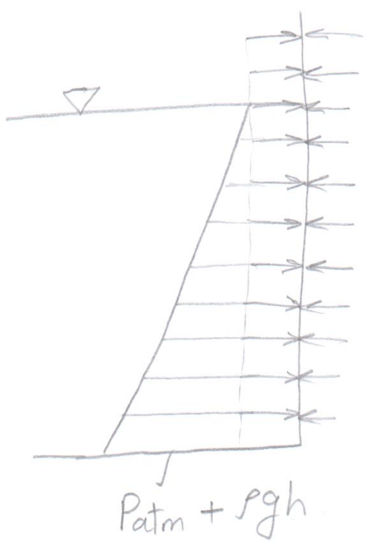
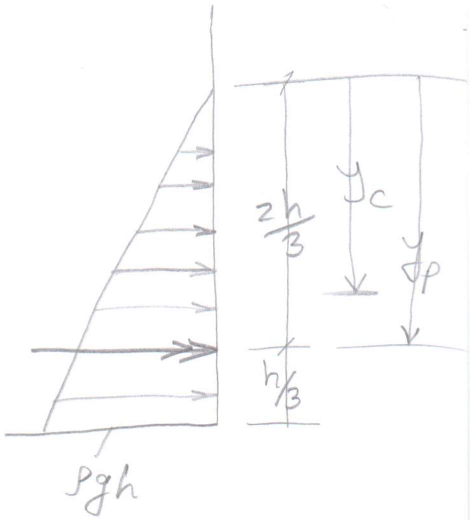
Hydrostatic forces on submerged plane surfaces

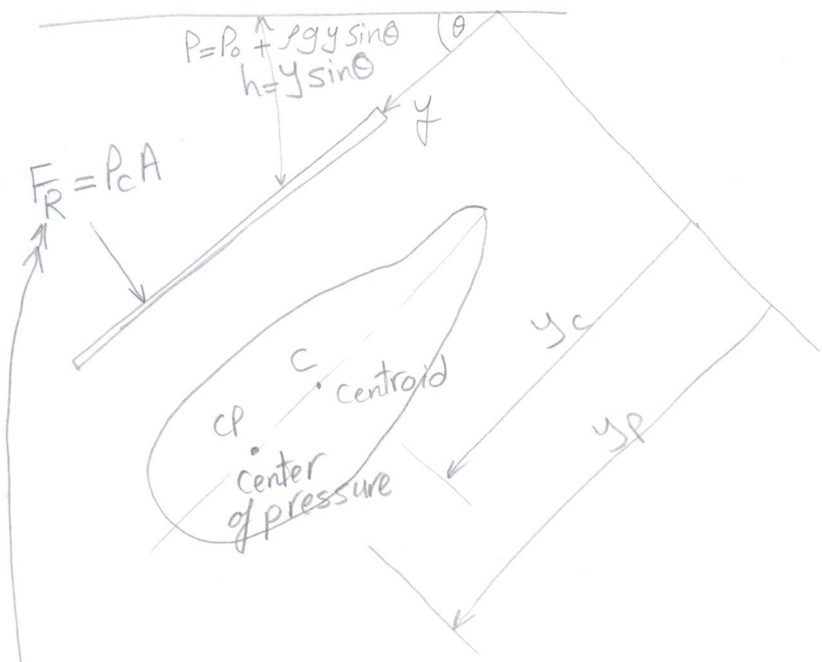
$P_{gage} = \rho g h$

y_c at centroid

y_p at center of pressure





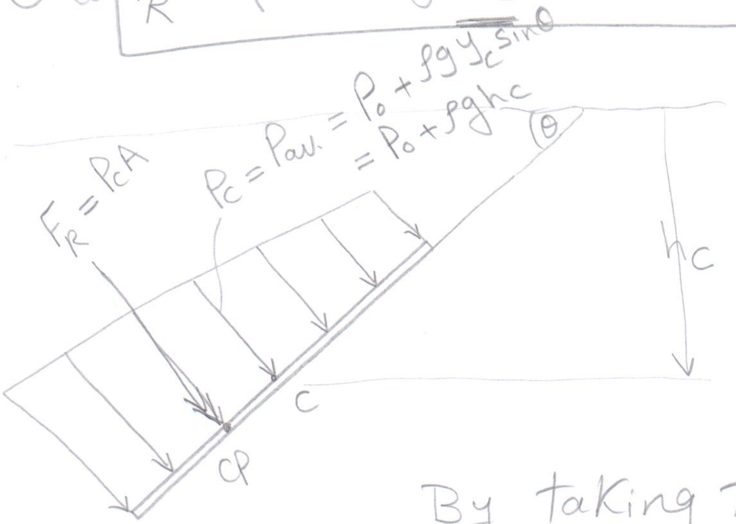


$$P = P_o + \rho g h$$
$$= P_o + \rho g y \sin\theta$$

$$F_R = \int_A P \, dA$$
$$= \int_A (P_o + \rho g y \sin\theta) \, dA$$
$$= P_o A + \rho g \sin\theta \underbrace{\int_A y \, dA}_{\text{first moment of area}}$$

as $y_c = \frac{1}{A}\int y \, dA$

① $\boxed{F_R = (P_o + \rho g y_c \sin\theta) A} \Longrightarrow$ resultant force



Next we need to determine its line of action which doesn't pass through the centroid of the surface where it lies underneath where the pressure is higher

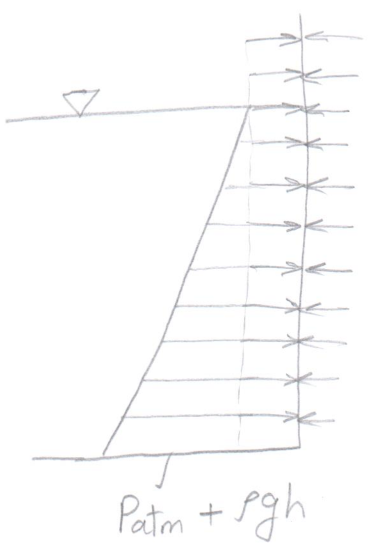
By taking the moment of F_R about x-axis to the moment of the distributed pressure force

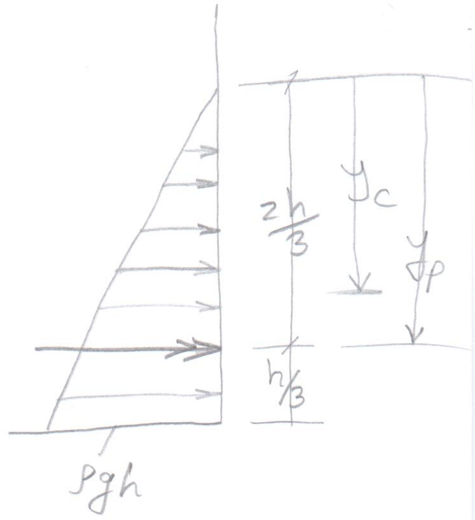
Hydrostatic Forces on submerged plane surfaces

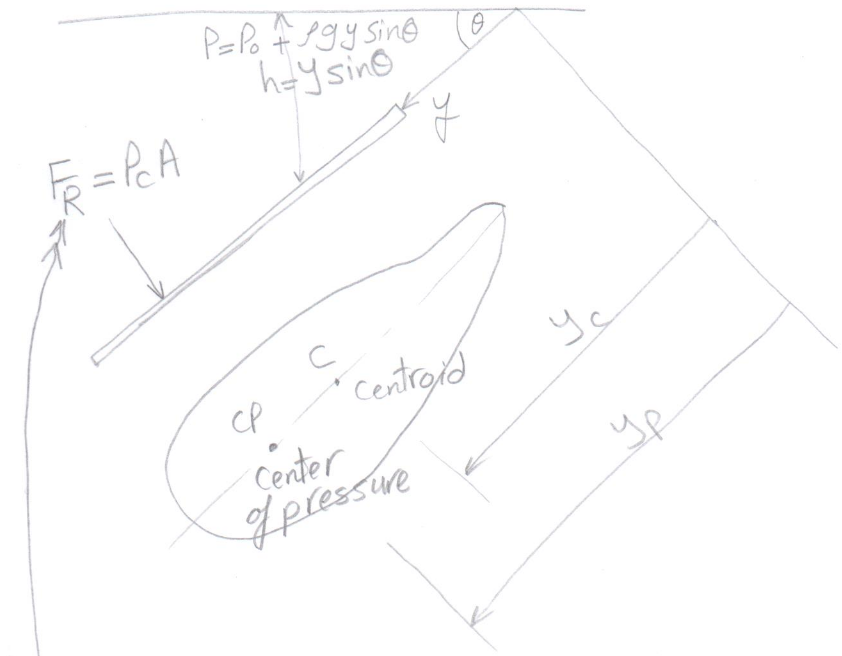
$P_{gage} = \rho g h$

y_c at Centroid

y_p at center of pressure







$$P = P_o + \rho g h$$

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$$F_R = \int_A P \, dA$$

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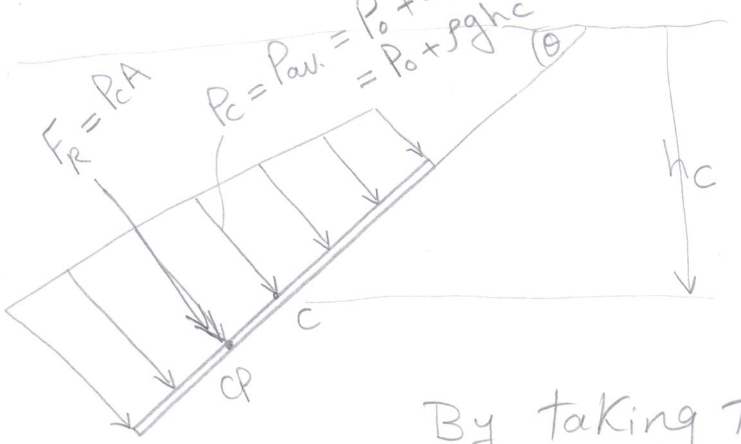
$$= P_o A + \rho g \sin\theta \underbrace{\int_A y \, dA}$$

first moment of area

as $y_c = \frac{1}{A} \int y \, dA$

(1) ∴ $\boxed{F_R = (P_o + \rho g y_c \sin\theta) A}$ ⟹ resultant force

Next we need to determine its line of action which doesn't pass through the centroid of the surface where it lies underneath where the pressure is higher



By taking the moment of F_R about x-axis to the moment of the distributed pressure force

$$y_p F_R = \int_A y P \, dA$$

$$= \int_A y(P_o + \rho g y \sin\theta) \, dA$$

$$= P_o \int_A y \, dA + \rho g \sin\theta \int_A y^2 \, dA \qquad I_{xx,o} = \int_A y^2 dA$$

$$\boxed{y_p F_R = P_o y_c A + \rho g \sin\theta \, I_{xx,o}} \qquad (2)$$

$$\boxed{I_{xx,o} = I_{xx,c} + y_c^2 A} \qquad (3) \text{ parallel axis theorem}$$

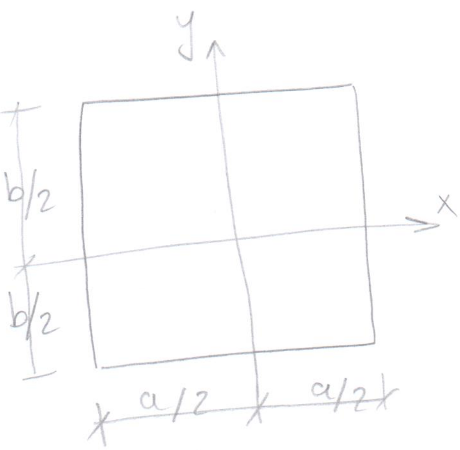
$I_{xx,c}$ → second moment of area about centroid

from eqns (1), (3) in (2)

$$\boxed{y_p = y_c + \frac{I_{xx,c}}{\left[y_c + \frac{P_o}{\rho g \sin\theta}\right] A}}$$

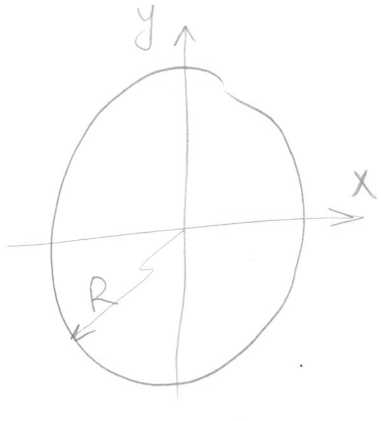
at $P_o = 0$

$$\boxed{y_p = y_c + \frac{I_{xx,c}}{y_c A}}$$



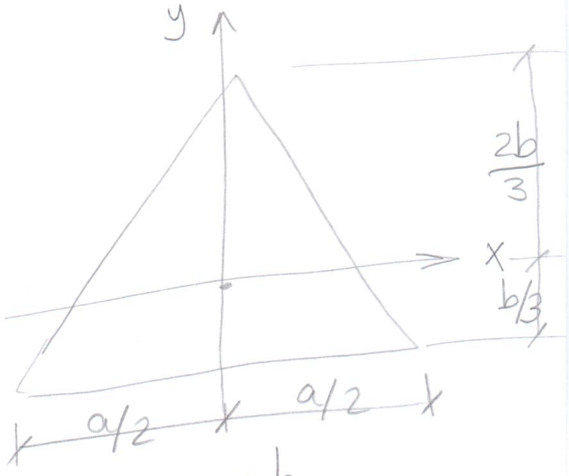
$A = ab$

$I_{xx,c} = \frac{ab^3}{12}$



$A = \pi R^2$

$I_{xx,c} = \frac{\pi}{4} R^4$



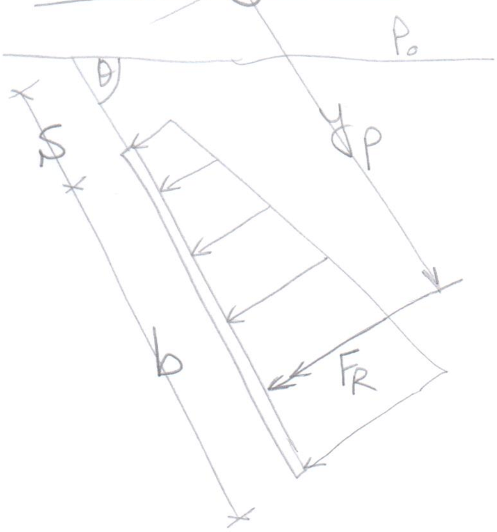
$A = \frac{ab}{2}$

$I_{xx,c} = \frac{ab^3}{36}$

special Case

submerged Rectangular Plate

tilted plate



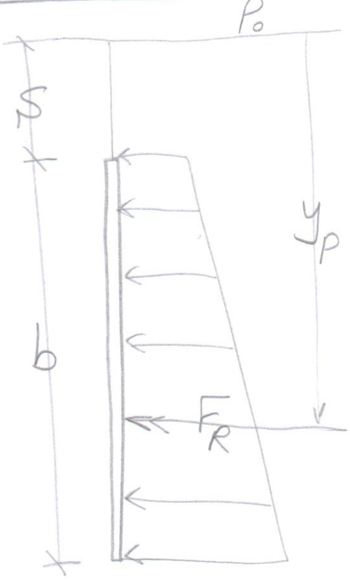
$$F_R = \left[P_o + \rho g\left(s + \frac{b}{2}\right)\sin\theta\right] ab$$

$$y_p = s + \frac{b}{2} + \frac{ab^3/12}{\left[s + \frac{b}{2} + \frac{P_o}{\rho g \sin\theta}\right] ab}$$

$$= s + \frac{b}{2} + \frac{b^2}{12\left(s + \frac{b}{2} + \frac{P_o}{\rho g \sin\theta}\right)}$$

IF $s = 0$ $\therefore F_R =$ ____ & $y_p =$ ____

vertical plate $\sin\theta = 1$



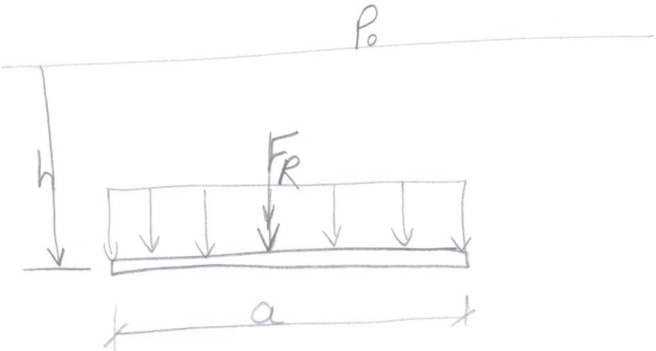
$$F_R = \left[P_o + \rho g\left(s + \frac{b}{2}\right)\right] ab$$

$$y_p = s + \frac{b}{2} + \frac{b^2}{12\left(s + \frac{b}{2} + \frac{P_o}{\rho g}\right)}$$

Horizontal plate ~~$\sin\theta = 0$~~

$$F_R = (P_o + \rho g h)\, ab$$

$$y_p = h$$

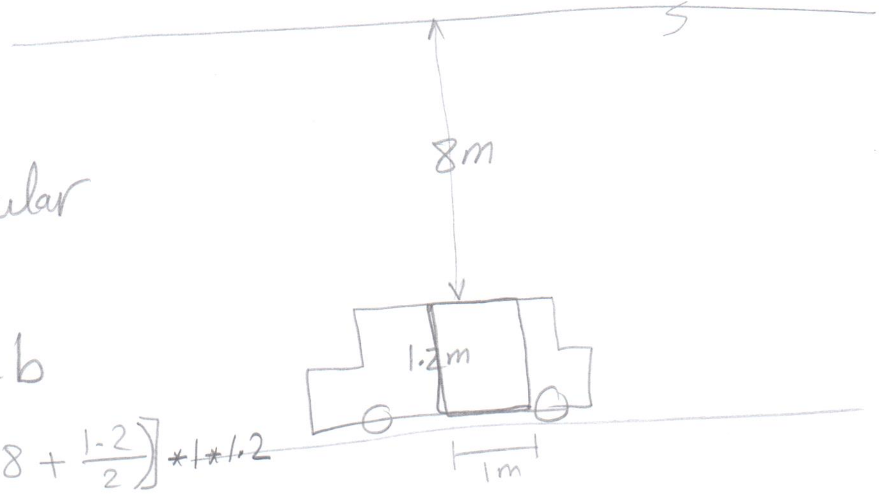


Example Pg 84

Hydrostatic Force Acting on the door of a submerged Car

– A heavy Car plunges into a lake during an accident and lands at the bottom of the lake on its wheels. The door is 1.2 m high & 1 m wide and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of pressure Center, and discuss if the driver can open the door

$F_R = ??\ ,\ y_p = ??$



Sol/n

$a = 1\,m$

$b = 1.2\,m$ & vertical rectangular plate

$s = 8\,m$

$$F_R = \left[P_0 + \rho g\left(s + \frac{b}{2}\right)\right] ab$$

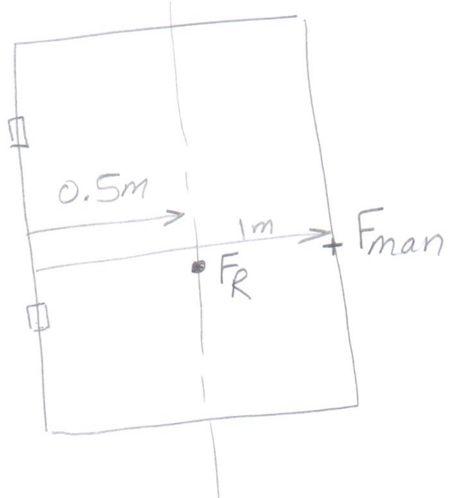
$$= \left[0 + 1000 * 9.8 * \left(8 + \frac{1.2}{2}\right)\right] * 1 * 1.2$$

$$= 101.3 * 10^3\ N = 101.3\ KN$$

$$y_p = s + \frac{b}{2} + \frac{b^2}{12\left(s + \frac{b}{2} + \frac{P_0}{\rho g}\right)}$$

$$= 8 + \frac{1.2}{2} + \frac{(1.2)^2}{12\left(8 + \frac{1.2}{2}\right)} = 8.61\,m$$

By taking moment about hinges



$F_R * 0.5 = F_{man} * 1$

$101.3 * 0.5 = F_{man}$

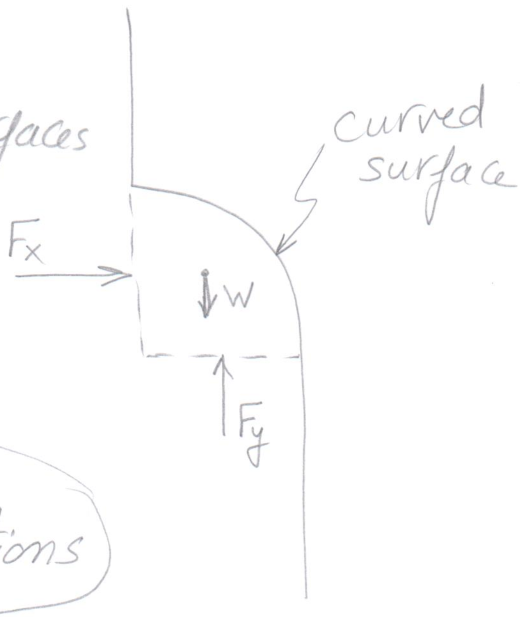
$F_{man} = 50.6\ KN$

A strong man can lift 100 Kg = 981 N ≃ 1 KN

about 50 times the effort the driver can possibly generate.

Hydrostatic Forces on submerged curved surfaces

Consider the free-body diagram of the liquid block enclosed by the curved suface and two plane surfaces (one horizontal & one vertical) passing through two ends of the curved surface



$$F_H = F_x$$
$$F_V = F_y \pm W$$

± depends on directions

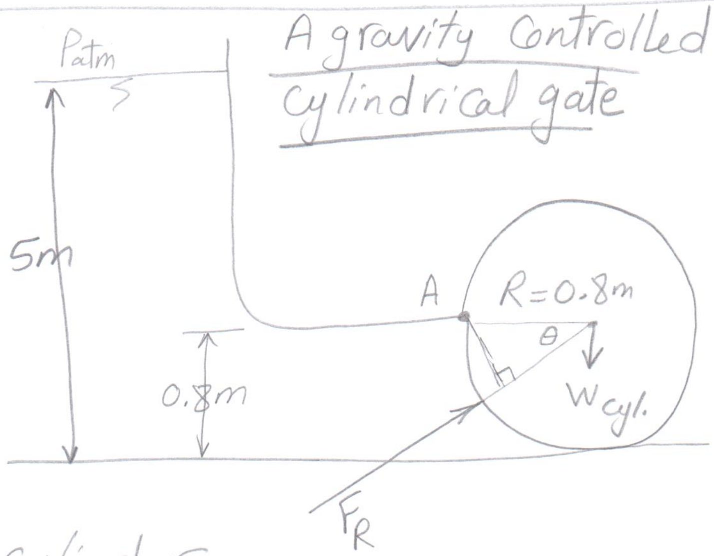
$$F_R = \sqrt{F_H^2 + F_V^2} \quad \& \quad \tan\alpha = \frac{F_V}{F_H}$$

The exact location of the line of action of F_R can be determined by taking a moment about an appropriate point.

Example Pg 87

A gravity controlled cylindrical gate

The gate opens by turning about the hinge at point A



Determine:

a) F_R & its line of action when gate opens

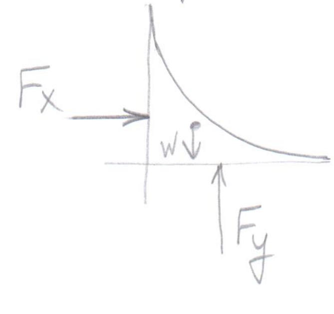
b) $W_{cyl.}$ per length of the cylinder

Soln

$$F_H = F_x = \left[\cancel{P_o}^{\,0\ atm} + \rho g\left(5 + \frac{R}{2}\right)\right] * A \qquad A = R*1$$

$$= 1000 * 9.8 * \left(4.2 + \frac{0.8}{2}\right) * 0.8 * 1$$

$$= 36.1\ KN$$

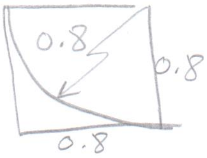


$$F_y = (\underbrace{\cancel{P_o}}_{=0\ atm} + \rho g h)\ A = (1000 * 9.8 * 5) * 0.8 * 1$$

$$= 39.2\ KN$$

$$W = mg$$
$$= \rho V * g$$

$V =$ مساحة مستطيل − مساحة ربع دائرة

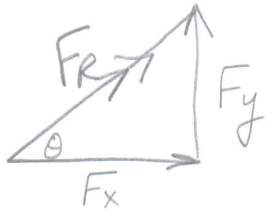


$$\rho = \frac{m}{V}$$
$$m = \rho V$$

$$= 1000 * \left[0.8^2 - \frac{1}{4} * \pi (0.8)^2\right] * 1 * 9.8$$
$$= 1.3\ KN$$

$$F_V = F_y - W$$ according to directions

$$= 39.2 - 1.3 = 37.9\ KN$$



$$F_R = \sqrt{F_H^2 + F_V^2} = 52.3\ KN$$

$$\tan\theta = \frac{F_V}{F_H} = 1.05 \qquad \therefore \theta = 46.4^\circ$$

passes through the center of the cylinder.

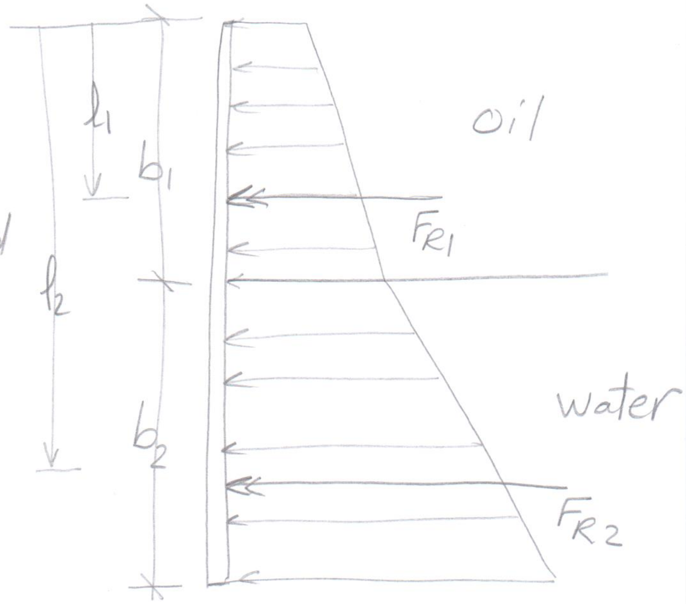
(b) $\Sigma M_A = 0$ the gate is about to open.

$$F_R * R \sin\theta = W_{cyl.} * R$$

$$W_{cyl.} = F_R \sin\theta = 52.3 * \sin 46.4 = 37.9\ KN$$

weight per unit length

Multi layered fluid



$$F_R = \Sigma F_{R,i} = \Sigma P_{c,i} A_i$$

The line of action can be determined by taking the moment

$$F_R * l = F_{R1} * l_1 + F_{R2}\, l_2$$