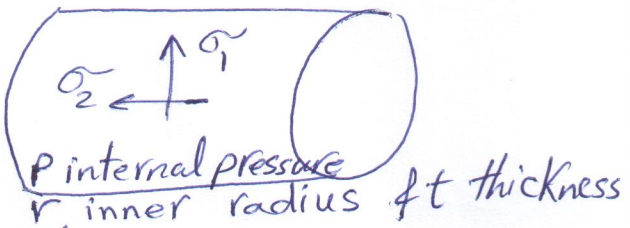
Thin-walled pressure vessels

* Cylindrical or spherical vessels are used to serve as boilers or tanks.
* thin wall means $\frac{\text{inner radius}}{\text{thickness}} = \frac{r}{t} \geq 10$

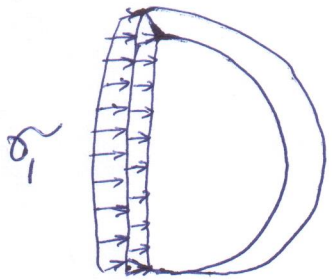
Cylindrical vessels

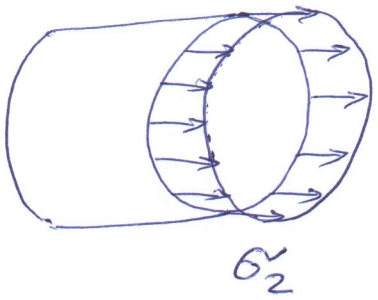
σ_1 in the circumferential or hoop direction

σ_2 in the longitudinal or axial direction



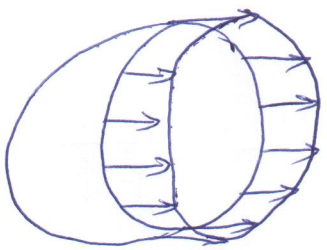
$$\sigma_1 = \frac{Pr}{t} \quad \& \quad \sigma_2 = \frac{Pr}{2t}$$





sphyrical vessels

$$\sigma_2 = \frac{Pr}{2t}$$



σ_2: longitudinal stress

P: internal guage pressure

r: inner radius

t: wall thickness

EXAMPLE 8.1

A cylindrical pressure vessel has an inner diameter of 1.2 m and a thickness of 12 mm. Determine the maximum internal pressure it can sustain so that neither its circumferential nor its longitudinal stress component exceeds 140 MPa. Under the same conditions, what is the maximum internal pressure that a similar-size spherical vessel can sustain?

SOLUTION

Cylindrical Pressure Vessel. The maximum stress occurs in the circumferential direction. From Eq. 8–1 we have

$$\sigma_1 = \frac{pr}{t}; \qquad 140\ \text{N/mm}^2 = \frac{p(600\ \text{mm})}{12\ \text{mm}}$$

$$p = 2.8\ \text{N/mm}^2 = 2.8\ \text{MPa} \qquad \textit{Ans.}$$

Note that when this pressure is reached, from Eq. 8–2, the stress in the longitudinal direction will be $\sigma_2 = \frac{1}{2}(140\ \text{MPa}) = 70\ \text{MPa}$. Furthermore, the *maximum stress* in the *radial direction* occurs on the material at the inner wall of the vessel and is $(\sigma_3)_{\max} = p = 2.8\ \text{MPa}$. This value is 48 times smaller than the circumferential stress (140 MPa), and as stated earlier, its effects will be neglected.

Spherical Vessel. Here the maximum stress occurs in any two perpendicular directions on an element of the vessel, Fig. 8–2*a*. From Eq. 8–3, we have

$$\sigma_2 = \frac{pr}{2t}; \qquad 140\ \text{N/mm}^2 = \frac{p(600\ \text{mm})}{2(12\ \text{mm})}$$

$$p = 5.6\ \text{N/mm}^2 = 5.6\ \text{MPa} \qquad \textit{Ans.}$$

NOTE: Although it is more difficult to fabricate, the spherical pressure vessel will carry twice as much internal pressure as a cylindrical vessel.