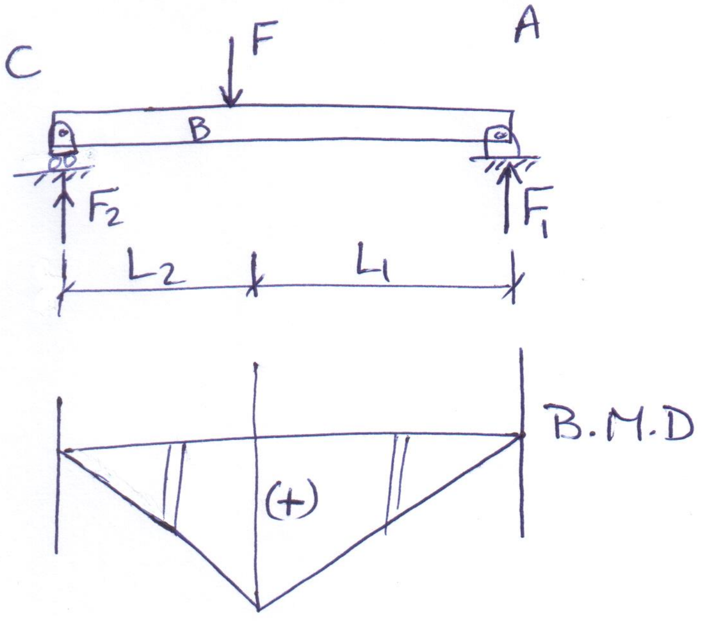
Bending moment



* Find F_1 & F_2

$\Sigma F_y = 0 \quad \uparrow +$

$F_1 + F_2 = F \longrightarrow (1)$

$\Sigma M_A = 0$

$F_2(L_1 + L_2) = F L_1$

$\therefore F_2 = \checkmark$

sub. in (1)

$\therefore F_1 = \checkmark$

* from left side of beam

$M_c = 0$

$M_B = F_2 * L_2 \Longrightarrow$ max. $\sigma_{b_{max}} = \dfrac{M_B C}{I}$

$M_A = F_2(L_1 + L_2) - F_1 L_1 = 0$

$$\boxed{\sigma_b = \frac{MC}{I}}$$

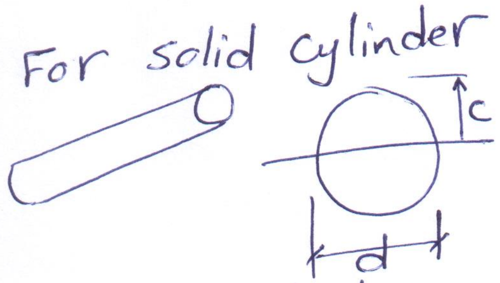
as

M: Moment

C: distance from neutral axis to a point farthest away from ~ ~.

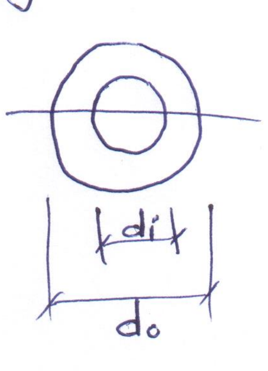
I: moment of inertia of the cross-sectional area about neutral axis

* For solid cylinder



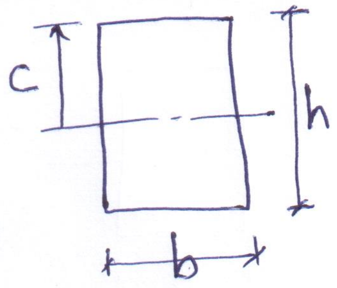
$c = \dfrac{d}{2}$, $I = \dfrac{\pi}{64} d^4$

* For hollow cylinder

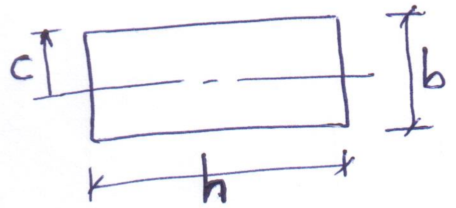


$c = \dfrac{d_o}{2}$, $I = \dfrac{\pi}{64}(d_o^4 - d_i^4)$

* For rectangular cylinder



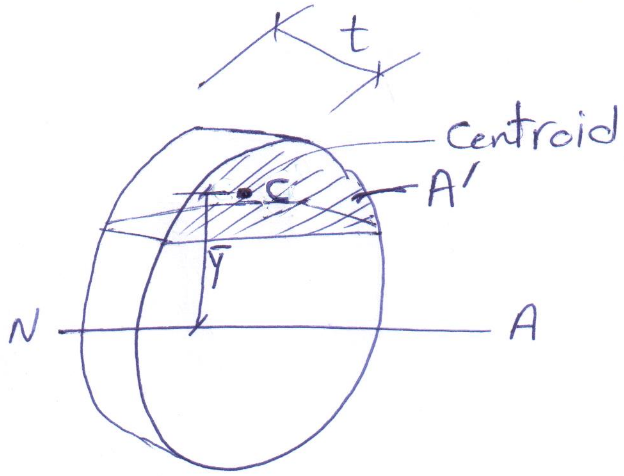
$c = \frac{h}{2}$, $I = \frac{1}{12} b h^3$



$c = \frac{b}{2}$, $I = \frac{1}{12} b^3 h$

Transverse shear stress

$$\boxed{\tau = \frac{VQ}{It}}$$



as

τ : shear stress

V : shear force

I : moment of area

t : member's width

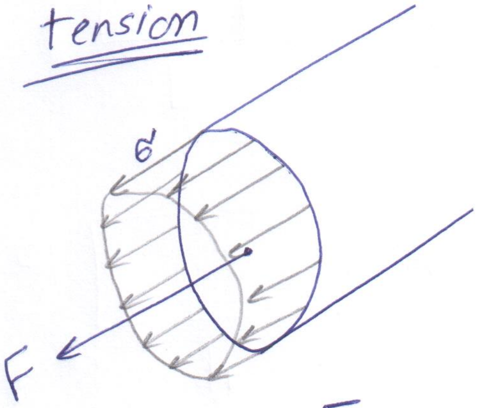
$$Q = \bar{y}' A'$$

as A' : the area of the top portion

$\bar{y}'$: the distance from the neutral axis to the centroid of A'

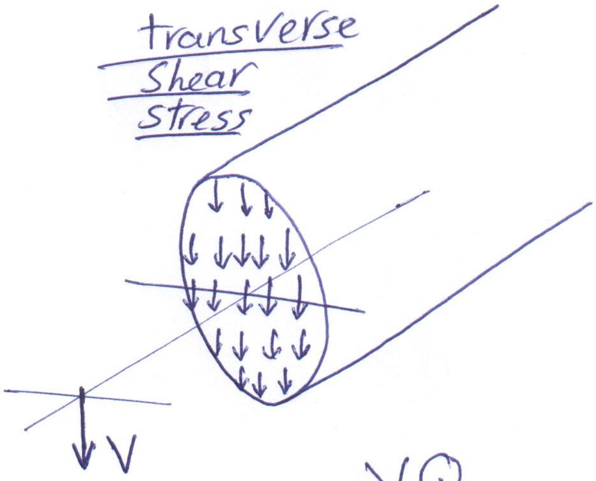
stress distribution

tension



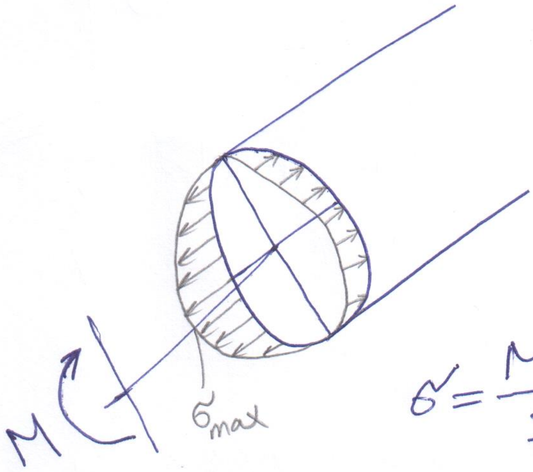
$$\sigma = \frac{F}{A}$$

transverse Shear stress



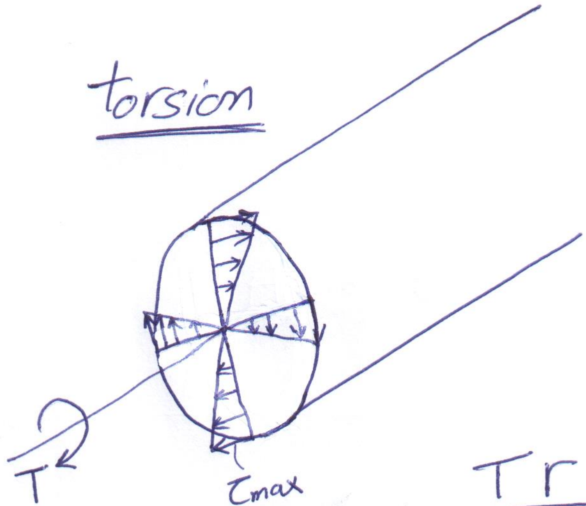
$$\tau = \frac{VQ}{It}$$

Bending



$$\sigma = \frac{My}{I}$$

torsion



$$\tau = \frac{Tr}{J}$$

Combined stresses

from the same Kinds

σ & σ

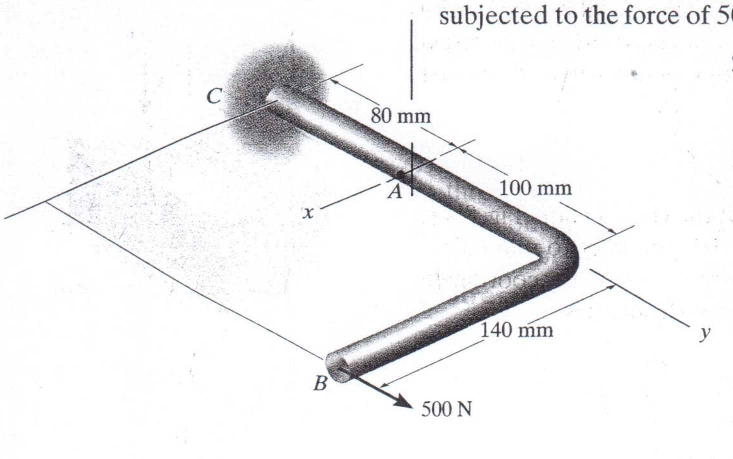
τ & τ

from different Kinds

τ & σ

EXAMPLE 8.7

The solid rod shown in Fig. 8–8*a* has a radius of 7.5 mm. If it is subjected to the force of 500 N, determine the state of stress at point A.



(a)

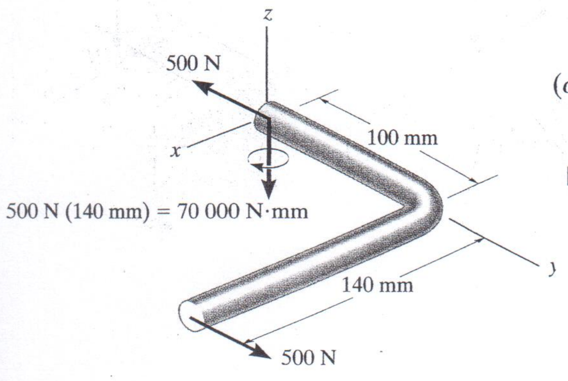
SOLUTION

Internal Loadings. The rod is sectioned through point A. Using the free-body diagram of segment AB, Fig. 8–8*b*, the resultant internal loadings are determined from the equations of equilibrium. Verify these results. In order to better "visualize" the stress distributions due to these loadings, we can consider the *equal but opposite resultants* acting on segment AC, Fig. 8–8*c*.

Stress Components.

Normal Force. The normal-stress distribution is shown in Fig. 8–8*d*. For point A, we have

$$(\sigma_A)_y = \frac{P}{A} = \frac{500\text{ N}}{\pi(7.5\text{ mm})^2} = 2.83\text{ N/mm}^2 = 2.83\text{ MPa}$$



(b)

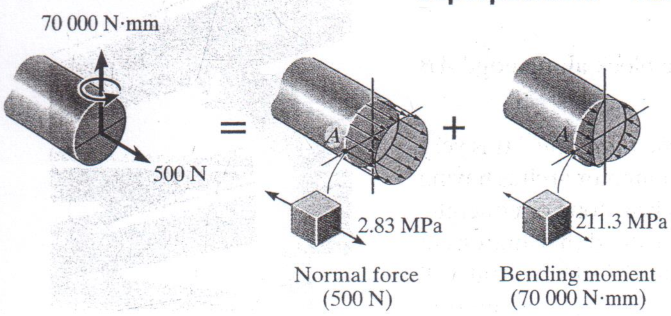
Bending Moment. For the moment, $c = 7.5$ mm, so the normal stress at point A, Fig. 8–8*e*, is

$$(\sigma_A)_y = \frac{Mc}{I} = \frac{70\ 000\text{ N}\cdot\text{mm}(7.5\text{ mm})}{[\frac{1}{4}\pi(7.5\text{ mm})^4]}$$

$$= 211.3\text{ N/mm}^2 = 211.3\text{ MPa}$$

Superposition. When the above results are superimposed, it is seen that an element of material at A is subjected to the normal stress

$$(\sigma_A)_y = 2.83\text{ Mpa} = 211.3\text{ MPa} = 214.1\text{ MPa} \qquad \textit{Ans.}$$



(c) (d) (e)

Fig. 8–8

EXAMPLE 8.8

The solid rod shown in Fig. 8–9*a* has a radius of 7.5 mm. If it is subjected to the force of 800 N, determine the state of stress at point *A*.

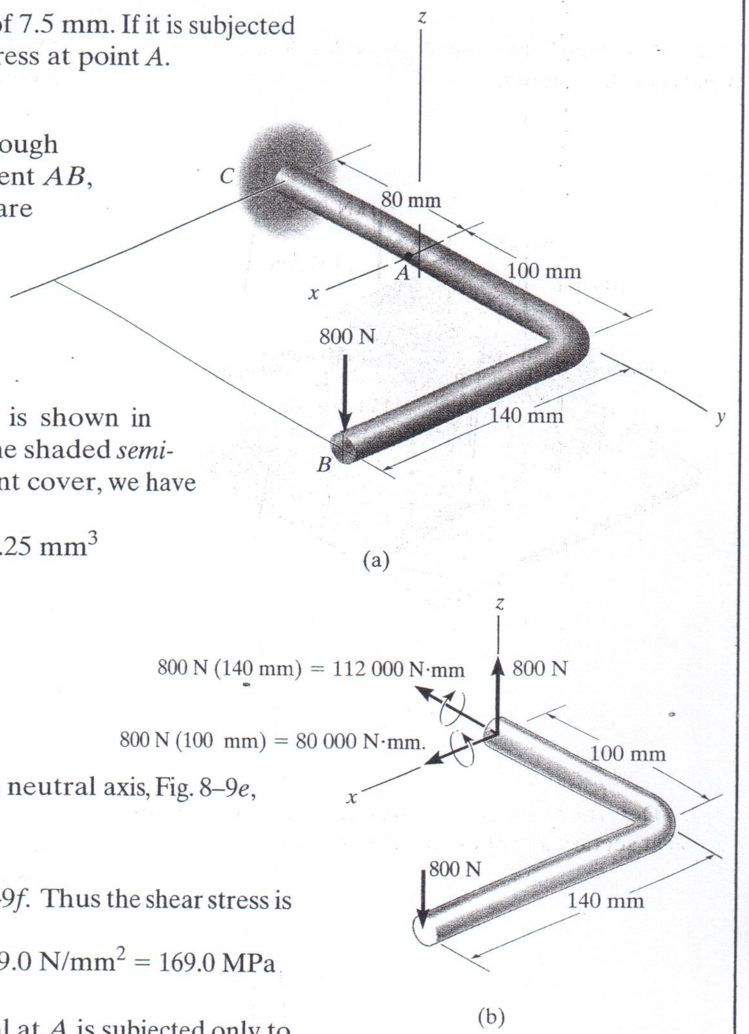


Fig. 8–9

SOLUTION

Internal Loadings. The rod is sectioned through point *A*. Using the free-body diagram of segment *AB*, Fig. 8–9*b*, the resultant internal loadings are determined from the six equations of equilibrium. Verify these results. The *equal but opposite resultants* are shown acting on segment *AC*, Fig. 8–9*c*.

Stress Components.

Shear Force. The shear-stress distribution is shown in Fig. 8–9*d*. For point *A*, *Q* is determined from the shaded *semi-circular* area. Using the table on the inside front cover, we have

$$Q = \bar{y}'A' = \frac{4(7.5\text{ mm})}{3\pi}\left[\frac{1}{2}\pi(0.75\text{ mm})^2\right] = 281.25\text{ mm}^3$$

so that

$$(\tau_{yz})_A = \frac{VQ}{It} = \frac{800\text{ N}(281.25\text{ mm}^3)}{[\frac{1}{4}\pi(7.5\text{ mm})^4]2(7.5\text{ mm})}$$

$$= 6.04\text{ N/mm}^2 = 6.04\text{ MPa}$$

Bending Moment. Since point *A* lies on the neutral axis, Fig. 8–9*e*, the normal stress is

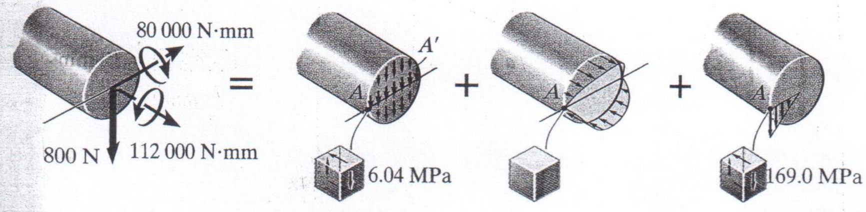
$$\sigma_A = 0$$

Torque. At point *A*, $\rho_A = c = 7.5$ mm, Fig. 8–9*f*. Thus the shear stress is

$$(\tau_{yz})_A = \frac{Tc}{J} = \frac{112\,000\text{ N}\cdot\text{mm }(7.5\text{ mm})}{[\frac{1}{2}\pi(7.5\text{ mm})^4]} = 169.0\text{ N/mm}^2 = 169.0\text{ MPa}$$

Superposition. Here the element of material at *A* is subjected only to a shear stress component, where

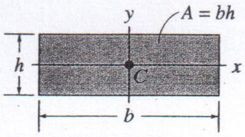
$$(\tau_{yz})_A = 6.04\text{ MPa} + 169.0\text{ MPa} = 175.0\text{ MPa} \qquad \textit{Ans.}$$



Shear force (800 N) | Bending moment (80 000 N·mm) | Torsional moment (112 000 N·mm)

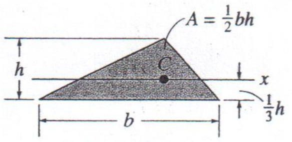
(c) (d) (e) (f)

Geometric Properties of Area Elements



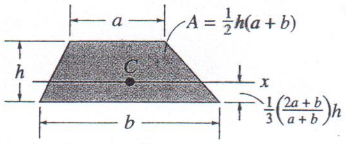
$$I_x = \frac{1}{12}bh^3$$
$$I_y = \frac{1}{12}hb^3$$

Rectangular area

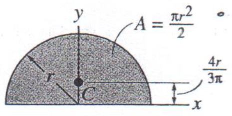


$$I_x = \frac{1}{36}bh^3$$

Triangular area

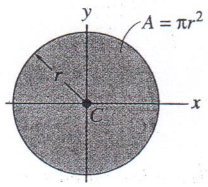


Trapezoidal area



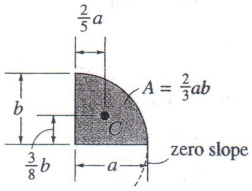
$$I_x = \frac{1}{8}\pi r^4$$
$$I_y = \frac{1}{8}\pi r^4$$

Semicircular area

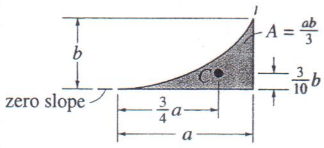


$$I_x = \frac{1}{4}\pi r^4$$
$$I_y = \frac{1}{4}\pi r^4$$

Circular area



Semiparabolic area



Exparabolic area