Problem Set (3)

1- The three components of velocity in a flow field are given by, $u = x^2 + y^2 + z^2$, $v = xy + yz + z^2$, and $w = -3xz - \frac{z^2}{2} + 4$

a) Determine the volumetric dilation rate and interpret the results.
b) Determine an expression for the rotation vector. Is this an irrotational flow field?

2- Determine an expression for the vorticity of the flow field described by, $\vec{V} = -4xy^3\,\hat{i} + y^4\,\hat{j}$. Is the flow irrotational?

3- A one-dimensional flow is described by the velocity field, $u = ay + by^2$, $v = w = 0$, where 'a' and 'b' are constants. Is the flow irrotational? For what combination of constants (if any) will the rate of angular deformation $\dot{\gamma}$ be zero?

4- A two-dimensional flow field described by, $\vec{V} = (2x^2 y + x)\,\hat{i} + (2xy^2 + y + 1)\,\hat{j}$, where the velocity is in m/s when 'x' and 'y' are in m. Determine the angular rotation of the fluid element located at $x = 0.5$ m and $y = 1$ m.

5- An incompressible viscous fluid is placed between two large parallel plates as shown in Figure P.5. The bottom plate is fixed and the upper plate moves with a constant velocity 'U'. For these conditions the velocity distribution between the plates is linear can be expressed as, $u = \frac{Uy}{b}$. Determine :

a) The volumetric dilatation rate.
b) The vorticity.
c) The rotation vector.
d) The rate of angular deformation.

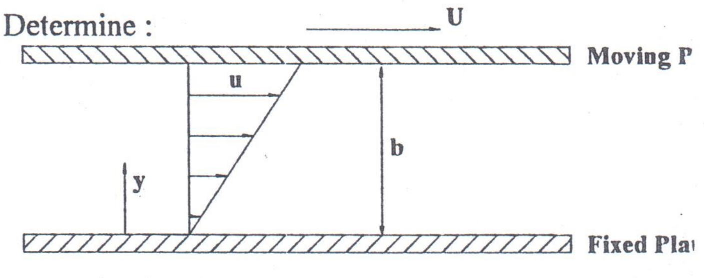

Figure P.5

6- For a certain incompressible flow field, it is suggested that the velocity components are given by the equation, $u = x^2 y$, $v = 4y^3 z$, and $w = 2z$ is physically possible flow field? Explain.

7- The velocity components of an incompressible, two-dimensional velocity field are given by the equation, $u = 2xy$ and $v = x^2 - y^2$. Show that the flow is irrotational and satisfies the conservation of mass.

8- For a certain incompressible two-dimensional flow field the velocity component in y-direction is given by the equation, $v = 3xy - x^2 y$. Determine the velocity component in the x-direction so that the continuity equation is satisfied.

9- The velocity components in an incompressible, two-dimensional flow field are given by, $u = x^2$, and $v = -2xy + x$. Determine, if possible, the corresponding stream function.

10- The stream function for a certain incompressible flow field is given by the equation $\psi = 2x^2 y - \frac{2y^3}{3}$. Show that the velocity field represented by the stream function satisfies the continuity equation.

11- The stream function for an incompressible two-dimensional flow field is, $\psi = ay^2 - bx$, where '*a*' and '*b*' are constants. Is this an irrotational flow? Explain.

12- The velocity components for an incompressible plane flow are,
$v_r = \left(\frac{A}{r}\right) + \left(\frac{B}{r^2}\right) \cos\theta$, $v_\theta = \left(\frac{B}{r^2}\right) \sin\theta$, where '*A*' and '*B*' are constants.
Determine the corresponding stream function.

13- The velocity potential for a certain flow field is, $\phi = -2\left(2x + y\right)$. Determine the corresponding stream function.

14- The velocity potential for a given two-dimensional flow field is,
$\phi = \left(\frac{5x^3}{3}\right) - 5xy^2$. Show that the continuity equation is satisfied and determine the corresponding stream function.

15- Determine the stream function corresponding to the velocity potential, $\phi = x^3 - 3xy^2$. Sketch the streamline $\psi = 0$, which passes through the origin.

16- A certain flow field is described by the velocity potential, $\phi = (A \ln r) + Br \cos\theta$ where '*A*' and '*B*' are positive constants. Determine the corresponding stream function and locate any stagnation points in this flow field.