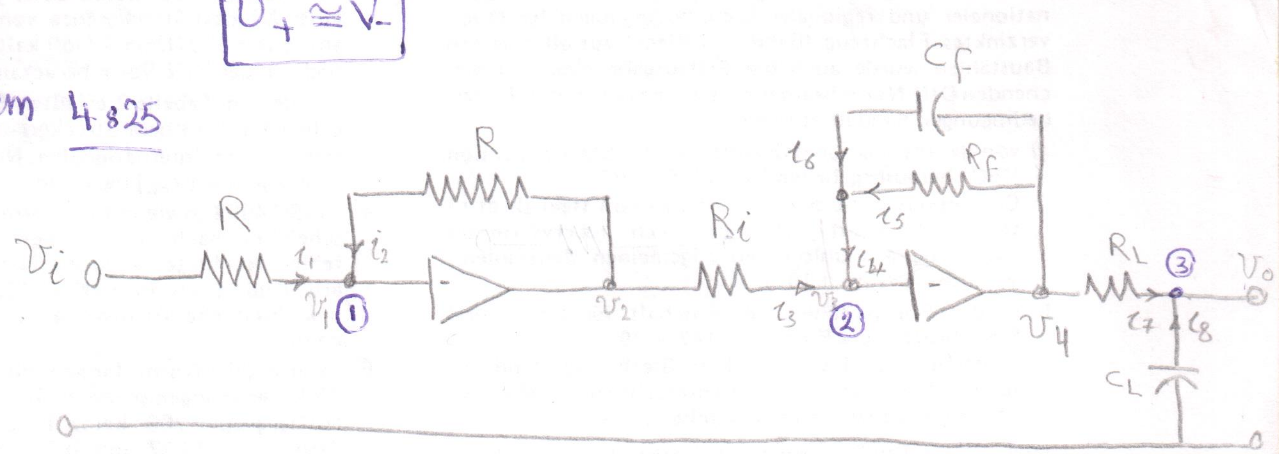
Dynamic systems analysis: Part (2) Lecture No. (1)

Ch. (4) / Electronices systems:

The op-amp equation:

$$\boxed{v_+ \cong v_-}$$

Ex: 2 Problem 4.25



$R_i = R_f = 10\,K\Omega$

$R_L = 500\ \Omega$

$C_f = 1\ \mu F$

$C_L = 10\ \mu f$

Req:

a] differential eqn.

b] static gain.

c] natural frequancy.

d] damping Factor.

" Solution "

at node (1):

KCL

$$i_1 + i_2 = 0$$

$$\frac{V_i - V_1}{R} + \frac{v_2 - v_1}{R} = 0$$

op-amp eqn $\boxed{V_1 = V_+ = zero}$

$$v_i = -v_2 \longrightarrow (1)$$

at node (2):

KCL

$$i_3 + i_4 = 0$$

$i_4 \rightarrow (i_5 + i_6)$

$$i_3 + i_5 + i_6 = 0$$

$$\frac{v_2 - v_3}{R_i} + \frac{v_4 - v_3}{R_f} + c\frac{d}{dt}(v_4 - v_3) = 0$$

op-amp eqn $\boxed{V_3 = V_+ = zero}$

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$$\frac{v_2}{R_i} + \frac{v_4}{R_f} + C_f \dot{v}_4 = 0 \longrightarrow (2)$$

$$\boxed{C_f \dot{v}_4 + \frac{v_4}{R_f} = \frac{v_i}{R_i}} \longrightarrow \boxed{3} \Rightarrow \cancel{\left(C_f D + \frac{1}{R_f}\right) v_4 = \frac{v_i}{R_i}}$$

at node (3)

KCL

$i_7 \rightarrow \circ \uparrow i_8$

$$i_7 + i_8 = 0$$

$$\frac{v_4 - v_o}{R_L} + C_L \frac{d}{dt}(0 - V_o) = 0$$

$$\boxed{C_L \dot{V}_o + \frac{V_o}{R_L} = \frac{V_4}{R_L}} \longrightarrow \boxed{4}$$

$$\cancel{C_L \dot{V}_o + \frac{v_o}{R_L} = \frac{1}{R_L}\left[\frac{v_i}{R_i\left[C_f D + \frac{1}{R_f}\right]}\right]}$$

diff w.r.t time & $* R_L$

$$v_4 = R_L C_L \dot{v}_o + v_o \longrightarrow (5)$$
$$\dot{v}_4 = R_L C_L \ddot{v}_o + \dot{v}_o \longrightarrow (6)$$

sub in eq$\underline{n}$ (3)

$$C_f\left[R_L C_L \ddot{v}_o + \dot{v}_o\right] + \frac{R_L C_L \dot{v}_o + v_o}{R_f} = \frac{v_i}{R_i} \Big] \rightarrow * R_f$$

$$\boxed{R_f C_f C_L R_L \ddot{v}_o + (R_f C_f + R_L C_L)\dot{v}_o + v_o = \frac{R_f}{R_i} v_i} \rightarrow \text{Req } \underline{(a)}$$

$$m\ddot{x} + b\dot{x} + Kx = F(t)$$

G : static gain

يعني في حالة عدم الحركة تكون $\boxed{0 = \ddot{x} \,(\dot{x}}$

$$G = \left(\frac{V_o}{V_i}\right)_{\dot{V}_o = \ddot{V}_o = 0} = \frac{R_f}{R_i} = \frac{10}{10} = 1$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{1}{R_f C_f R_L C_L}} = 141.42 \text{ rad/sec}$$

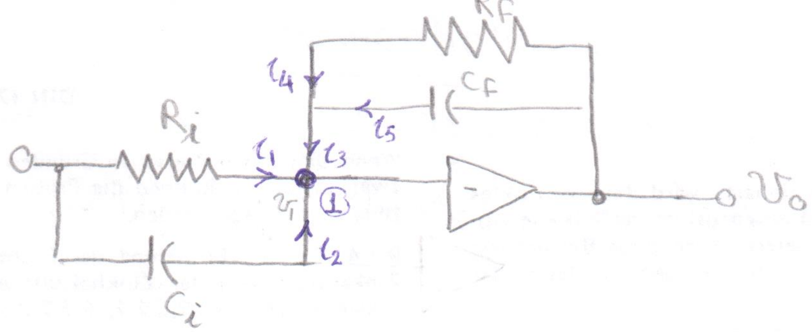
زيتا

$$\zeta = \text{damping factor} = \frac{b}{b_c} = \frac{R_f C_f + R_L C_L}{2 * R_f R_L C_f C_L * \omega_n} = 1.06$$

$\boxed{2m\omega_n}$ $\ddot{x}$ معامل $\rightarrow$ $2\zeta \cdot \omega_n$

Ex 3

Prob/4-27



Req$^{s}_{t}$ a) diff. eq$^{n}_{t}$

b) Transfer function $\dfrac{V_i(s)}{V_o(s)}$

" Solution "

at node (1)

KCL

$$i_1 + i_2 + i_3 = 0$$

$$i_3 = i_4 + i_5$$

$$\therefore i_1 + i_2 + i_4 + i_5 = 0$$

$$\frac{v_i - v_1}{R_i} + C_i \frac{d}{dt}(v_i - v_1) + \frac{(v_o - v_1)}{R_f} + C_f \frac{d}{dt}(v_o - v_1) = 0$$

$\boxed{V_+ = V_1 = 0}$ → op-amp eq$^{n}_{t}$

$$\frac{V_o}{R_f} + C_f \dot{v}_o = -\left[\frac{v_i}{R_i} + C_i \dot{v}_i\right]$$

$$\boxed{R_f C_f \dot{v}_o + V_o = -\frac{R_f}{R_i}\left[R_i C_i \dot{v}_i + V_i\right]} \rightarrow (a)$$

Take {L.T}

$$(R_f C_f s + 1) V_o(s) = -\frac{R_f}{R_i}\left[R_i C_i s + 1\right] V_i(s)$$

$$\frac{V_i(s)}{V_o(s)} = -\frac{R_f C_f s + 1}{\frac{R_f}{R_i}\left[R_i C_i s + 1\right]}$$

Electromeahanical systems

DC. motors are essential in Control system Two Types of D.C motors

1] Armature - Controlled DC motors (more Popular).

2] Field Controlled DC motors.

* Elemental Relations of Electromechanical systems:

The mechanical motion of the rotor relative to the stator affects the electromotive force (e.m.f) Voltage developed within the motor:

There are two critical relations for el-me systems.

① The back emf Voltage e_b across a DC motor is proportional to the angular speed of the motor rotor.

$$\boxed{e_b = K_e \omega = K_e \dot{\theta}}$$

② The Torque developed by the motor is proportional to the current.

$$\boxed{T = K_i \cdot i_a}$$

Where:

e_b = back emf Voltage

θ = angular displacement of rotor ($\dot{\theta} = \omega$)

T = torque applied to the rotor (developed by motor)

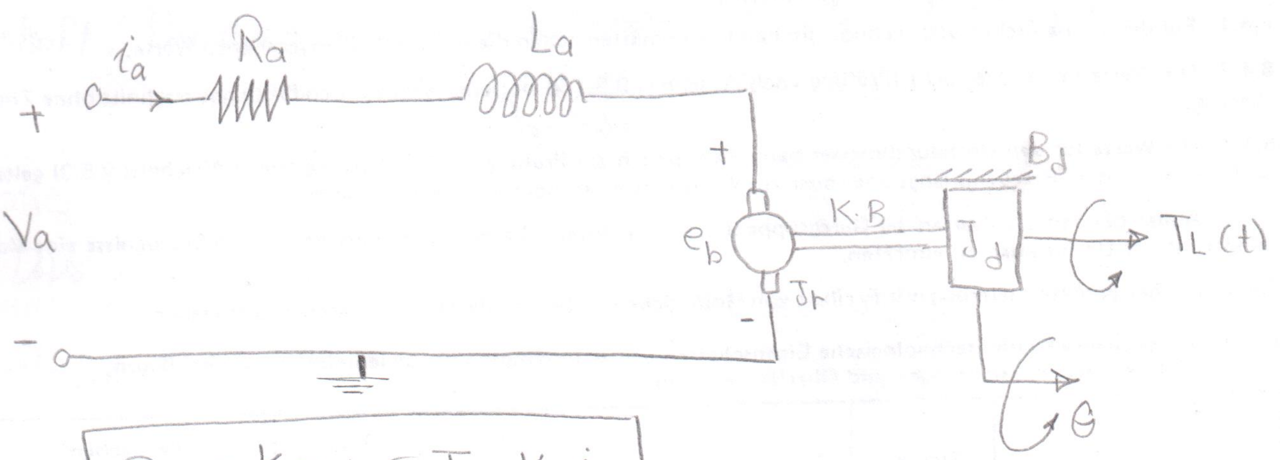
K_e = emf Constant of the motor $V/K_{r.p.m}$

K_t = torque Constant of the motor $\frac{in - ozf}{A}$

K_e & K_t are the same [Listed in the specifications sheet of motor]

They have the same numerical Value if expressed in [MKS] or [CGS]

I Armature - Controled DC motors :



$$\boxed{e_b = K_e \omega \quad \& \quad T = K_t i_a}$$

The shaft is assumed mass less

B = Coef. of torsional Viscous damping of the shaft

K = torsional stiffness of the shaft

B_d = Coef. of Viscous damp associated with the dish

i_f = Field Current → Stator

J_d = mass moment of inertia of the disk $Kg \cdot m^2$

J_r = mass moment of inertia of the rotor

i_a = armature current

L_a = " inductance

R_a = " Resistance

V_a = " voltage [i/P

a] A simple model "Rigid shaft"

Lecture (2)

The shaft, the rotor and the disk lumped as a single rigid body

$$\boxed{J = J_r + J_d}$$

i/p one v_a

o/p two i_a and θ

$\Delta S = (JS + B_J)(L_a \cdot S + R_a) + K_e K_t$

$= J L_a S^2 + (J R_a S + B_J L_a) S + B_J R_a + K_e K_t$

$\div J L_a$

$S^2 + \frac{(\quad)}{J L_a} S + \frac{(\quad)}{J L_a}$

* for electrical circuit

KVL $\quad \sum V_{drop} = 0.0$

$v_{ra} + v_{La} + e_b - v_a = 0$

$R_a \cdot i_a + L_a \frac{di_a}{dt} + \overset{e_b}{K_e \cdot \dot{\theta}} = v_a$

$$\boxed{L_a \dot{i}_a + R_a i_a + K_e \dot{\theta} = V_a} \rightarrow (1)$$

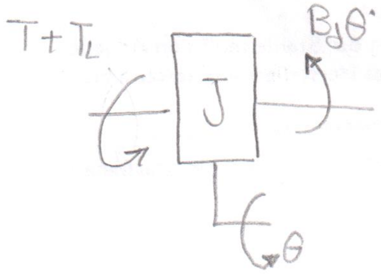
$\boxed{\omega = \dot{\theta}}$

$\dot{\omega} = \ddot{\theta}$

* for mechanical Part

$\sum M = J\ddot{\theta}$

$T + T_L - B_J \dot{\theta} = J\ddot{\theta}$



$$\boxed{J\ddot{\theta} + B_J \dot{\theta} - K_t i_a = T_L(t)} \rightarrow (2)$$

$J\dot{\omega} + B_J \omega - K_t i_a = T_L \rightarrow (2)$

$L_a \dot{i}_a + R_a i_a + K_e \cdot \omega = V_a \rightarrow (1)$ } Take {L.T}

$(JS + B_J) W(s) - K_t I_a(s) = T_L(s) \rightarrow (3)$

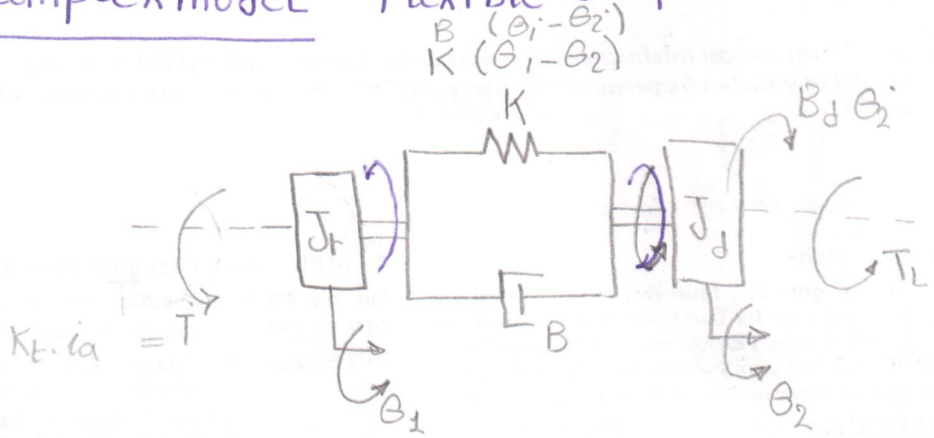
$(L_a \cdot S + R_a) I_a(s) + K_e W(s) = V_a(s) \rightarrow (4)$ } Kramer method

$$W(s) = \frac{\begin{vmatrix} T_L(s) & -K_t \\ V_a(s) & L_a s + R_a \end{vmatrix}}{\begin{vmatrix} Js + B_J & -K_t \\ K_e & L_a s + R_a \end{vmatrix} = \Delta(s)} \quad , \quad I_a(s) = \frac{\begin{vmatrix} Js + B_J & T_L(s) \\ K_e & V_a(s) \end{vmatrix}}{\Delta(s)}$$

$$\Delta(s) = \left(S^2 + \frac{B_J L_a + J R_a}{J L_a} S + \frac{B_J R_a + K_e K_t}{J L_a}\right) \cdot J L_a$$

$$\dot{\omega}(s) = \frac{T_L(s)\,(L_a \cdot s + R_a) + K_t V_a(s)}{\Delta s}$$

b] A more complex model "flexible damped shaft"



* for electrical circut : (KVL) $\sum V_{drop} = 0.0$

$$\boxed{L_a \dot{i}_a + R_a i_a + K_e \dot\theta_1 = v_a} \rightarrow (1)$$

* for mechanical Part :

rotor

$$J_r \ddot\theta_1 = -B(\dot\theta_1 - \dot\theta_2) - K(\theta_1 - \theta_2) + K_t\, i_a$$

$$\boxed{J_r \ddot\theta_1 + B\dot\theta_1 - B\dot\theta_2 + K\theta_1 - K\theta_2 = K_t \cdot i_a} \rightarrow (2)$$

Disk

$$J_d \ddot\theta_2 = B(\dot\theta_1 - \dot\theta_2) + K(\theta_1 - \theta_2) + T_L - B_d \dot\theta_2$$

$$\boxed{J_d \ddot\theta_2 + (B + B_d)\dot\theta_2 - B\dot\theta_1 + K\theta_2 - K\theta_1 = T_L} \rightarrow (3)$$