An Analytical Study on Centrifugal Pump Performance Under Different Rotational speeds

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***Abstract*— Understanding the local flow characteristics of centrifugal pumps under diverse piping system flow conditions is essential. This study conducts analytical research on a radial flow single-stage centrifugal pump operating at five different rotational speeds: 1000, 1200, 1400, 1600, and 1800 rpm. All equations are implemented in MATLAB to forecast pump performance. Additionally, losses have been accounted for, including leakage loss and impeller loss. The pump's discharge, head, efficiency, and the power necessary to operate the pump were evaluated. This study can be used for any type of fluids. However, only a single phase is involved. The petroleum and chemical industries may gain advantages from the results of this research.**

***Keywords*— Centrifugal pump; Pump performance; Analytical Analysis.**

I. INTRODUCTION

The performance attributes of a centrifugal pump for transporting pure water at varying pump speeds are being evaluated. Omar et al. [1] employed empirical and theoretical equations for external and internal energy loss to formulate a theoretical method for calculating the head, efficiency, and power of the pump, while changing fluid properties, rotating speeds, and input geometries for two distinct impeller configurations. The aim of Perissinotto et al. [2] was to investigate the formation of an oil-water emulsion within the transparent volute and impeller of a prototype electrical submersible pump (ESP). Flow visualization techniques generated images depicting the behavior of oil droplets injected into water at different flow rates and rotational speeds. The two observed patterns are "concentrated drops" and "dispersed drops." Zhao et al. [3] investigated the influence of centrifugal pump rotating speed, liquid flowrate, and gas volume ratio entering on pump flow patterns. They identified four prevalent flow patterns: segregated flow, bubble flow, gas pocket flow, and aggregation bubble flow. Perissinottoa et al. [4] conducted an experimental investigation on the act of water droplets within an oil medium inside an impeller of a centrifugal pump. Similarly, they [5] examined the behavior of oil droplets in an aqueous medium. They conducted their experiments at various water flow rates and pump speeds. They additionally recorded the flow patterns utilizing a high-speed camera. Abo Elyamin et al. [6] conducted a numerical investigation determining that an impeller with seven blades is optimal at a rotational speed of 2800 rpm.

The main objectives of the present work are studying pump performance characteristics at different pump rotational speeds.

II. ANALYTICAL ANALYSIS

The essential equation of turbo-machines with infinite blade is declared by Euler [7] as:

$$H_{inf} = \frac{(u_2 c_{2u} - u_1 c_{1u})}{g} \tag{1}$$

u_1 & u_2 = Circumferential velocities (m/s)

c_{1u} & c_{2u} = The circumferential components of absolute velocity at inlet (m/s)

g = Gravitational acceleration

After presenting the slip factor γ and the blade blockage τ_2, the theoretical head will be [8]:

$$H_{th} = \frac{u_2^2}{g}\left(\gamma - \frac{Q_{La}}{A_2 u_2 \tan\beta_{2B}}\left[\tau_2 + \frac{A_2 d_{1m}^* \tan\beta_{2B}}{A_1 \tan\alpha_1}\right]\right) \tag{2}$$

$$A_1 = \frac{\pi}{4}(d_1^2 - d_n^2) \tag{3}$$

$$A_2 = \pi d_{2b} b_2 \tag{4}$$

Q_{La} = Pump flow (m^3/s)

β = Angle between relative velocity vector and the negative direction of circumferential velocity

α = Angle between direction of circumferential and absolute velocity

Pump performance is evaluated with a loss correlation examination in the pump. In the current study, the next internal losses are considered from [9]:

2.1. Leakage loss

Leakage loss happens according to smaller circulation in gaps between pump's fixed and rotating parts. Leakage loss reduces the efficiency due to the flow increment in the impeller compared to the flow in the pump. For computing the leakage loss, the pressure change over the seal at the inlet of the impeller ΔH_{sp} has to be recognized [8]:

$$\Delta H_{sp} = H_P - k^2 \frac{u_2^2}{2g}\left(1 - \frac{d_{sp}^2}{d_2^2}\right) \tag{5}$$

Where

$$H_P = \frac{u_2^2 - u_1^2 + w_1^2 - w_2^2}{2g} - Z_{La} \quad (6)$$

w_1 & w_2 = Relative velocities (m/s)
Z_{La} = Sum of hydraulic losses in Impeller

$$k = 0.9\, y_{sp}^{0.087} \quad (7)$$

K = Rotation factor

$$y_{sp} = Re_{u2}^{0.3} \frac{s d_{sp}}{d_2^2} \sqrt{\frac{s}{l_{sp}}} \quad (8)$$

y_{sp} = Annular seal, leakage flow
Re = Reynolds number
d_2 = Diameter at impeller blade trailing edge
S = Gap width

$$R_{e_{u2}} = \frac{u_2 r_2}{v} \quad (9)$$

v = Kinematic viscosity

$$c_{ax} = \sqrt{\frac{2g\Delta H_{sp}}{\xi_{EA} + \lambda \frac{l_{sp}}{2s}}} \quad (10)$$

C = Axial coefficient
ξ_{EA} = Inlet + outlet loss
λ = Friction coefficient

The friction coefficient is calculated as:
For laminar flow *Resp* < 2300

$$\lambda = \frac{64}{Re_{sp}} \quad (11)$$

For turbulent flow 4000 < Re_{sp} <10^8:

$$\lambda_0 = \frac{0.31}{\left(\log\left[A + \frac{6.5}{Re_{sp}}\right]\right)^2} \quad (12)$$

As

$$Re_{sp} = \frac{2\, s\, c_{ax}}{v} \quad (13)$$

The rotation impact in turbulent flow is experimentally calculated using a determined factor $\frac{\lambda}{\lambda_0}$, [8]:

$$\frac{\lambda}{\lambda_0} = \left[1 + 0.19\left(\frac{Re_{u2}}{Re_{sp}}\right)^2\right]^{0.375} \quad (14)$$

The friction coefficient is considered iteratively [8]. Then, the leakage flow is considered as follows:

$$Q_{sp} = \pi d_{sp}\, s\, c_{ax} \quad (15)$$

c_{ax} = Axial velocity in gap

2.2. Impeller loss,

$$L_{sh,La} = C_{sh} \frac{(w_1 - w_{1q})^2}{2g} \quad (16)$$

$L_{sh,La}$ = Impeller loss (m)
C_{sh} = Shock loss coefficient (0.5 – 0.7)

$$d_{h,La} = \frac{2(a_2 b_2 + A_{q1})}{a_1 + b_1 + a_2 + b_2} \quad (17)$$

$$L_e = \frac{r_2 - r_1}{cos\beta_{2B}} \quad (18)$$

$$L_{fr,La} = 4C_{fr,La} \frac{l_e}{d_{h.La}} \frac{{w_{av}}^2}{2g} \quad (19)$$

$$Re_{La} = \frac{w_{av} l_e}{v} \quad (20)$$

$$L_{D,La} = 0.25 \frac{w_1^2}{2g} \quad (21)$$

2.3. Efficiencies and Power

The fraction of pump to impeller flows is the volumetric efficiency:

$$\eta_v = \frac{Q_{La}}{Qi} = \frac{Q_{La}}{Q_{La} + Q_{sp}} \quad (22)$$

η_v = Volumetric efficiency

Impeller flow can be considered as:

$$\varepsilon_1 = (z_{La}/\pi)\left((e_1/\sin\beta_{1B})/d_1\right) \quad (23)$$

$$\varepsilon_2 = (z_{La}/\pi)\left((e_2/\sin\beta_{2B})/d_2\right) \quad (24)$$

ε = Roughness

$$Q_i = C_{2m}\pi d_2 b_2 \varepsilon_2 = C_{1m}\pi d_1 b_1 \varepsilon_1 \quad (25)$$

Q_i = The impeller flow (m^3/s)

The mechanical efficiency is given as:

$$\eta_{me} = 1/1 + \left(\frac{\eta_h \eta_v (P_{me} + P_{RR})}{\rho g H_{th} Q_{La}}\right) \quad (26)$$

The full equation of hydraulic efficiency is calculated using power of the pump and is given by:

$$\eta_h = \frac{\rho g H_r (Q_{La} + Q_{sp})}{(P - P_m - P_{RR})} \quad (27)$$

η_h = Hydraulic efficiency

Actual head H_r is:

$$H_r = H_{th} - (L_{sh,La} + L_{fr,La} + L_{D,La}) \quad (28)$$

The overall power of the pump is:

$$P = \frac{\rho g H_r Q}{\eta_h \eta_v} + P_m \quad (29)$$

The overall efficiency is given as:

$$\eta = \eta_v \eta_h \eta_m \quad (30)$$

III. RESULTS

Analytical analysis is undertaken to forecast the performance of the centrifugal pump. The equations referenced in the analytical analysis are considered. All external and internal losses are computed utilizing MATLAB. The pump's actual head, efficiency, and power are assessed.

Fig. 1 (a), (b), and (c) show the analytical relations head, efficiency, and shaft Power and flowrate for pure water at 1000 rpm, respectively. Fig. 1 (a) declares the pump head and flowrate relation. As the flowrate increases, pump head decreases. The variation of the overall efficiency with flow rate is shown in Fig. 1 (b). As the flowrate increases, pump efficiency increases till a maximum value then it starts to decrease. Fig. 1 (c) shows the pump shaft power and flow rate relation. Shaft power increases linearly as the flowrate increases.

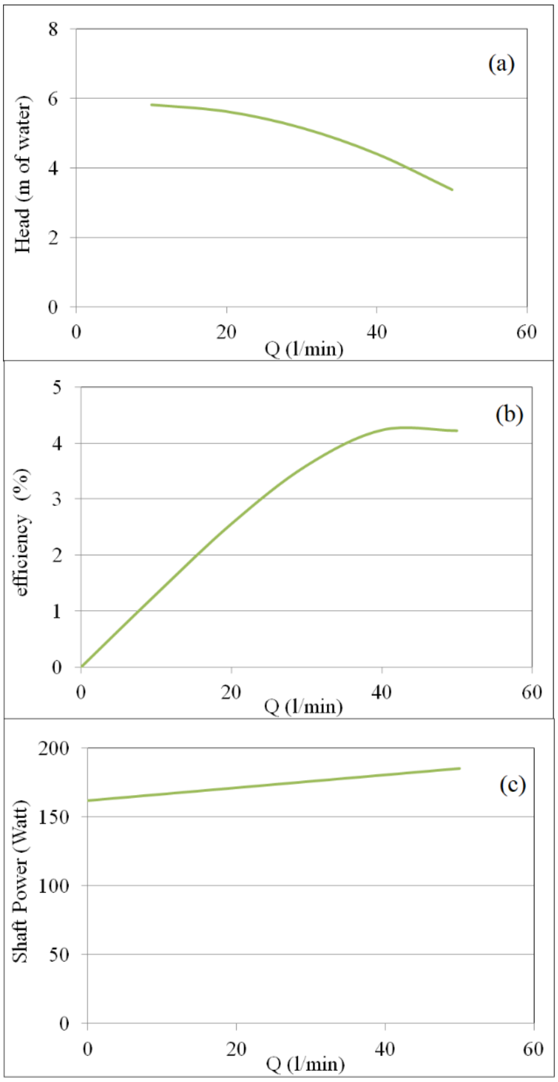

Fig. 1. Analytical relations between (a) Head, (b) efficiency (c) Shaft Power and flowrate for pure water at 1000 rpm, respectively.

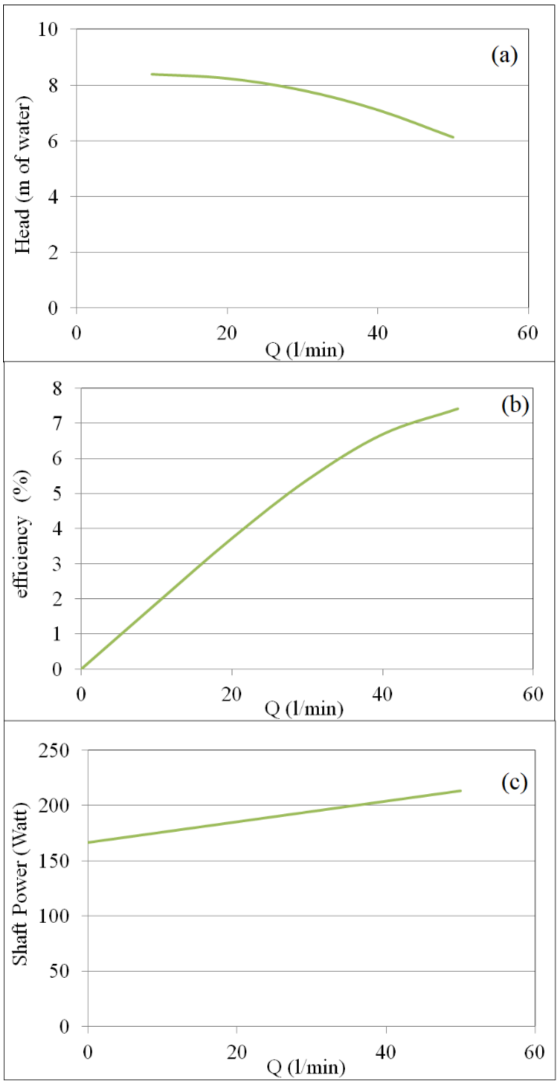

Fig. 2. Analytical relations between (a) Head, (b) efficiency (c) Shaft Power and flowrate for pure water at 1200 rpm, respectively.

Moreover, Figs. 1 to 5 show the analytical relations for pure water at rotational speeds of 1000, 1200, 1400, 1600, and 1800 rpm, respectively. It can be recognized from figures that all values have the same trend with their values increasing with increasing rotational speeds. Rotational speed and flow rate typically have a linear relationship. Generally speaking, a higher flow rate results from a faster spin. As the rotating speed increases, so does the total pump head quadratically. Doubling the rotational speed can result in a fourfold increase in the head, presuming no other changes are made. Rotational speed causes a large rise in pump shaft power. It is proportional to the speed increase's cube. This may require a more powerful motor and result in increased operating costs.

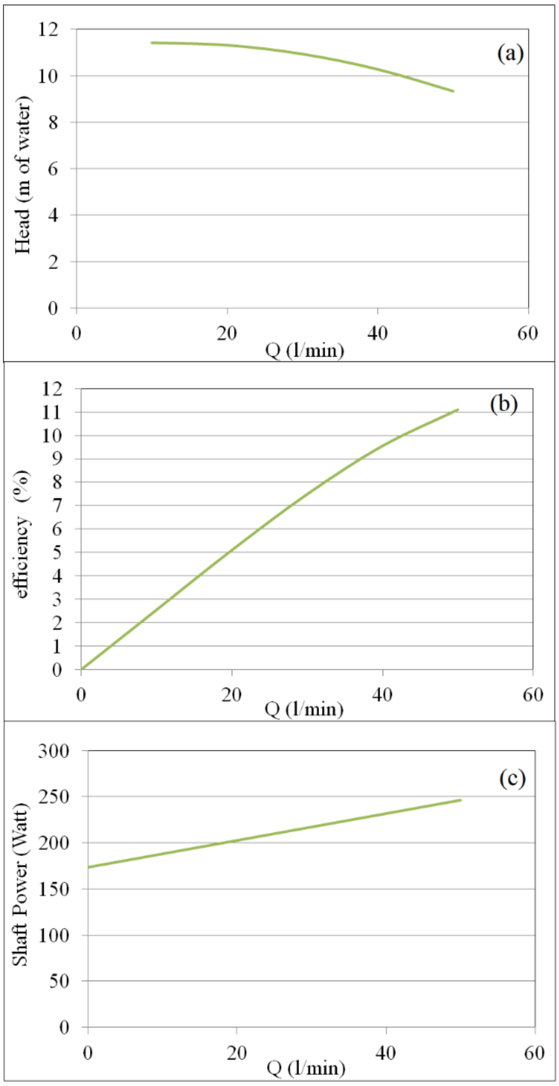

Fig. 3. Analytical relations between (a) Head, (b) efficiency (c) Shaft Power and flowrate for pure water at 1400 rpm, respectively.

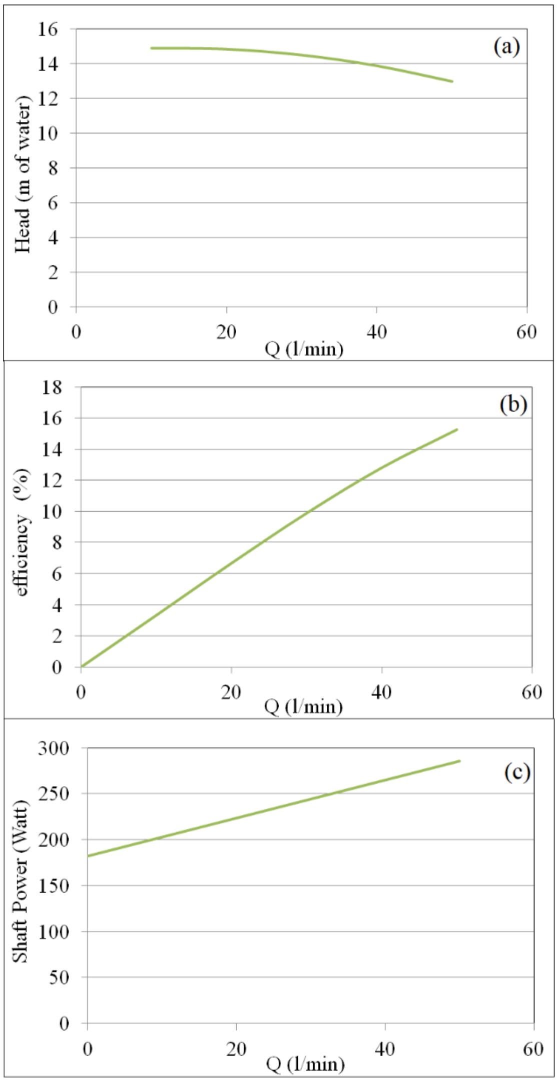

Fig. 4. Analytical relations between (a) Head, (b) efficiency (c) Shaft Power and flowrate for pure water at 1600 rpm, respectively.

IV. VALIDATION WITH PREVIOUS EXPERIMENTAL WORK

In this section, the present study is validated using previous experimental study of [10]. Fig. 6 shows a comparison between the present analytical results and Afify et al. [10]'s experimental results for pure water at 1400 rpm. Fig. 6 (a), (b), and (c) show the relations between head, efficiency, and shaft Power and flowrate, respectively. All the relations show the same trend. Disk and volute losses are not considered in the present analytical study, which could explain the discrepancy in the results.

V. CONCLUSIONS

This study analytically examines the performance of a variable speed centrifugal pump for pure water. The pump operates at five distinct rotational speeds: 1000, 1200, 1400, 1600, and 1800 rpm. Performance pump curves are generated for each case through analytical analysis. Results indicate that for pure water, an increase in pump rotational speed leads to an elevation in head, efficiency, and shaft power.

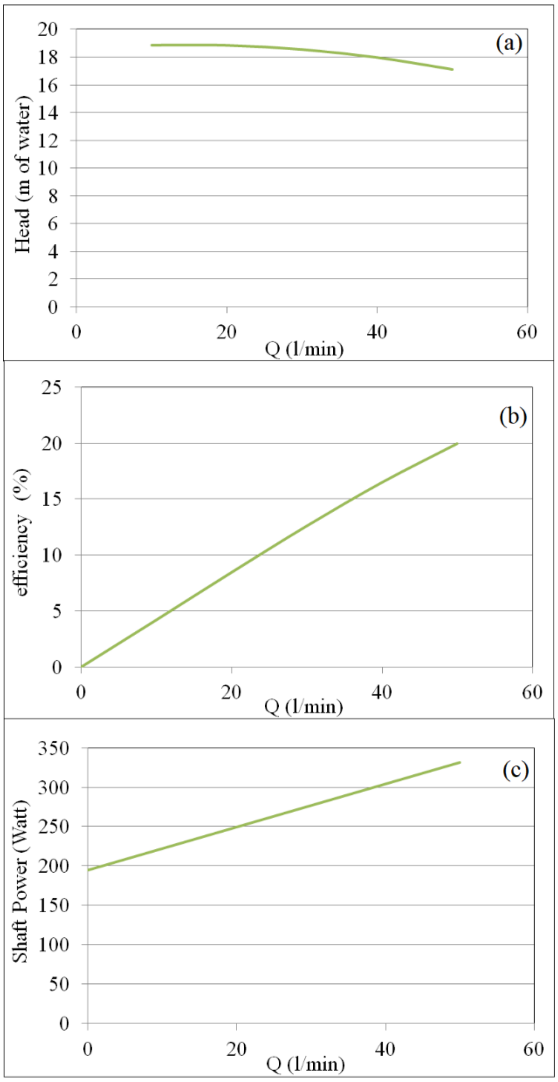

Fig. 5. Analytical relations between (a) Head, (b) efficiency (c) Shaft Power and flowrate for pure water at 1800 rpm, respectively.

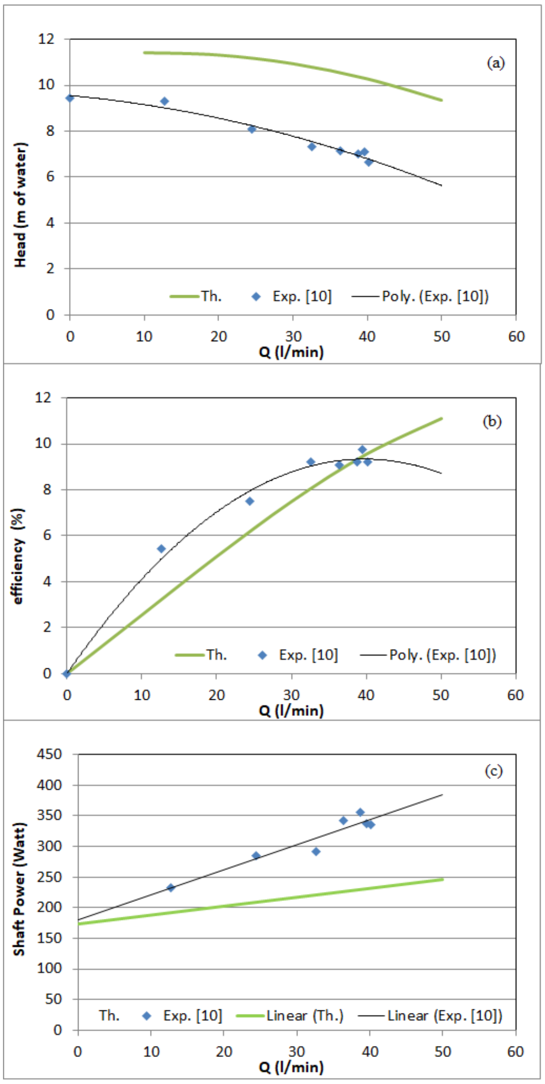

Fig. 6. Analytical relations between (a) Head, (b) efficiency (c) Shaft Power and flowrate for pure water at 1400 rpm, respectively.

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