α thermal diffusivity
σ Stefan-Boltzmann constant
ρ density
τ time constant

PROBLEMS

6.1 Several metals are being considered for use in an application that conducts heat from one location to another through a solid rod of metal at room temperature. *Prepare* a table that lists the thermal conductivity of the following metals: aluminum, steel (use iron), copper, and silver. *Which* metal has the best thermal conductivity?

If the best conductor were used with a given rod diameter d^*, this would represent a reference thermal resistance per unit length. Now *calculate* what diameter d of rod would be required in the other metals to have the same thermal resistance per unit length as the best conductor. Based upon the density of each metal, *what* will be the weight per unit length for each? *Which* metal has the lowest weight for the given thermal resistance?

Obtain a rough estimate for the cost per pound of each of the preceding metals from industrial suppliers, and *calculate* the relative cost of each metal that is required to have the same resistance as discussed in the previous paragraph. *Which* metal has the lowest cost for the given thermal resistance?

Which metal would you use and why?

6.2 *Perform* an analysis similar to that in Problem 6.1 for the following insulators used as thin sheets: asbestos, glass fibers, plywood, and wool. Now, however, we are looking for the worst conductor (i.e., the best insulator), the lowest weight for a given insulation, and the lowest cost for a given insulation.

6.3 *Derive* Eq. 6.9 to confirm Eq. 6.10.

6.4 The actual convection coefficient can be calculated for a variety of situations using the dimensionless parameters of the Nusselt number, Prandtl number, Reynolds number, and Grashof number (Ernst Nusselt, German engineer, 1882–1951; Ludwig Prandtl, German engineer, 1875–1953; Osborne Reynolds, English engineer, 1842–1912, Franz Grashof, German engineer, 1826–1893). *Refer* to a textbook on heat transfer, and *state* the definitions for these terms and the parameters they use.

6.5 The Nusselt number is a dimensionless ratio for the convection coefficient. Continue Problem 6.4, and *state* the relationship between the Nusselt number and the Prandtl and Grashof numbers for the following convection situations: free convection from a vertical flat plate, horizontal flat plate, vertical cylinder, and horizontal cylinder.

6.6 *Calculate* the amount of heat lost (in watts) through an uncovered window in a home. The temperature difference between the air inside and outside of the house is 20°C, with moderate convection on each side of the glass. The size of the glass is 0.75 m by 1.1 m, and the thickness is 3 mm.

6.7 It is desired to replace a 1/8 inch glass window with a sheet of Plexiglas®. *What* thickness of Plexiglas® will be required to have the same insulation as the window has?

6.8 A dual-pane window glass in a home is used to reduce the heat loss due to thermal conduction. You are asked to compare three situations: (1) the heat loss through a single pane of glass, (2) the heat loss through two panes of glass touching each other, and (3) two panes of glass separated with an air space. The glass is 3 mm thick, and the air gap in part 3 is 10 mm. *Perform* your calculations on a per-unit area basis. *How* much reduction in heat transfer from part 1 is observed in parts 2 and 3. *Is* the air gap worthwhile compared to two panes?

6.9 The radiator hose on an automobile is made of something similar to Buna™ rubber. The internal diameter of the hose is 50 mm, its length is 200 mm, and its thickness is 6 mm. The water on the inside is maintained at 100°C, and the outside air temperature is 50°C. *Calculate* how much heat is conducted through the hose.

6.10 A 3/4 inch iron pipe (ID = 0.824 inch, OD = 1.050 inches) is used to transmit hot water. The internal water temperature is 120°F, and the ambient air temperature is 75°F. Consider a pipe that is 5 feet in length.

a. *Calculate* how much heat is transferred from the water to the air.

b. If a spongy rubber tape that is 0.100 inch thick is placed on the outside of the pipe, *calculate* how much heat is transferred.

c. *What* is the reduction in heat loss with the use of the rubber insulation?

6.11 A steel plate is heated with a torch. When we are interested in cooling the plate, we can allow normal free convection, we can blow on the plate with our breath, or we can squirt a mist of water on the surface of the plate from a spray bottle. Looking at Table 6.3, *comment* on how fast the plate can be cooled with these three approaches.

6.12 *Derive* equation 6.20.

6.13 A copper ball 1 inch in diameter is heated to 500°F with a torch. *Calculate* how much heat will be transferred at this temperature by radiation. Assume that the ball is placed in the center of a room with walls at 75°F. *Calculate* how much heat would be transferred by convection if we were to use the upper end of free convection from Table 6.3. *What* convection coefficient did you select? *What* is the equivalent convection coefficient due to radiation from Fig. 6.11 (or Eq. 6.20)? *How* do these two coefficients compare with each other and *which* mode of heat transfer dominates in this case?

6.14 Shown in Figure P6.14 is a swimming pool operating in cool weather. The pool has a heater that provides a heat input of q_h watts. The water depth varies from 1.2 m to 3 m, and the pool is 5 m wide by 10 m long. A pump runs continuously to keep the water thoroughly mixed.

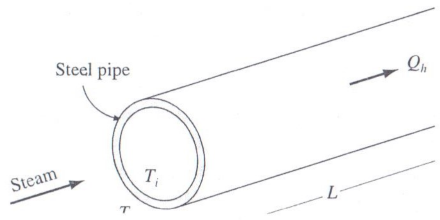

Figure P6.14 Swimming pool dynamics.

a. *Calculate* the Biot number, and *justify* the assumption that the pool can be treated as a single-lumped capacitance model.

b. Assuming that the significant heat transfer is the convection at the water surface and that no heat transfers out the sides and bottoms, *write* the modeling equations for the system using a single-lumped capacitor model, and *derive* a differential equation for the temperature of the water as a function of the ambient temperature and the heat input.

c. *Calculate* the value of the time constant of the system. If a period of one time constant would be enough time to "take the chill off," *would* you recommend leaving the heater on all the time or turning it off overnight?

d. *State* the expression for the steady-state temperature of the water. If the ambient temperature were 20°C and the desired water temperature were 25°C, *what* size heater would be required (in watts)?

e. Consider the heat transfer to the ground in the system, and *derive* the differential equation for the temperature of the pool as a function of the ambient temperature, the ground temperature, and the heat input. *State* the expressions for the time constant and the steady-state temperature of the water. If the pool is constructed with concrete 100 mm thick, *is* the heat transfer to the ground significant? *Which* model should we use?

6.15 Shown in Figure P6.15 is a boiler used in a thermal energy plant. *Write* the modeling equations for the system, and *derive* a differential equation for the temperature inside the boiler as a function of the ambient temperature and the heat input. *State* the expressions for the time constant and the steady-state temperature.

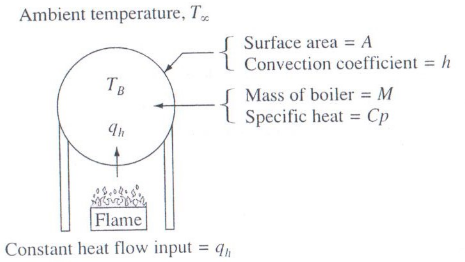

Figure P6.15 Thermal plant boiler.

6.16 Shown in Figure P6.16 is a mass suspended at the end of a rod. The mass is exposed to ambient temperature and experiences heat transfer from the rod. The rod is insulated, but is exposed to a constant temperature source T_0 at the other end. *Write* the modeling equations for this system, and *derive* the transfer function for the temperature of the mass as a function of the ambient temperature and the temperature of the source. *State* the expressions for the time constant and steady-state temperature.

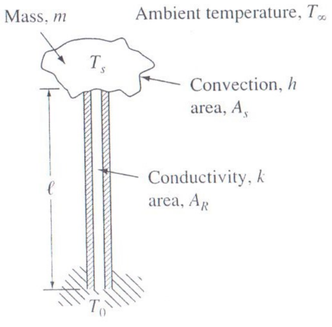

Figure P6.16 Suspended-mass thermal system.

6.17 A tank 75% (by volume) full of liquid propane is taken from a warehouse and placed in the direct sunlight on a hot summer day. We are concerned about the internal temperature (and therefore the internal pressure) of the tank as it sits in the sun. We want to know how long it takes to heat up and what is its final steady-state temperature.

Model the tank as a sphere 1 foot in diameter. The walls of the tank are steel 0.100 of an inch thick. The temperature inside the warehouse is 90°F, and the outside air temperature is 100°F. (The tank is in Texas!) The solar insolation for this particular day is 900 watts/m^2 = 285 (Btu/hr)/ft^2. You should consider the sun as a constant heat flow source of energy over the exposed surface area of the sphere. The density of liquid propane is 30.8 lbm/ft^3. The density of steel is 490 lbm/ft^3. The specific heat of liquid propane is 0.58 Btu/(lbm °F). The specific heat of steel is 0.11 Btu/(lbm °F). The wind velocity is such that you should use a convection coefficient for high free convection or low forced convection.

Model this system, and *derive* a differential equation for the internal temperature of the propane as a function of the solar heat input and the ambient temperature (with a given initial temperature). *Calculate* the time constant for the system, and *state* how long it will take for the tank to be at its steady-state temperature. *What* will the steady-state temperature be for the given conditions?

6.18 *Derive* the differential equation for the temperature at the center of a can of soft drink that is sitting upright. *What* is the Biot number for the wall of the aluminum can, and *do* you have to model the thermal conductivity of the wall of the can? *What* is the Biot number for the fluid in the can, and can you use a single-lump capacitance model?

Suppose that the can is placed upright in a frost-free refrigerator. *Measure* the dimensions of the can yourself, and use the properties of water to approximate the soft drink. If the air circulation fan inside the refrigerator is running, *what* do you estimate the convection coefficient to be? Using an analytic solution to the differential equation, *plot* the response of the internal temperature over time if the can is taken from room temperature (75°F) and placed in the refrigerator (40°F). *What* is the time constant of the system? *What* is the settling time. *How* long will it take the drink to reach an internal temperature of 50°F? If the can were placed in the freezer section of the refrigerator (10°F), *how* long should it remain there to cool to 50°F?

6.19 *Work* problem 6.18 using digital simulation (see Chapter 9).

6.20 *Perform* an experiment to determine the temperature response of water placed in a soft drink can as described in problem 6.18. Use any suitable thermometer to measure the temperature at the center of the volume of water. Before measuring the water temperature, *measure* the temperature of the air inside the refrigerator so that you will know what the steady-state temperature of the water will be. *Record* the observed temperature every 5 to 10 minutes for the first hour and every 15 minutes thereafter. *Plot* your response using a spreadsheet, and compare it to your original guess of the convection coefficient. After seeing the data, *can* you use better values for the convection coefficient, etc., to match the data more closely? *Plot* your revised theoretical response, and *compare* all three responses.

Perform the same experiment as in the previous paragraph, except remove the can that has been in the refrigerator for a long time (several hours) and let it heat to room temperature. (*Measure* the temperature of the room beforehand.) Record the temperature every 3 to 6 minutes for the first hour and every 10 minutes thereafter. *Do* you see any difference in the initial response (the first 30 minutes) of heating compared to cooling? Based upon the data you have recorded, is the convection coefficient constant?

discussed in Chapter 2. The temperature T can be expressed in degrees Celsius or Kelvin, or in degrees Fahrenheit or Rankine (Anders Celsius, Swedish astronomer, 1701–44; Lord Kelvin is William Thomson, British physicist, 1824–1907; Gabriel Daniel Fahrenheit, German physicist, 1686–1736; William John M. Rankine, English engineer, 1820–72). The heat transfer will be the variable that relates to the flow variable or the rate variable. The heat flow rate Q_h has the units of power, e.g., joules per second or Btu (British thermal units) per hour.

Thermal conduction, convection, and radiation all display an algebraic relation between the temperature and the heat flow; therefore, these effects represent thermal resistance and have no dynamic effects.

6.2.1 Thermal Conduction

Thermal conduction is the ability of solid or continuous media to conduct heat. The heat transfer Q_h is related to the temperature gradient in the direction of heat flow dT/dx as shown by Figure 6.1 (The figure illustrates one-dimensional heat transfer; a more general case would be two or three-dimensional heat transfer.)

The steady-state relationship between the temperature and heat flow in a material is given by **Fourier's law of heat conduction** (named after the French mathematician and physicist Joseph Fourier, 1768–1830). In this one-dimensional relationship, the heat transfer per unit area is related to the temperature gradient in the direction of heat flow by the equation

$$\frac{Q_h}{A} = -k_t \frac{dT}{dx} \tag{6.1}$$

Note that the negative sign indicates that there is a temperature drop in the direction of flow.

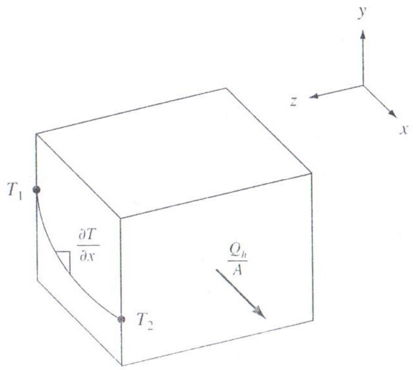

Figure 6.1 Conduction heat transfer in a continuous material.

6.1

6.2

Look closely at the surface of the can during the initial heating, and see whether you can *justify* any differences in the time constant over the initial response.

6.21 It is claimed that if you place a soft drink can on its side in the refrigerator, it will cool faster than if it were placed upright. *What* theoretical justification can you offer for this contention?

6.22 A fish tank is fabricated using 4 mm Plexiglas® walls. The water on the inside of the tank is maintained at a constant temperature of 26°C. The ambient room temperature is 22°C. We are interested in modeling the transient heat transfer through the walls of the tank if there is a sudden change in room temperature. The inside of the tank has water that is circulated with a pump and therefore should have a convection coefficient on the lower end of forced convection with water. The outside of the tank has air that should have a convection coefficient on the upper end of free convection with air.

a. *Calculate* the Biot number for the inside and outside surfaces. Based upon these numbers, *can* lumped parameter modeling be used to model the temperature distribution in the Plexiglas®? *Can* you assume that the convection heat transfer is so good in the water, that the water temperature on the inside surface is constant?

b. *Based* upon the Biot number, how many lumps should be considered in the lumped-parameter analysis?

6.23 A window glass has 25°C air temperature on the inside and 20°C air temperature on the outside. The convection coefficient on both sides is 30 W/(m^2 °C). The thickness of the glass is 7.5 mm, and the thermal conductivity of the glass is 0.75 W/(m °C). The windowpane is 0.8 m by 1.1 m. It is desired to calculate the transient temperature distribution in the glass with a sudden change in outside temperature in order to determine the internal stresses caused by a temperature gradient.

a. *Calculate* the Biot number to determine whether a single lumped-parameter analysis is possible. If a multiple lumped-parameter analysis is necessary, *how* many lumps should be used?

b. *Set up* the differential equations to simulate the transient response of all temperatures of interest.

c. If the outside temperature were suddenly changed from 20°C to 15°C, use digital simulation to *calculate* the transient response of all temperatures. *Plot* your results.

6.24 Using numerical integration, *solve* the state-space differential equations described in Example 6.5, and *plot* your results as a function of time. Use $T_{i\infty}$ = 50°C, $T_{o\infty}$ = 25°C, with all initial conditions = 25°C. *What* is the observed settling time of this system? *Compare* the observed settling time to the time constant for an individual internal node.