

## Modeling of a regenerative system

(1)

### Nodes of operation

(1) Electric motor is on & the magnetic clutch is disengaged (off)

(a) Starting mode:

at first the inertia of the motor must appear in the system

(b) normal operation mode

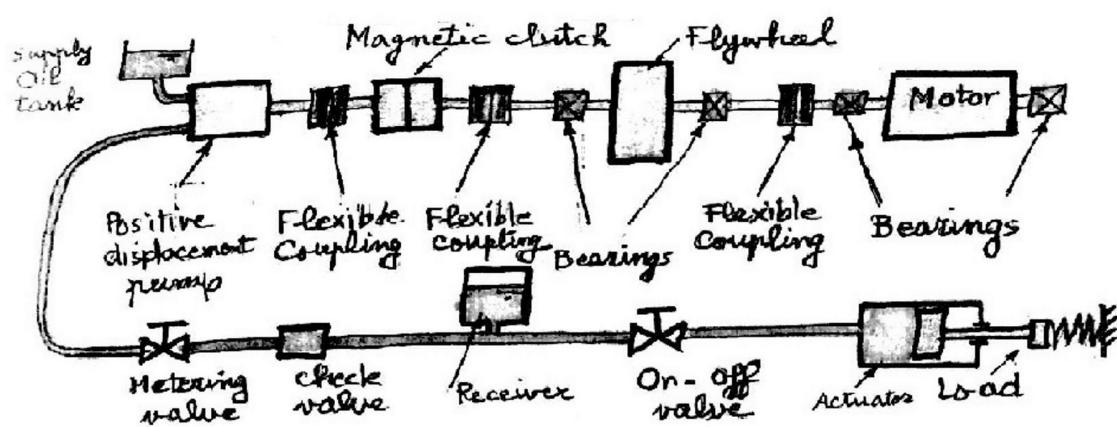
The motor drives the flywheel through a flexible coupling which stores this energy.

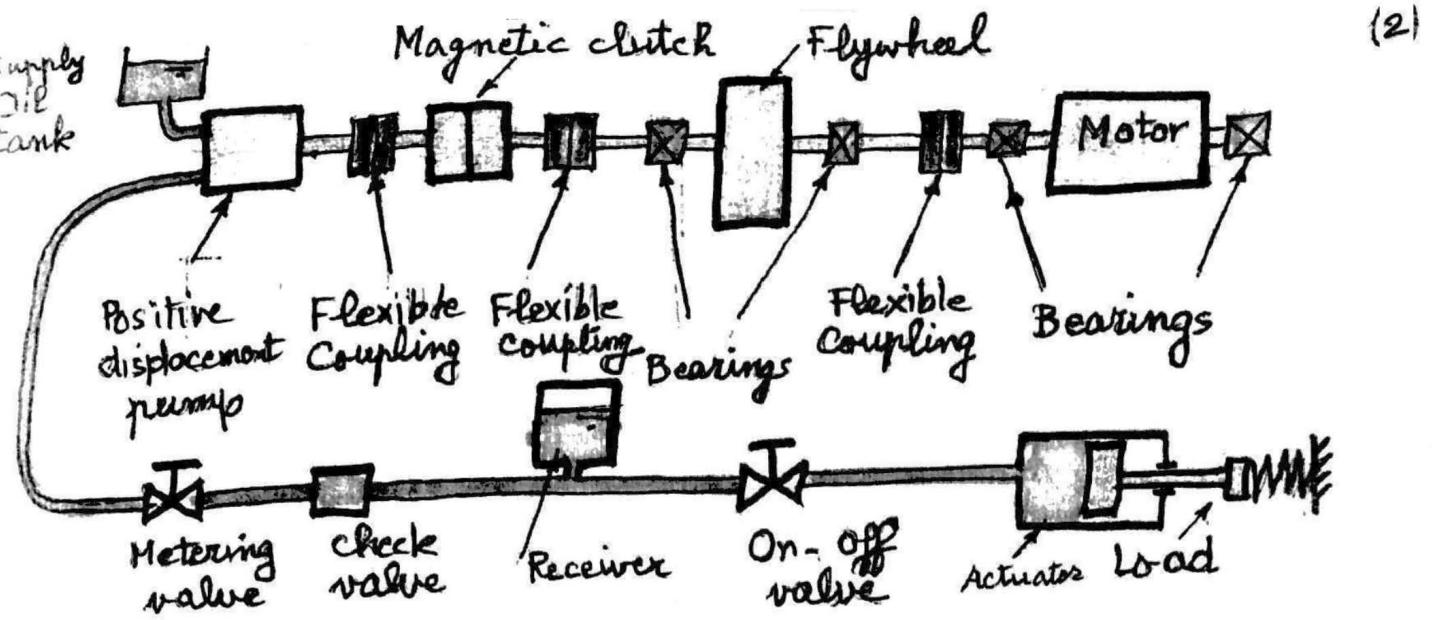
(2) Electric motor is off and magnetic clutch is engaged (on)

in this position the flywheel discharge its stored energy to drive the positive displacement pump which generates the necessary hydraulic power to charge the receiver and no flow in the actuator side as the on-off valve in the closed position.

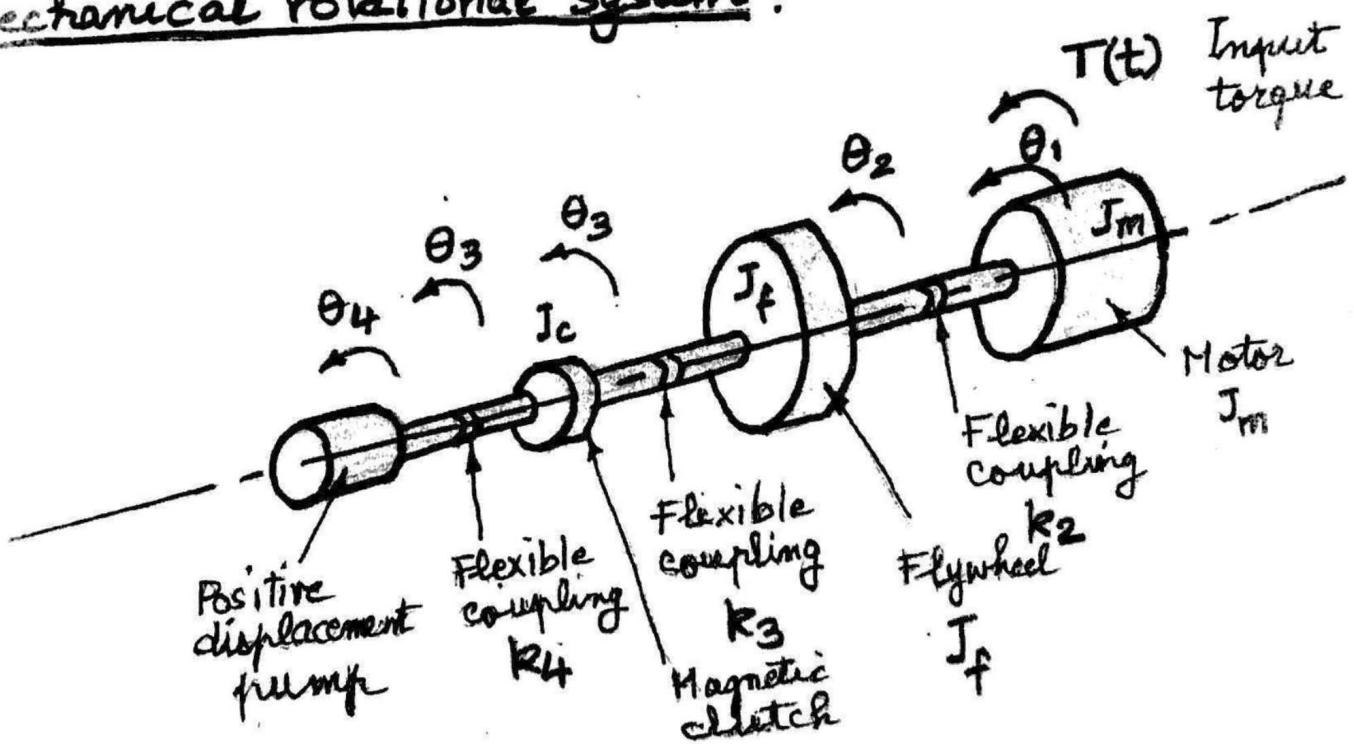
(3) Receiver is full & Pump is off and no energy in flywheel

here the receiver discharge the stored hydraulic under pressure to linear motion in the actuator as on-off valve in the open position.





### Mechanical rotational system :



### Magnetic clutch off :

$$T(t) + k_2(\theta_{02} - \theta_{01}) = J_m \ddot{\theta}_{01} \quad (1)$$

$$- k_2(\theta_{02} - \theta_{01}) = J_f \ddot{\theta}_{02} \quad (2)$$

Initial conditions :

$$t=0, \theta_{01} = \theta_{02} = \dot{\theta}_{01} = \dot{\theta}_{02} = 0$$

To be solved for  $\theta_{01}$  &  $\theta_{02}$  in the domain  
 $t = 0 \rightarrow t_0$

starting mode

(3)

"Some general comments"

$$T(t) + k_2(\theta_{02} - \theta_{01}) = J_m \ddot{\theta}_{01} \quad (1)$$

$$-k_2(\theta_{02} - \theta_{01}) = J_f \ddot{\theta}_{02} \quad (2)$$

Initial conditions:

$$t=0, \theta_{01} = \dot{\theta}_{02} = \ddot{\theta}_{01} = \ddot{\theta}_{02} = 0$$

Simplified solution:

Assume a simple form of induction motor,

$$\dot{\theta}_{01} = \omega_m (1 - e^{-t/t_0}) \quad (3)$$

and  $T(t) = T$  (4)

Adding eqn (1) and (2)

$$T(t) = J_m \ddot{\theta}_{01} + J_f \ddot{\theta}_{02} \quad (4)$$

Assuming the stiffness of the couplings to be rigid enough to let  $\dot{\theta}_{02} \approx \dot{\theta}_{01}$  and consequently  $\ddot{\theta}_{02} = \ddot{\theta}_{01}$

Eqn. (4) becomes,

$$T(t) = (J_m + J_f) \ddot{\theta}_{02} \quad (5)$$

From eqn (5)

$$\ddot{\theta}_{02} = \frac{T}{J_m + J_f} \quad (6)$$

(4)

Integrating eqn. (3) once we have,

$$\text{with } \theta_{01} = \theta_{02}, \quad \theta_{02} = \omega_m(t + t_0 e^{-t/t_0}) + C_1$$

with  $\theta_{01}(0) = 0$ , we have,

$$\theta_{02} = \omega_m(t + t_0 e^{-t/t_0 - t_0}) \quad (7)$$

But from eqn. 3,

$$\ddot{\theta}_{02} = \frac{\omega_m}{t_0} e^{-t/t_0},$$

Assuming constant acceleration at start,

$$\ddot{\theta}_{02} = \ddot{\theta}_{02}(0) = \frac{\omega_m}{t_0},$$

Therefore from eqn. (6) we have,

$$\frac{T}{J_m + J_f} = \frac{\omega_m}{t_0}$$

Therefore,

$$\omega_m = \frac{T t_0}{J_m + J_f}$$

Eqn(7) becomes,

$$\boxed{\theta_{02} = \frac{T t_0}{J_m + J_f} (t + t_0 e^{-t/t_0 - t_0})} \quad (8)$$

Note: The rest of the program in simulink  
is for the Mechanical and pump system.

(5)

Magnetic clutch on :

$$k_2(\theta_2 - \theta_1) = J_m \ddot{\theta}_1 \quad (3)$$

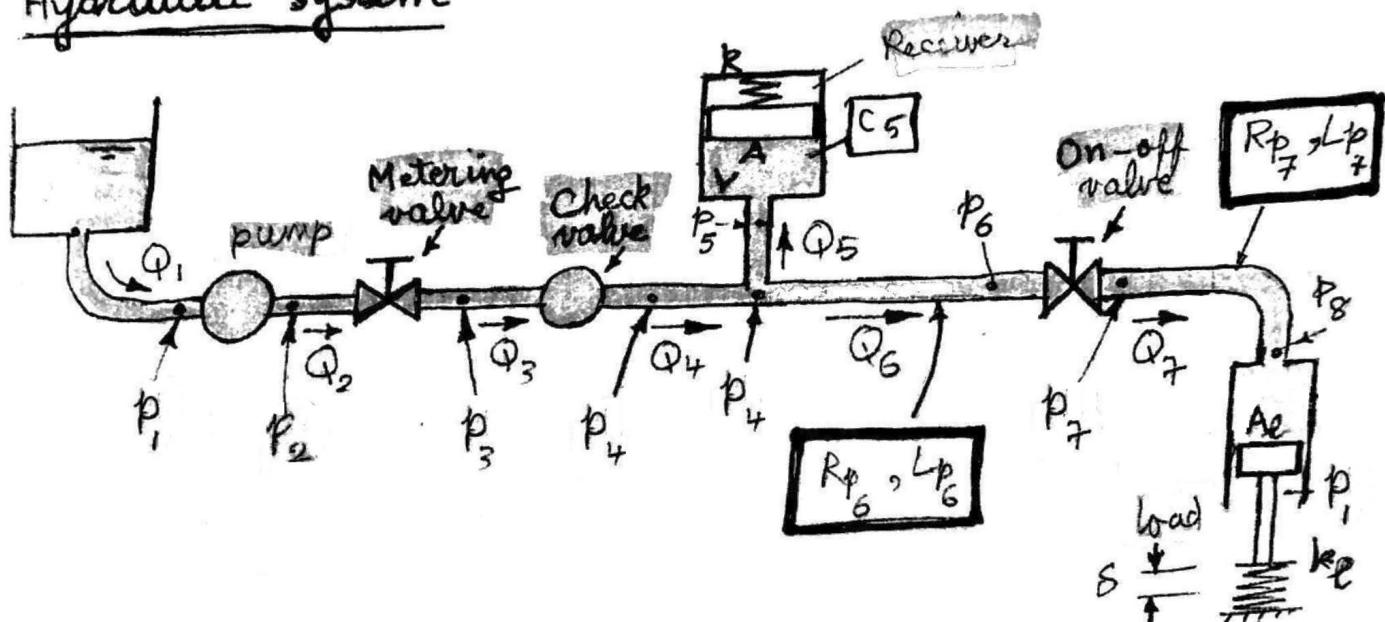
$$-k_2(\theta_2 - \theta_1) + k_3(\theta_3 - \theta_2) = J_f \ddot{\theta}_2 \quad (4)$$

$$-k_3(\theta_3 - \theta_2) + k_4(\theta_4 - \theta_3) = J_c \ddot{\theta}_3 \quad (5)$$

Initial conditions :

$$t = t_0, \theta_1 = \theta_{01}, \theta_2 = \theta_{02}, \theta_3 = 0, \theta_4 = 0,$$

$$\dot{\theta}_1 = \dot{\theta}_{01}, \dot{\theta}_2 = \dot{\theta}_{02}, \dot{\theta}_3 = 0, \dot{\theta}_4 = 0.$$

Hydraulic systemnote

$k_2, k_3, k_4$  : stiffness of flexible coupling

$J_m, J_f, J_c, J_p$  : moment of inertia of motor, flywheel, clutch and pump

$k_m, k_c, k_v$  : valve coefficients of metering, check, on-off valves

$A$  : Receiver Area ( $m^2$ ) ,  $k_r$  : receiver spring stiffness

$k_L$  : Actuator spring stiffness ,  $m$  : piston mass ,  $P_1$  : Atmospheric pressure

\* The eqns. used in the case of on-off valve closed are: (6)

1- The pressure at inlet to the pump is assumed atmospheric to avoid any sort of negative pressure at inlet to the pump to avoid cavitation.

2- The equation for the efficiency of the pump was corrected to become,

$$\frac{(P_2 - P_1)Q_2 - k_4(\theta_4 - \theta_3)}{\dot{\theta}_4 \eta_t} = J_p \ddot{\theta}_4 \quad (1)$$

$$Q_3 = \frac{1}{k_m} \sqrt{|P_2 - P_3|} \quad (2)$$

$$Q_4 = \frac{1}{k_c} |P_3 - P_4| \quad (3)$$

$$\text{With } Q_3 = Q_4 = Q_5$$

$$Q_5 = C_5 (\dot{P}_4 - \dot{P}_5) \text{ with } C_5 = \frac{A^2}{k_r} \quad (4)$$

$$Q_6 = Q_7 = 0 \quad (\text{On-off valve closed}) \quad (5)$$

\* The equations in the case of on-off valve opened are:

$$\ddot{s} = (P_8 - P_1) \frac{A_e}{m} - \frac{k_f}{m} s \quad (1)$$

\* Compressibility of the fluid is assumed negligible  $B=0$ , in order to eliminate the problems associated with compressibility. Therefore,  $Q_5$  is to be calculated from:

$$Q_5 = A_6 \dot{S} \quad (2)$$

$$P_6 = P_4 - L_{P_6} Q_5 - R_{P_6} Q_5 \quad (3)$$

$$P_7 = P_6 - (Q_5 k_v)^2 \quad (4)$$

$$P_8 = P_7 - L_{P_7} Q_5 - R_{P_7} Q_5 \quad (5)$$

$$\text{With } P_5 = P_4 \quad (6)$$

- \* It seems that the problem is apparently working but it needs further investigations.
- \* The problem also needs more reliable experimental data.

$$L_{P_6} = \frac{f l_{P_6}}{A_{P_6}} \quad \begin{aligned} l_{P_6} &: \text{pipe (6) length} \\ A_{P_6} &: \text{pipe (6) Area} \end{aligned} \quad \times f: \text{Hydraulic oil density}$$

$$L_{P_7} = \frac{f l_{P_7}}{A_{P_7}} \quad \begin{aligned} l_{P_7} &: \text{pipe (7) length} \\ A_{P_7} &: \text{pipe (7) Area} \end{aligned}$$

$$R_{P_6} = \frac{128 \nu l_{P_6}}{\pi d_{P_6}^4} \quad \begin{aligned} \nu &: \text{Dynamic oil viscosity} \\ d_{P_6} &: \text{pipe (6) diameter} \end{aligned}$$

$$R_{P_7} = \frac{128 \nu l_{P_7}}{\pi d_{P_7}^4} \quad d_{P_7} : \text{pipe (7) diameter}$$