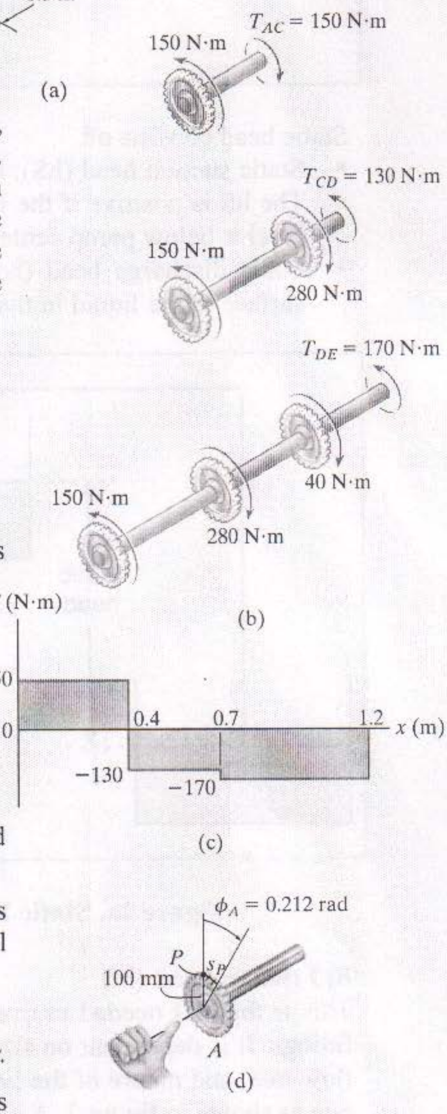
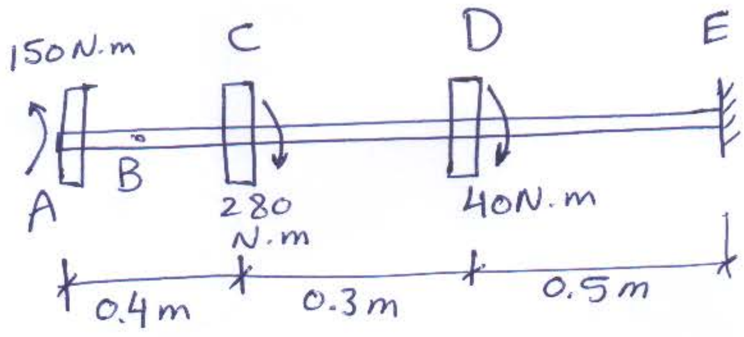


Fig. 5–19

Ex. 5-5 Pg 205



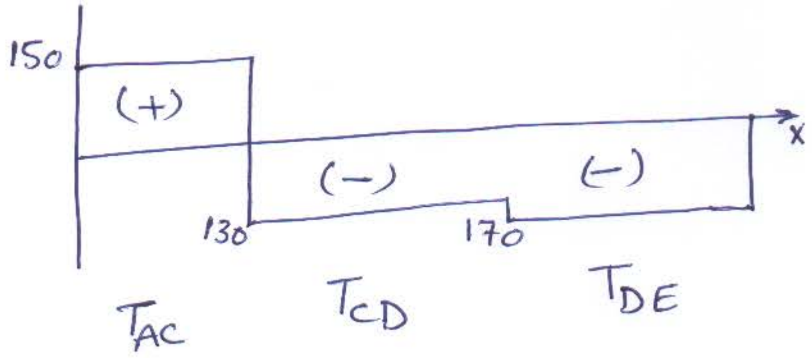
$G = 80 * 10^3$ MPa

$d = 14$ mm

$\theta_A = ??$ arc length

max. shear stress

$\tau_{max} = ??$



soln

$$J = \frac{\pi}{32} d^4 = 3771.48 \ mm^4$$

$$\theta_A = \sum \frac{TL}{GJ} = \frac{T_{AC} L_{AC}}{GJ} + \frac{T_{CD} L_{CD}}{GJ} + \frac{T_{DE} L_{DE}}{GJ}$$

$$= \frac{10^3 * 10^3}{80 * 10^3 * 3771.48} \left[150 * 0.4 - 130 * 0.3 - 170 * 0.5 \right]$$

$$= -0.2121 \ rad$$

$R_A = 100$ mm

$$\text{arc length} = \theta_A R_A$$

$$= -0.2121 * 100 = 21.2 \ mm$$

$$\tau_{max} = \frac{T_{max} r_{max}}{J}$$

$$= \frac{170 * 7 * 10^3}{3771.48} = 315.53 \ MPa$$

from graph

$$T_{max} = 170 \ N.m = 170 * 10^3 \ N.mm$$

$$r_{max} = \frac{d}{2} = \frac{14}{2} = 7 \ mm$$

Q_6 sh #3

$\frac{d}{D} = 0.75$ $\quad L = 4\,m$ $\quad$ Power = 1MW

Power = $10^9 \frac{N.mm}{sec}$

$N = 120$ rpm $\quad \therefore \omega = \frac{2\pi * 120}{60} = 4\pi$ rad/sec

$\tau_{max} \leq 70$ MPa $\quad \theta_{max} \leq 1.75$ rad $\quad G = 80 * 10^3$ MPa

determine (a) $D = ??$ $\quad$ (b) τ_{max} , $\theta_{max} = ??$

Sol$^{\underline{n}}$

$$\text{power} = T * \omega$$

$$T = \frac{\text{Power}}{\omega} = \frac{10^9}{4\pi} = 79.577 * 10^6 \text{ N.mm}$$

$$J = \frac{\pi}{32}\left[D^4 - (0.75D)^4\right] = 0.067 D^4$$

Assume max. shear stress

$$\tau_{max} \leq 70 \text{ MPa}$$

$$\frac{T_{max}\, r_{max}}{J} \leq 70$$

$$\frac{79.577 * 10^6 * D/2}{0.067\, D^4} \leq 70$$

$$\text{mm} \quad 203.952 \leq D \longrightarrow (1)$$

Assume max. angle of twist

$$\theta_{max} \leq 1.75$$

$$\frac{T_{max}\, L}{G\, J} \leq 1.75$$

$$\frac{79.577 * 10^6 * 4 * 10^3}{80 * 10^3 * 0.067 D^4} \leq 1.75$$

$$\text{mm} \quad 76.324 \leq D \longrightarrow ②$$

take max. value of D

ⓐ $\therefore D = 204$ mm

ⓑ

$$\tau_{max} = \frac{Tr}{J} = \frac{79.577 * 10^6 * 204/2}{0.067 (204)^4}$$

$$= 69.95 \text{ MPa}$$

$$\theta_{max} = \frac{TL}{GJ} = \frac{79.577 * 10^6 * 4 * 10^3}{80 * 10^3 * 0.067 * (204)^4}$$

$$= 0.034 \text{ rad}$$

$$= 1.965^\circ$$