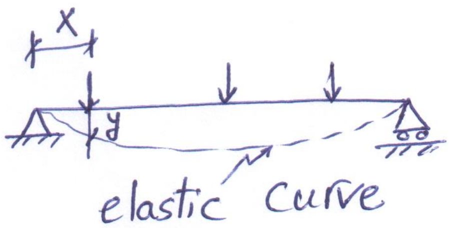
Deflection

equation of the elastic curve



$$\boxed{\frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{M}{EI}}$$

ρ: radius of curvature

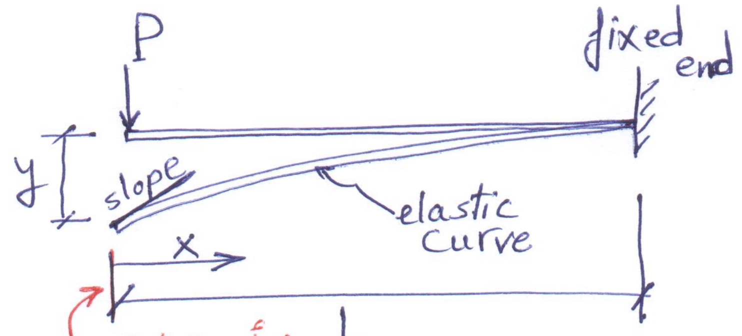
M: bending moment

I: second moment of area

---

* Double integration method

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$



Ex.

$$EI\frac{d^2y}{dx^2} = M = -Px$$

نبدأ من الطرف الأقل في عدد القوى

$$\circlearrowright+ \quad EI\frac{d^2y}{dx^2} = -Px$$

تحديد الاتجاه اختياري

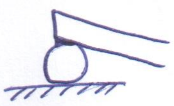
$$EI\frac{dy}{dx} = -\frac{Px^2}{2} + C_1 \longrightarrow (1)$$

$\frac{dy}{dx}$: slope

$$EIy = -\frac{Px^3}{6} + C_1x + C_2 \longrightarrow (2)$$

y : deflection

Boundary Conditions



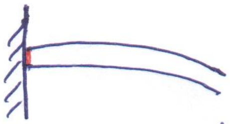
roller

$y = 0$

$M = 0$

roller

$y = 0$



fixed end

$\frac{dy}{dx} = 0$

$y = 0$

hinge pin

$y = 0$

$M = 0$

pin

$y = 0$

Free end

$\frac{d^3y}{dx^3} = 0$

$M = 0$

internal pin or hinge

$M = 0$

B.C.

* at $x = L \therefore \frac{dy}{dx} = 0$ sub. in ①

$$0 = \frac{-PL^2}{2} + C_1 \qquad \therefore \boxed{C_1 = \frac{PL^2}{2}}$$

* at $x = L \therefore y = 0$ sub. in ②

$$0 = \frac{-PL^3}{6} + \frac{PL^2}{2} * L + C_2 \qquad \therefore \boxed{C_2 = \frac{-PL^3}{3}}$$

* at $x = 0 \therefore y = y_{max}$

$$\boxed{EI\,y = \frac{-P}{6}x^3 + \frac{PL^2}{2}x - \frac{PL^3}{3}}$$

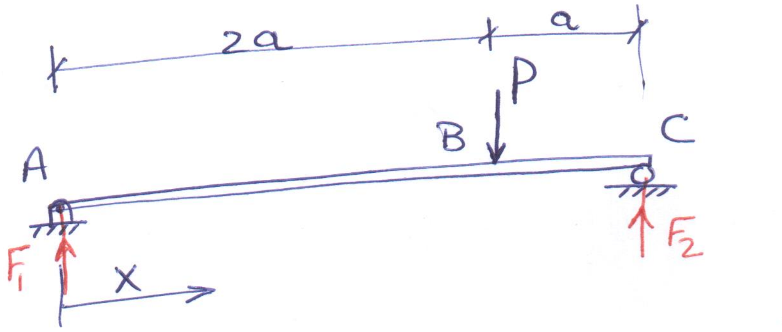
$$EI\,y_{max} = 0 + 0 - \frac{PL^3}{3}$$

$$\boxed{y_{max} = \frac{-PL^3}{3EI}}$$

Ex. 12.3 Pg 582

$EI = \text{Const.}$

$y_{max} = ??$



sol$\underline{n}$

$$\Sigma F_y = 0$$

$$F_1 - P + F_2 = 0 \qquad \therefore F_1 + F_2 = P \longrightarrow ①$$

$$\circlearrowleft + \ \Sigma M_c = 0$$

$$F_1 3a - P(3a - 2a) = 0 \qquad \therefore \boxed{F_1 = \frac{P(3a-2a)}{3a}} \Rightarrow \boxed{F_1 = \frac{P}{3}} \qquad 3a = L$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$EI\frac{d^2y}{dx^2} = M = \frac{P}{3}x - P\langle x - 2a\rangle$$

$\langle - \rangle$ القوسين ما بينهما موجب فقط ولو أصبح سالب نساويه بالصفر

$$EI\frac{dy}{dx} = \frac{P}{6}x^2 - \frac{P}{2}\langle x-2a\rangle^2 + C_1 \longrightarrow (2)$$

$$EIy = \frac{P}{18}x^3 - \frac{P}{6}\langle x-2a\rangle^3 + C_1x + C_2 \rightarrow (3)$$

B.C.

* at A $x=0 \therefore y=0$ sub. in (3)

$$0 = 0 - 0 + 0 + C_2 \qquad \therefore \boxed{C_2 = 0}$$

* at C $x = L = 3a \therefore y = 0$ sub. in (3)

$$0 = \frac{P}{18}(3a)^3 - \frac{P}{6} * a^3 + C_1 * 3a + 0 \qquad \div 3a$$

$$0 = \frac{P}{18} * 9a^2 - \frac{P}{18}a^2 + C_1$$

$$\boxed{C_1 = -\frac{P}{18} * 8a^2} \Rightarrow \boxed{C_1 = -\frac{4Pa^2}{9}}$$

* y_{max} at $\frac{dy}{dx} = 0$ sub. in eqn (2)

$$0 = \frac{\cancel{P}}{6}x^2 - \frac{\cancel{P}}{2}\langle x-2a\rangle^2 - \frac{4\cancel{P}a^2}{9} \qquad * 18$$

$$8a^2 = 3x^2 - 9\langle x-2a\rangle^2 \longrightarrow (4)$$

assume $x < 2a$ Sub. in (4)

$$8a^2 = 3x^2 - 0$$

because $\langle x-2a$ = -ve $\therefore \langle x-2a\rangle = 0$

$$\sqrt{\frac{8}{3}}a = x$$

$\therefore x = 1.63a$ which is $< 2a$ $\therefore$ Correct.

sub. in (3)

$$\therefore y_{max} = \frac{1}{EI}\left[\frac{P}{18} * (1.63a)^3 - 0 - \frac{4Pa^2}{9} * (1.63a) + 0\right]$$

$$y_{max} = \frac{Pa^3}{EI} * (-0.4838) = \frac{-Pa^3}{2EI}$$

sheet 6

No 4

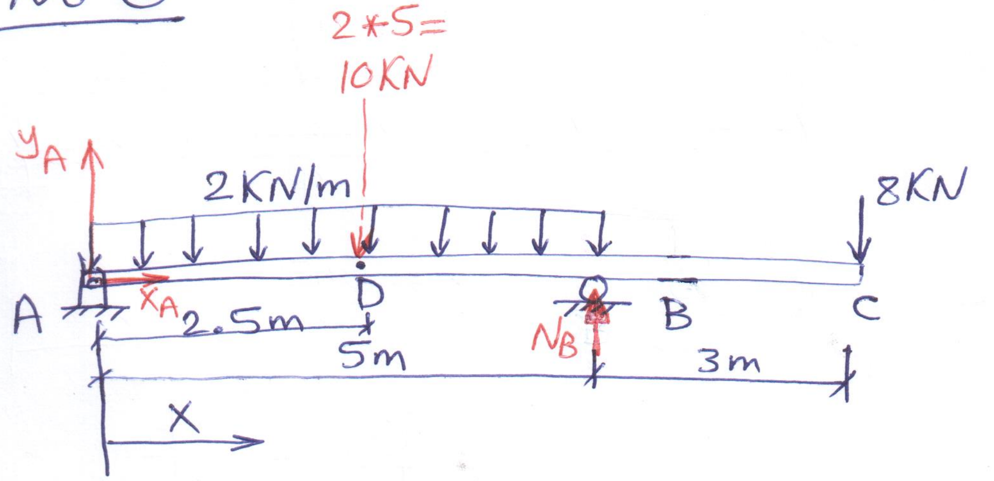
EI = Const.

determine :-

a) slope at A & B

b) deflection at C & D

soln



$\xrightarrow{+}$ $\Sigma F_x = 0$ $\therefore X_A = 0$

$+\uparrow$ $\Sigma F_y = 0$ $\quad y_A - 10 + N_B - 8 = 0$

$$y_A + N_B = 18 \longrightarrow (1)$$

$\circlearrowleft^{+}$ $\Sigma M_A = 0$ $\quad 10 * 2.5 - N_B * 5 + 8 * 8 = 0$

$$N_B = 17.8 \text{ KN}$$

sub. in (1) $\therefore y_A = 0.2 \text{ KN}$

$$\frac{d^2y}{dX^2} = \frac{M}{EI}$$

$$EI\frac{d^2y}{dX^2} = M = y_A * X - 2X * \frac{X}{2} + N_B \langle X-5 \rangle$$

$$EI\frac{d^2y}{dX^2} = 0.2X - X^2 + 17.8\langle X-5 \rangle$$

$$EI\frac{dy}{dX} = 0.1X^2 - \frac{X^3}{3} + \frac{17.8}{2}\langle X-5 \rangle^2 + C_1 \longrightarrow (2)$$

$$EI\,y = \frac{0.1}{3}X^3 - \frac{X^4}{12} + \frac{17.8}{6}\langle X-5 \rangle^3 + C_1X + C_2 \longrightarrow (3)$$

B.C. at A $X=0$, $y=0$ sub. in (3)

$$0 = 0 - 0 + 0 + C_2 \qquad \therefore \boxed{C_2 = 0}$$

* at B $\quad X = 5 \quad y = 0 \quad$ sub. in (3)

$$0 = \frac{0.1}{3}(5)^3 - \frac{(5)^4}{12} + 0 + C_1 * 5 + 0$$

$$C_1 = -\frac{0.1}{15}(5)^3 + \frac{(5)^4}{5*12} = -0.833 + 10.4167 = 9.583$$

$$\boxed{C_1 = 9.583}$$

deflection

$$\boxed{EIy = \frac{0.1}{3}X^3 - \frac{X^4}{12} + \frac{17.8}{6}\langle X-5\rangle^3 + 9.583X}$$

slope

$$\boxed{EI\frac{dy}{dx} = 0.1X^2 - \frac{X^3}{3} + \frac{17.8}{2}\langle X-5\rangle^2 + 9.583}$$

a) slope at A $\quad X = 0$

$$\left.\frac{dy}{dx}\right|_A = \frac{1}{EI}\left[0 - 0 + 0 + 9.583\right]$$

$$\left.\frac{dy}{dx}\right|_A = \frac{9.583}{EI}$$

* slope at B $\quad X = 5$

$$\left.\frac{dy}{dx}\right|_B = \frac{1}{EI}\left[0.1(5)^2 - \frac{(5)^3}{3} + 0 + 9.583\right] = \frac{\quad}{EI}$$

b) deflection at D $\quad X = 2.5$

$$y_D = \frac{1}{EI}\left[\frac{0.1}{3}(2.5)^3 - \frac{(2.5)^4}{12} + 0 + 9.583 * 2.5\right] = \frac{\quad}{EI}$$

* deflection at C $\quad X = 8$

$$y_c = \frac{1}{EI}\left[\frac{0.1}{3}(8)^3 - \frac{(8)^4}{12} + \frac{17.8}{6}(3)^3 + 9.583(8)\right]$$

$$= \frac{\quad}{EI}$$