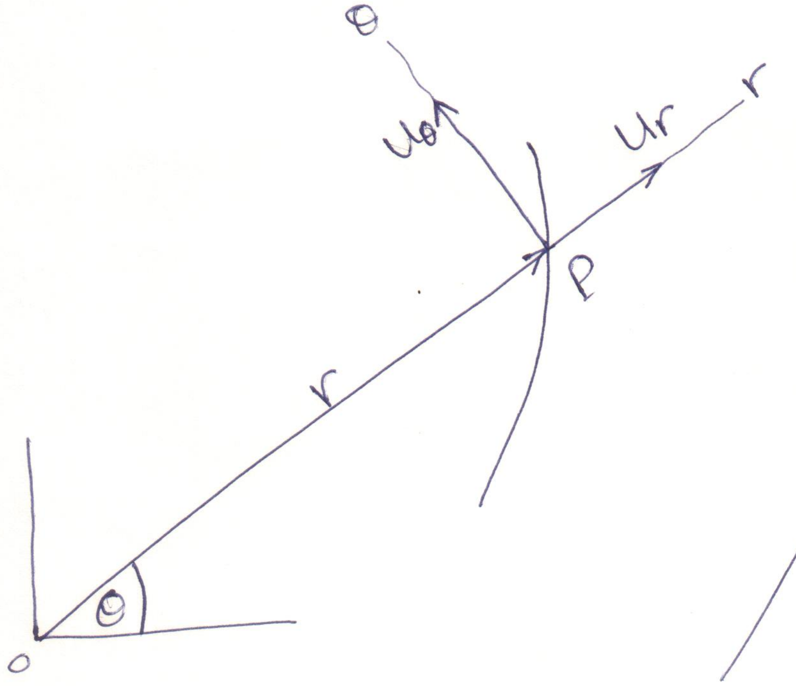
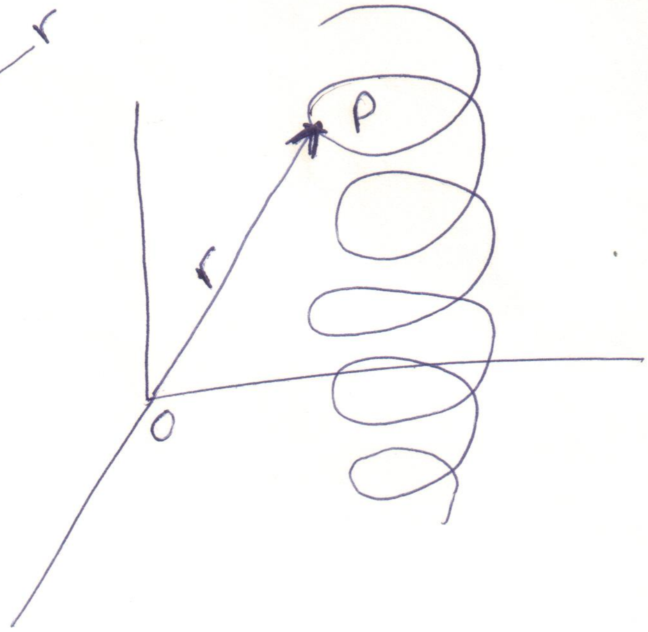
Curvilinear Motion

cylindrical Components r, θ, z

polar Co-ordinates r, θ

cyl. في مستوى واحد



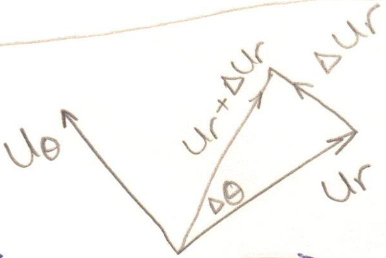


position $\quad \vec{r} = r\,\vec{U_r}$

$$\vec{v} = \dot{\vec{r}} = \dot{r}\,\vec{U_r} + r\,\dot{\vec{U_r}}$$

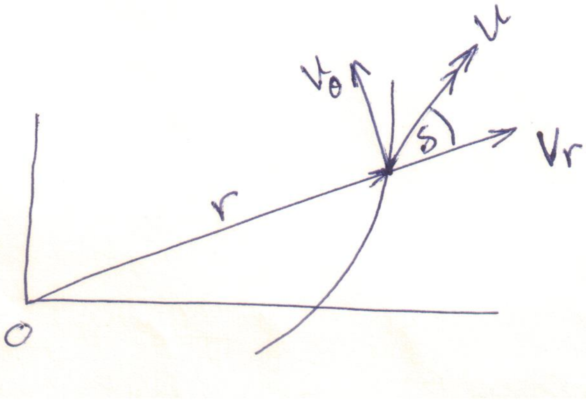
$\Delta U_r \approx 1(\Delta\theta)$

& $\Delta\theta$ in the direction of U_θ



$$\dot{\vec{U_r}} = \lim_{\Delta t \to 0}\left(\frac{\Delta U_r}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(\frac{\Delta\theta}{\Delta t}\right)\vec{U_\theta} = \dot{\theta}\,\vec{U_\theta}$$

$$\left.\begin{aligned} \vec{v} &= \dot{r}\,\vec{U_r} + r\dot{\theta}\,\vec{U_\theta} \\ &= V_r\,\vec{U_r} + V_\theta\,\vec{U_\theta} \end{aligned}\right\} \quad \begin{aligned} v &= \sqrt{\dot{r}^2 + (r\dot{\theta})^2} \\ \delta &= \tan^{-1}\frac{r\dot{\theta}}{\dot{r}} \end{aligned}$$



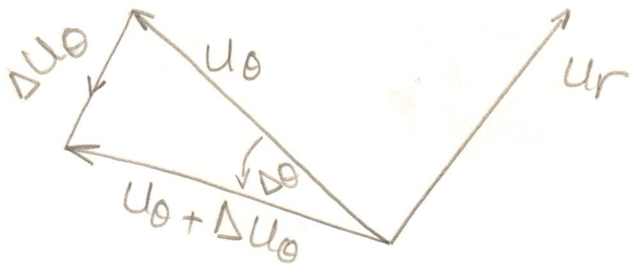
$$\vec{a} = \dot{\vec{v}}$$

$$= \ddot{r}\,\vec{u}_r + \dot{r}\,\dot{\vec{u}}_r + \dot{r}\dot{\theta}\,\vec{u}_\theta + r\ddot{\theta}\,\vec{u}_\theta + r\dot{\theta}\,\dot{\vec{u}}_\theta$$

$$= \ddot{r}\,\vec{u}_r + \dot{r}\dot{\theta}\,\vec{u}_\theta + \dot{r}\dot{\theta}\,\vec{u}_\theta + r\ddot{\theta}\,\vec{u}_\theta - r\dot{\theta}^2\,\vec{u}_r$$

$$\Delta u_\theta = u_\theta\,\Delta\theta$$

$$\approx 1(\Delta\theta)$$



$$\dot{\vec{u}}_\theta = \lim_{\Delta t \to 0}\left(\frac{\Delta u_\theta}{\Delta t}\right)$$

$$= -\lim_{\Delta t \to 0}\left(\frac{\Delta\theta}{\Delta t}\right)\vec{u}_r$$

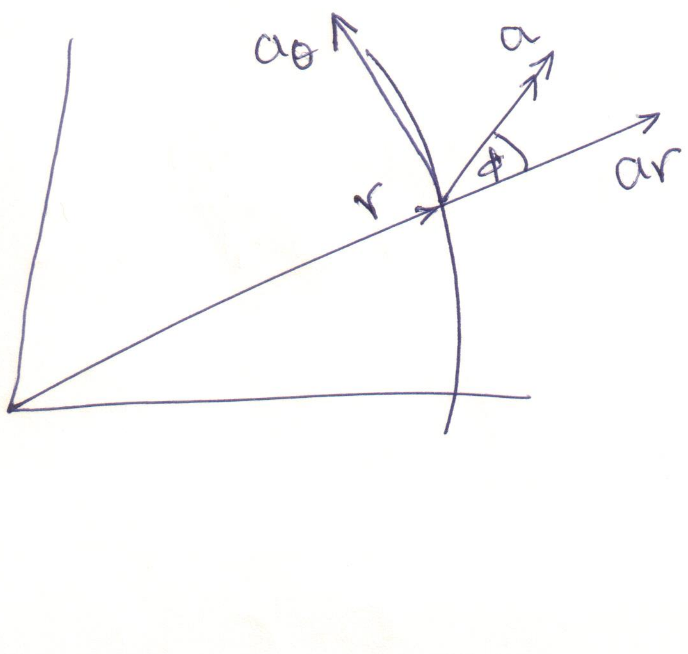
$$= -\dot{\theta}\,\vec{u}_r$$

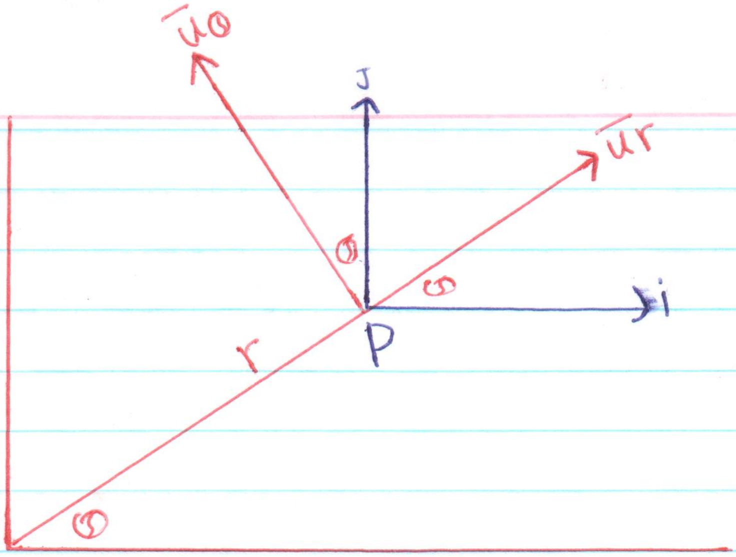
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\vec{u}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\vec{u}_\theta$$

$$= a_r\,\vec{u}_r + a_\theta\,\vec{u}_\theta$$

$$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (2\dot{r}\dot{\theta} + r\ddot{\theta})^2}$$

$$\phi = \tan^{-1}\frac{(2\dot{r}\dot{\theta} + r\ddot{\theta})}{(\ddot{r} - r\dot{\theta}^2)}$$





$$\bar{u}_r = \cos\theta\,\bar{i} + \sin\theta\,\bar{J} \longrightarrow (1)$$

$$u_\theta = -\sin\theta\,\bar{i} + \cos\theta\,\bar{J} \longrightarrow (2)$$

بتفاضل المعادلة (1) بالنسبة للزمن واستخدام المعادلة (2)

$$\frac{d\bar{u}_r}{dt} = -\frac{d\theta}{dt}\sin\theta\, i + \frac{d\theta}{dt}\cos\theta\, J$$

$$\therefore \frac{d\bar{u}_r}{dt} = \frac{d\theta}{dt}\, u_\theta$$

بالمثل: بتفاضل المعادلة (2) بالنسبة للزمن واستخدام المعادلة (1)

$$\frac{d\bar{u}_\theta}{dt} = -\dot{\theta}\,\bar{u}_r$$

$$\bar{r} = r\,\bar{u}_r$$

يمكن التعبير عن متجه موضع النقطة P على الصورة

$$\therefore \bar{V} = \dot{r}\,\bar{u}_r + r\frac{d\bar{u}_r}{dt}$$

$$\boxed{\bar{V} = \dot{r}\,\bar{u}_r + r\dot{\theta}\,u_\theta}$$