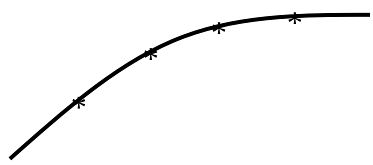
Fluid Kinematics

(Hydrodynamics)

Kinematics describes motion in terms of displacements, velocities, and accelerations regardless to the forces which are associated with these variables.

* Definitions :

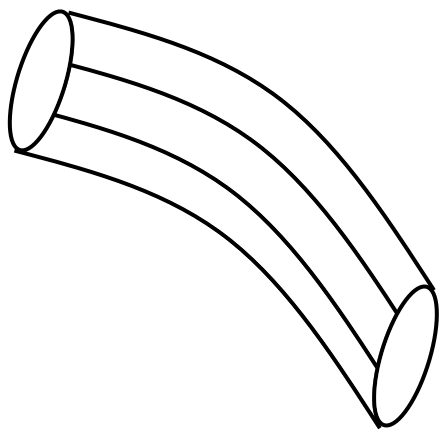
*** Streamline :** is a smooth imaginary curve represents one particle in the flow. The tangent of this line gives the direction of velocity at any point.



- Streamlines can never intersect.
- They can never have sudden change in direction .

Intersection or sudden change in direction means that there is a point where the velocity vector has two directions in the same time which is impossible .

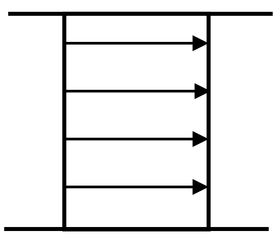
*** Stream tube :** is a tube formed of an infinite number of streamlines which are drawn passing through a closed curve in the flow.



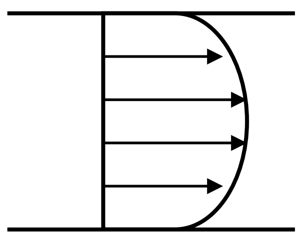
- No flow can go in or out the sides of this tube.

* Types of flow :

1 – Ideal and real flow



Ideal flow

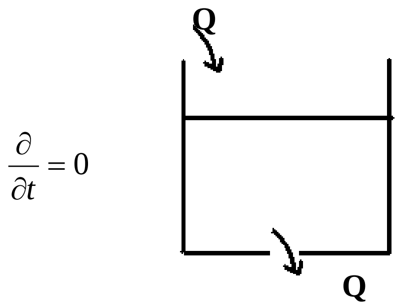


Real flow

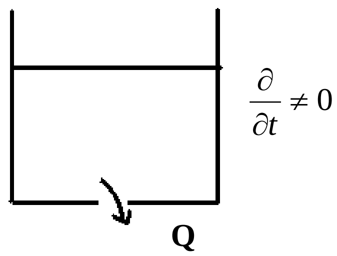
*** Ideal flow:** means frictionless flow, no energy is lost, viscosity is considered zero.

*** Real flow:** viscosity can't be neglected, there is friction. Friction causes some of the mechanical energy to be converted into heat energy and can't be restored.

2 – Steady and unsteady flow (with respect to time) (from time to time)



steady flow



unsteady flow

*** Steady flow:** pressure, velocity, flow rate (flow parameters) are constant with respect to time.

*** Unsteady flow:** any of the flow parameters change with time.

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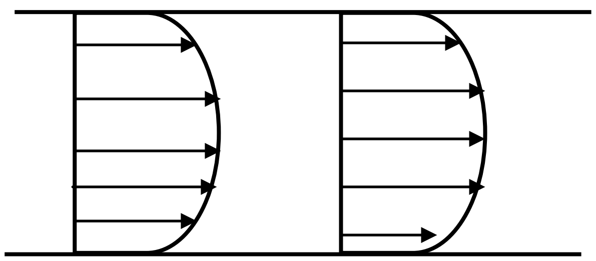
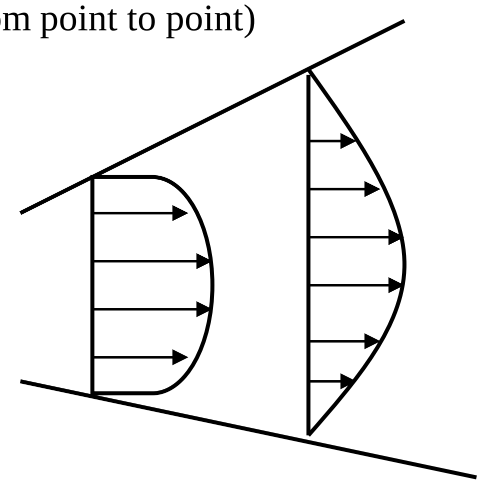
3 – Uniform and Non-uniform flow (from point to point)



Uniform flow



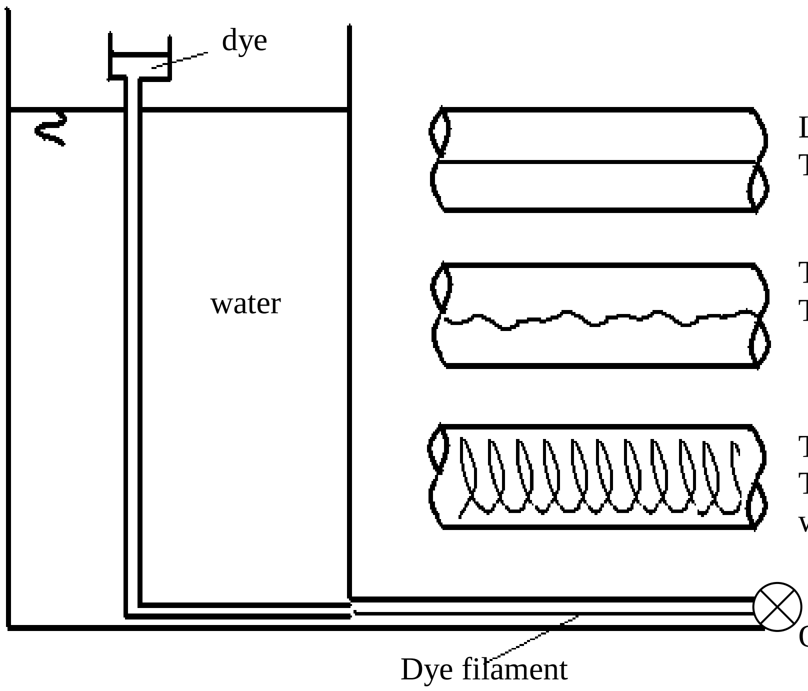
Non-uniform flow

*** Uniform flow:** the velocity at a given instant is the same in magnitude and direction at every point in the flow.

*** Non-uniform flow:** the velocity at a given instant changes from point to point.

4 - Laminar, Transient and Turbulent flow

$$\text{Re} = \frac{\rho v d}{\mu}$$



Laminar flow Re < 2000
The particle moves in parallel layers.

Transient flow 2000 < Re < 4000
The dye filament begins to oscillate.

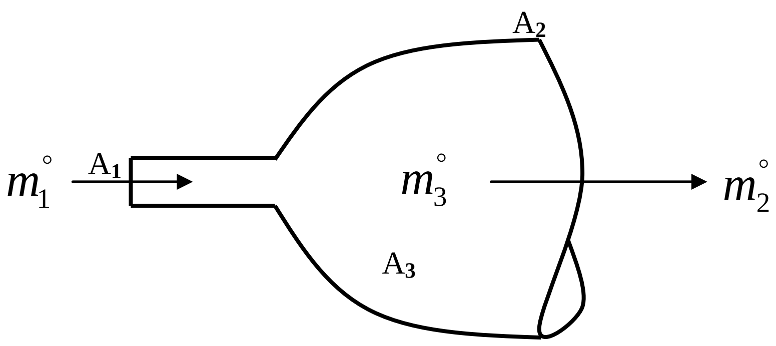
Turbulent flow Re > 4000
The dye color is diffused over the whole cross-section.

Equations of motion :

1 – Continuity equation.

2 – Bernoulli's equation.

1 – Continuity equation:



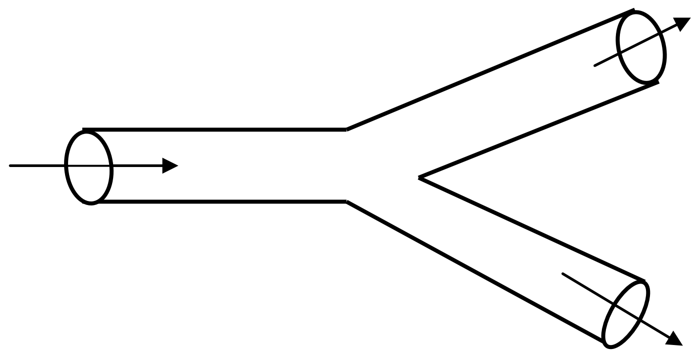
Mass of fluid entering per unit time = Mass of fluid leaving per unit time

$$\overset{\circ}{m}_1 = \overset{\circ}{m}_2 = \overset{\circ}{m}_3 = const$$ mass flow rate

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \rho_3 A_3 V_3$$

* For liquids (incompressible fluid) $\rho_1 = \rho_2 = \rho_3 = const.$

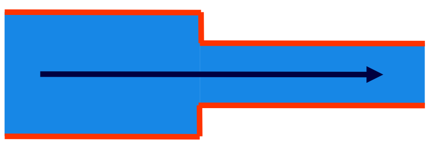
$$\boxed{A_1 V_1 = A_2 V_2 = A_3 V_3 = Q}$$ discharge (volume flow rate)



$Q_1 = Q_2 + Q_3$

Ex.:

Assuming the water moving in the pipe is an ideal fluid, relative to its speed in the 1" diameter pipe, how fast is the water going in the 1/2" pipe?



a) 2 v_1 b) 4 v_1 c) 1/2 v_1 c) 1/4 v_1

Solution

Using the continuity equation

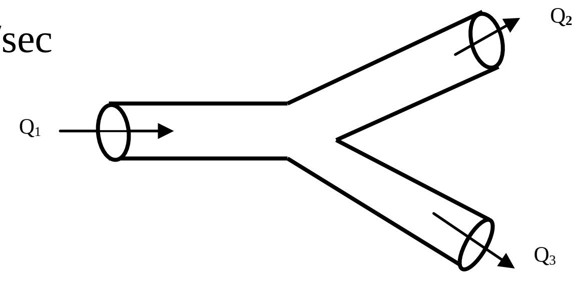
$$A_1 \cdot v_1 = A_2 \cdot v_2$$

$$\rightarrow \quad v_2 = \frac{A_1}{A_2} \cdot v_1 = \left(1 \cdot \frac{2}{1}\right)^2 \cdot v_1 = 4v_1$$

Ex.:

If pipe 1 diameter = 50mm, mean velocity 2m/s, pipe 2 diameter = 40mm takes 30% of total discharge and pipe 3 diameter = 60mm. What are the values of discharge and mean velocity in each pipe?

Solution



$A_1\ v_1 = \frac{\pi}{4}(50x10^{-3})^2 x2 = 3.93x10^{-3}\ m^3/sec$

$Q_2 = 0.3\ x\ Q_1 = 1.178x10^{-3}\ m^3/sec$

$Q_2 = A_2\ v_2$

$1.178x10^{-3} = \frac{\pi}{4}(40x10^{-3})^2 x\ v_2$

$v_2 = 0.9375\ m/s$

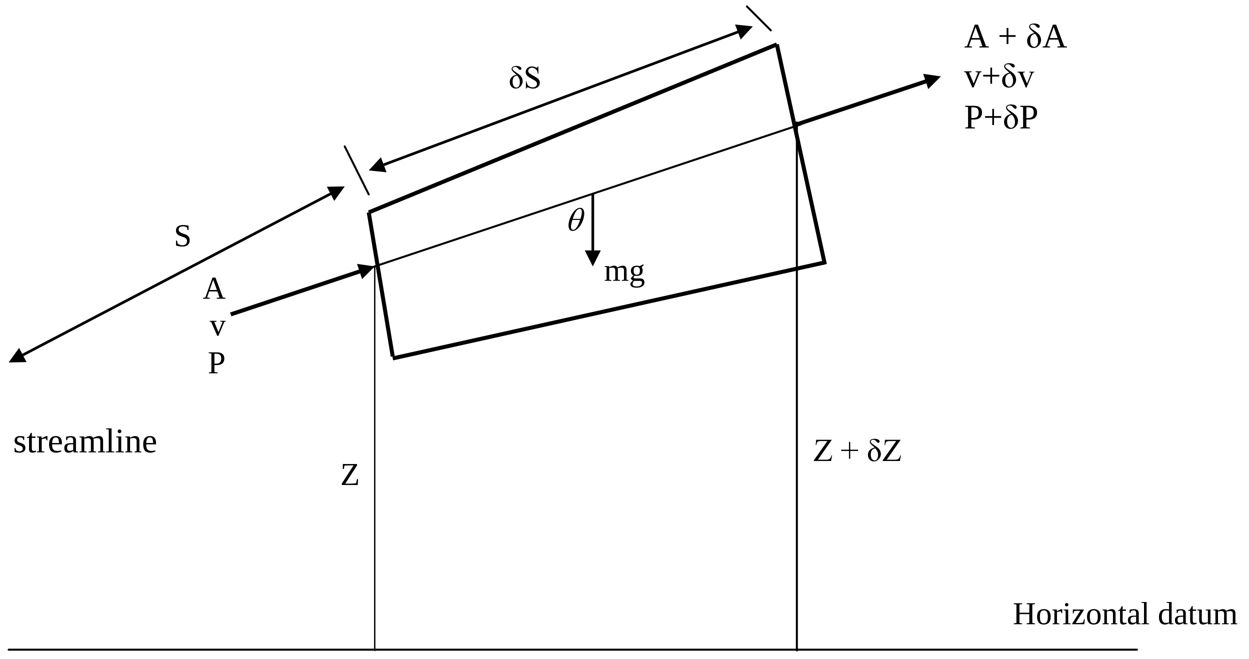
$Q_1 = Q_2 + Q_3$

$3.93x10^{-3} = 1.178x10^{-3} + Q_3 \qquad \therefore Q_3 = 2.752x10^{-3}\ m^3/sec$

$Q_3 = A_3\ v_3$

$2.752x10^{-3} = \frac{\pi}{4}(60x10^{-3})^2 x\ v_3 \qquad \therefore v_3 = 0.9733\ m/sec$

2 – Bernoulli's equation (energy equation)



$$E = Z + \frac{P}{\rho g} + \frac{V^2}{2g} = const$$

E : total energy per unit weight (m)

Z : potential energy per unit weight (m)

$\frac{P}{\rho g}$: pressure energy per unit weight (m)

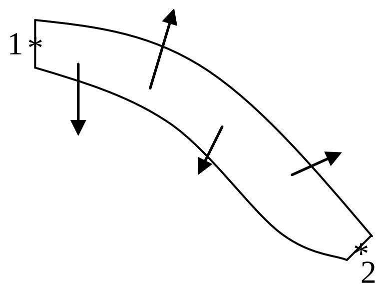
$\frac{V^2}{2g}$: kinetic energy (velocity energy) per unit weight (m)

* For ideal flow :

$$E_1 = E_2$$

$$Z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

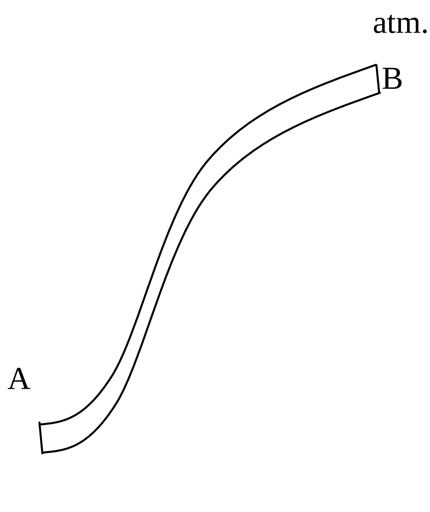
* For Real flow



$$E_1 - h_{\text{loss 1 to 2}} = E_2$$

$$E_1 = E_2 + h_{\text{loss 1 to 2}}$$

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} + h_{loss1to2}$$



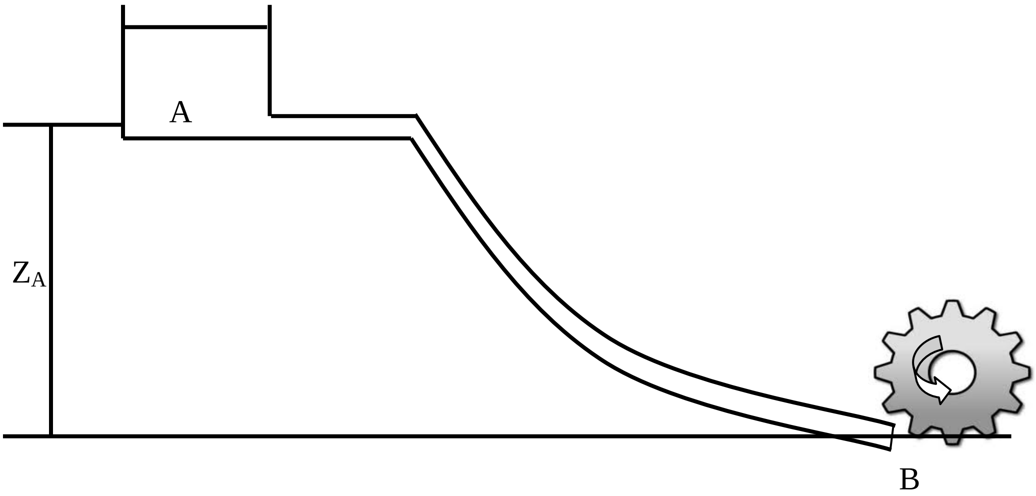
$$\frac{P_A}{\rho g} + Z_A + \frac{V_A^2}{2g} = \frac{P_B}{\rho g} + Z_B + \frac{V_B^2}{2g} + h_{loss1to2}$$

= 0 atm

$A_A = A_B$

$\therefore V_A = V_B$

$$\frac{P_A}{w} = \left(Z_B - Z_A\right) + h_{loss1to2}$$



$$\frac{P_A}{\rho g} + Z_A + \underbrace{\cancel{\frac{V_A^2}{2g}}}_{=0} = \underbrace{\cancel{\frac{P_B}{\rho g}}}_{=0} + \underbrace{\cancel{Z_B}}_{=0} + \frac{V_B^2}{2g} + h_{lossAtoB}$$

vel. Inside Tank   atm.   dotum

$$\frac{P_A}{\rho g} + Z_A = \frac{V_B^2}{2g} + h_{lossAtoB}$$

$$\boxed{E_1 + h_p = E_2 + h_T + h_{\text{loss 1 to 2}}}$$

$P_{atm} = 0$

Same dia $\therefore$ same vel.

vel .inside tank = 0

Point on datum Z = 0

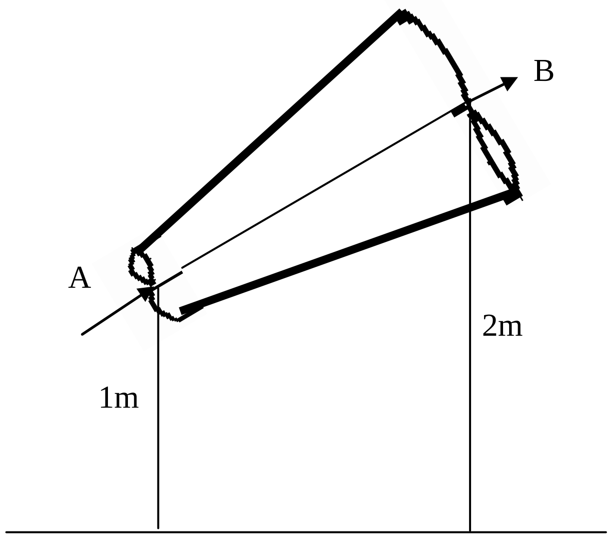
Ideal flow $h_{loss} = 0$

<u>Ex.:</u>

$d_{\mathbf{A}} = 1$ cm $d_{\mathbf{B}} = 5$ cm

$h_{\mathbf{A}} = 2$m $h_{\mathbf{B}} = 5$m

$$\frac{V_A^2}{2g} = 5m$$



<u>Determine</u>:

1 – The velocity at B

2 –Direction of flow

Solution take g = 10 m/s²

$\frac{V_A^2}{2g} = 5m$ $V_A = \sqrt{2*10*5} = 10m/s$

C.E from A to B $A_A V_A = A_B V_B$

$$\frac{\pi}{4}(1)^2 * 10 = \frac{\pi}{4}(5)^2 * V_B$$

$$\therefore V_B = \frac{10}{25} = 0.4m/\sec$$

$$E_A = h_A + Z_A + \frac{V_A^2}{2g} = 2 + 1 + 5 = 8m$$

$$E_B = h_B + Z_B + \frac{V_B^2}{2g} = 5 + 2 + \frac{(0.4)^2}{2*10} = 7.008m$$

$$E_A > E_B$$

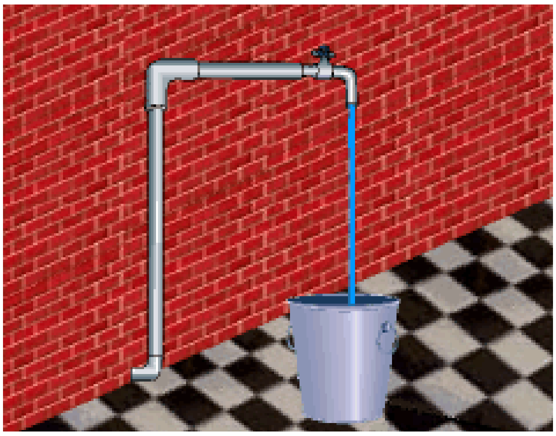
∴ The flow from A to B

$$h_{loss} = E_A - E_B$$

$$= 8 - 7.008 = 0.992 \cong 1m$$

Application of Bernoulli`s equation:

Simple Flowrate Measurement



The simplest technique to determine the steady flow rate of a liquid is to measure the amount of liquid collected over a period of time. For example, one could collect the liquid in a container of known size. If the time needed to fill the container is recorded, then the flow rate can be easily determined from the equation $\boxed{Q = \text{Volume} / \text{Time}}$

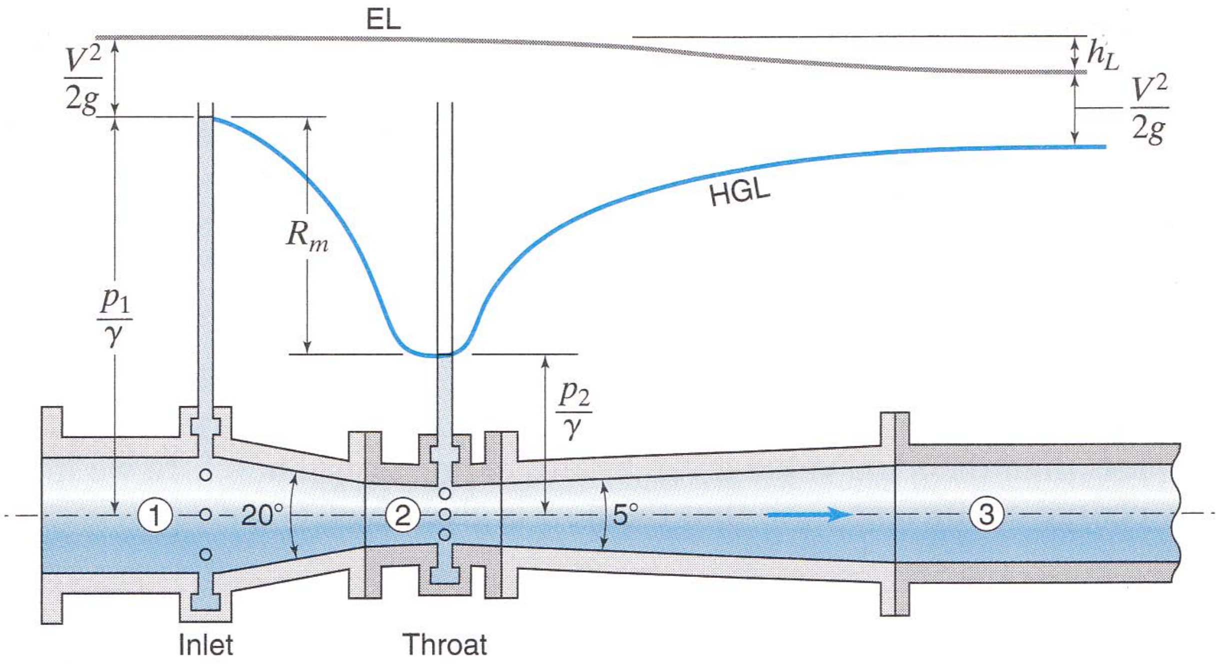
Measurements of flow rate:

A change in the cross-section area of a stream tube has been seen to produce an accelerated flow (change in flow velocity) and fall of pressure**.** By an excellent meter, flow may be calculated from pressure measurements, such as:

1. Venturi meter.
2. Orifice meter.

1. Venturi meter:

The **converging tube** is an efficient device for converting pressure head to velocity head, while the **diverging tube** converts velocity head to pressure head. Both of them may be combined to form a **Venturi tube**. As shown in Fig., it consists of a tube with a **constricted throat** (*smooth entrance cone of angle about 20 ^{o} as converging part*), which produces an increase in velocity accompanied by a reduction in pressure, followed by a gradually diverging portion of 5^{0} to 7^{0} cone angle in which the velocity is transformed back into pressure with slight friction loss.



As there is a definite relation between *the pressure differential and the rate of flow*, the tube may be made to serve as a metering device known as a **"Venturi Meter".** The pressure difference between the inlet section (1) and the throat section (2) is usually measured using **differential manometer.** Writing the Bernoulli equation between sections (1) (inlet) and (2) (throat), we have:

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_{loss_{1\to 2}}$$

$$V_2^2 - V_1^2 = 2g\left[\left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + (z_1 - z_2) - h_{loss_{1\to 2}} \right]$$

Continuity Eq.:

$$A_1V_1 = A_2V_2 = Q \quad \text{then,} \quad V_1 = \frac{Q}{A_1} \quad \& \quad V_2 = \frac{Q}{A_2}$$

Then, Bernoulli equation between sections (1) (inlet) and (2) (throat), can be rewritten as:

$$Q^2\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) = 2g\left[\frac{P_1 - P_2}{\rho g} + (z_1 - z_2) - h_{loss_{1\to 2}}\right]$$

$$Q^2 = \left(\frac{A_1^2 A_2^2}{A_1^2 - A_2^2}\right) 2g\left[\frac{P_1 - P_2}{\rho g} + (z_1 - z_2) - h_{loss_{1\to 2}}\right]$$

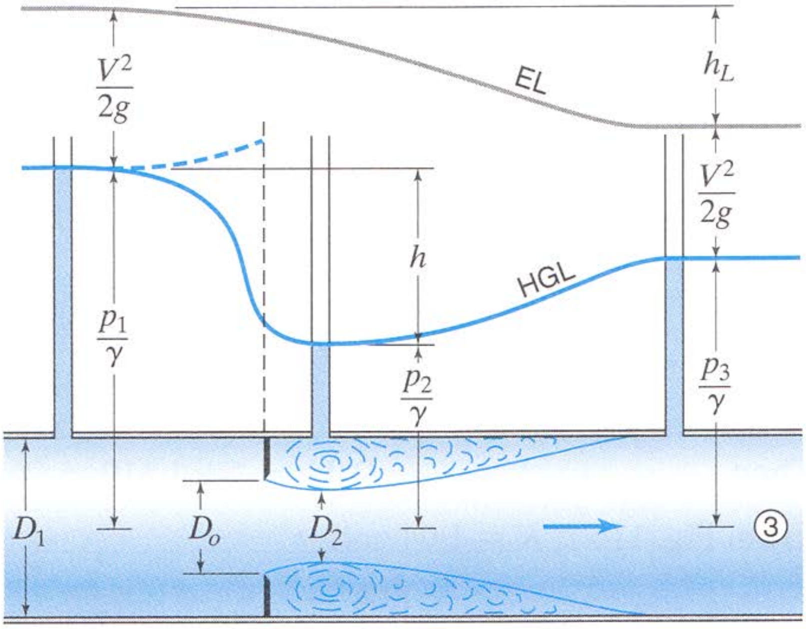
$$Q = \sqrt{\left(\frac{A_1^2 A_2^2}{A_1^2 - A_2^2}\right) 2g\left[\frac{P_1 - P_2}{\rho g} + (z_1 - z_2) - h_{loss_{1\to 2}}\right]}$$

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g(H - h_{loss_{1\to 2}})} \quad \text{or} \quad Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gH}$$

Where: $H = \left(\frac{P_1 - P_2}{\rho g} + z_1 - z_2\right)$ and $\mathbf{C_d}$ is a **discharge coefficient**

2. Orifice meter:

We can use an orifice in a pipeline as a meter in the same manner as the venturi tube. The orifice meter consists of a concentric sharp edged circular hole in a thin plate which is clamped between two flanges in a pipe line.



Thin-plate orifice in a pipe. (Scale distorted: the region of eddying turbulence will usually extend $4D_1$ to $8D_1$ downstream, depending on the Reynolds number.)

The flow characteristics of the orifice is similar to the flow in a venturi-meter **except** that the minimum section of the stream tube A_2 occurs down stream from the orifice (section 2) owing to the formation of **vena contraction**. The ratio between the minimum area A_2 and the area of orifice A_o is known as **coefficient of contraction *Cc***, i.e., $C_c = \frac{A_2}{A_o}$

Writing the Bernoulli equation between sections (1) and (2) (downstream), we have: $\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_{loss_{1\to 2}}$

For the same horizontal level $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_{loss_{1\to 2}}$

Continuity Eq.: $A_1V_1 = A_2V_2 = Q$ i.e., $V_1 = V_2\frac{A_2}{A_1} = \frac{C_cA_o}{A_1}V_2$

$$\therefore V_2 = \sqrt{2g\left(\frac{P_1 - P_2}{\rho g} + \frac{V_1^2}{2g} - h_{loss_{1\to 2}}\right)}$$

$$V_2 = C_v\sqrt{2g\left(\frac{P_1 - P_2}{\rho g} + \frac{V_1^2}{2g}\right)} = C_v\sqrt{2gH}$$

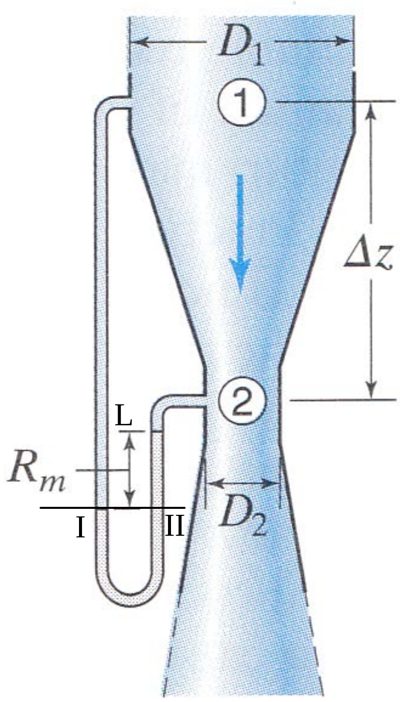
Where $H = \left(\frac{P_1 - P_2}{\rho g} + \frac{V_1^2}{2g} \right)$ and C_v is a **velocity coefficient**.

Then, $Q = A_2 V_2 = C_c A_o C_v \sqrt{2gH} = C_d A_o \sqrt{2gH}$

Where, $C_d = C_c \times C_v$ -------------- discharge coefficient.

Example:

Find the discharge rate of water through the venturi tube of discharge coefficient $\mathbf{C_d}$ = 0.988 that shown in Figure, if $\mathbf{D_1}$ = 800 mm, $\mathbf{D_2}$ = 400 mm, $\mathbf{\Delta z}$ = 2 m, and ***Rm*** =150 mmHg.. Use [ρ_w = *1000kg/m*^{3} , ρ_{Hg} = *13600kg/m*^{3} , and *g = 9.807m/sec*^{2}]



$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gH}$$

$$A_1 = \frac{\pi}{4} D_1^2 = 0.5 m^2 \quad \& \quad A_2 = \frac{\pi}{4} D_2^2 = 0.125 m^2$$

$$H = \frac{P_1 - P_2}{\rho g} + \Delta z$$

For U_tube manometer

$$P_I = P_{II}$$

$$P_1 + \rho_w g (R_m + L + \Delta z) = P_2 + \rho_w g L + \rho_m g R_m$$

$$P_1 - P_2 = \rho_w g L + \rho_m g R_m - \rho_w g R_m - \rho_w g L - \rho_w g \Delta z$$

$$P_1 - P_2 = \rho_m g R_m - \rho_w g R_m - \rho_w g \Delta z$$

$$P_1 - P_2 + \rho_w g \Delta z = \rho_w g R_m \left(\frac{\rho_m}{\rho_w} - 1 \right)$$

$$\frac{P_1 - P_2}{\rho_w g} + \Delta z = R_m \left(\frac{\rho_m}{\rho_w} - 1 \right) = H$$

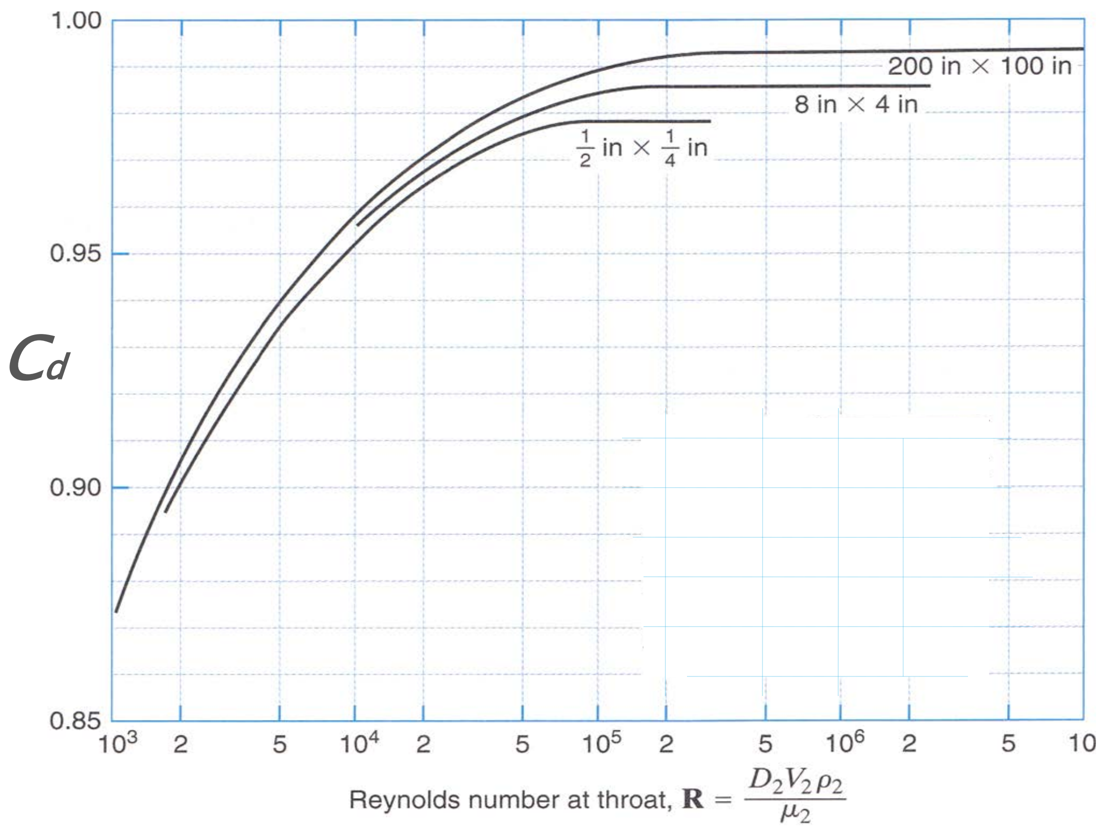
Then $H = 0.15 \left(\frac{13600 g}{1000 g} - 1 \right) = 1.89 m$

Now; $Q = C_d \dfrac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gH}$

$$Q = 0.988 \times \frac{0.5 \times 0.125}{\sqrt{(0.5)^2 - (0.125)^2}} \sqrt{2 \times 9.807 \times 1.89} = 0.777\, m^3/\text{sec}$$

Unless specific information is available for a given venturi tube, we can assume the value of **Cd** is about 0.99 for **large tubes** and about 0.97 or 0.98 for **small ones**, provided the flow is such as to give **Reynolds numbers** greater than about 10^5.

So, the dimensional analysis of a venturi tube indicates that the **coefficient of discharge** C_d should be a function of **Reynolds number** and of the **geometric parameters** D_1 and D_2. Values of venturi tube coefficients are shown in figure.



Losses in pipes

Types of losses

1 – Friction losses :

This type of losses exists for any flow as a result of fluid viscosity and velocity difference between fluid layers. As a result of friction, part of the fluid's mechanical energy is converted into heat energy (dissipated into atmosphere) and is considered as an energy loss.

2 – Eddy losses :

This type of losses occurs due to any change in the velocity vector (magnitude or direction). This change causes some of energy to be transferred from main flow to eddies formed at corners. This part of energy is considered as energy losses.

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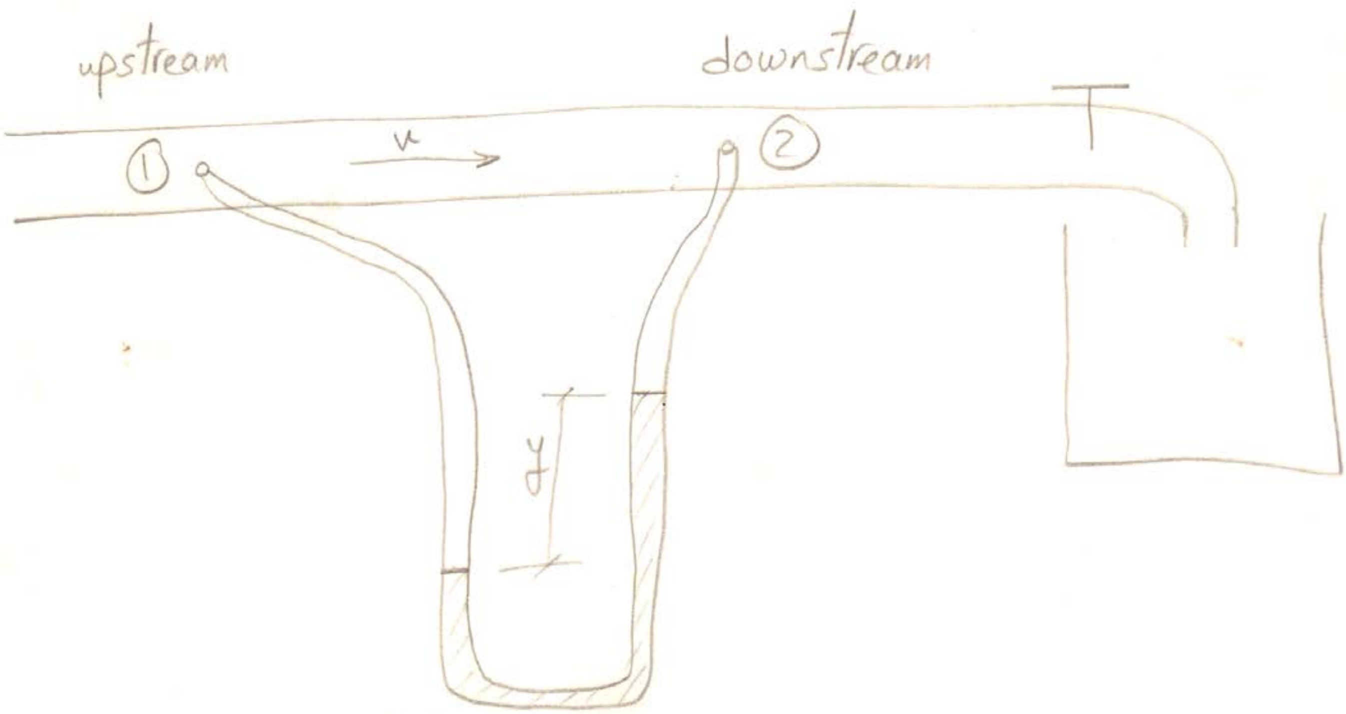
*** Friction losses** ماسورة ليس بها تغيير بالمساحة لحساب



$$E_1 = E_2 + h_{loss1to2}$$

$$\frac{P_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_{loss1to2}$$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_{loss1to2}$$

$$h_{f.l.} = \frac{P_1 - P_2}{\rho g} \quad \text{---} \quad (1)$$

تم تسميتها $h_{f.l.}$ لانها كلها friction losses وليس بها eddy losses

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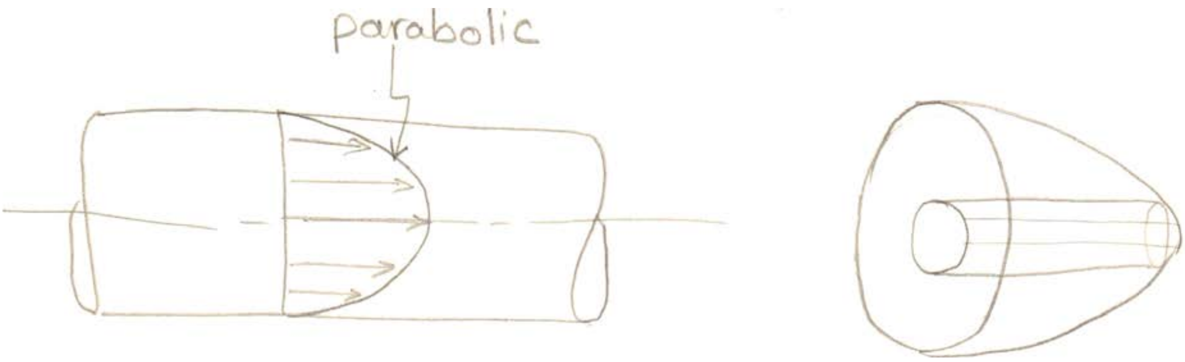
*** Friction losses in laminar flow   Re < 2000**



Velocity distribution for laminar flow

$$F_{vis} = \mu\ A\ \frac{du}{dy} \quad \text{---} \quad (2)$$

From (1) & (2)

$$\boxed{h_{f.l.} = \frac{32\mu\ v\ L}{\rho\ g\ d^2}}$$

Where L is the pipe length

$h_{f.l.}\ \alpha\ v$ (linear)

$$h_{f.l} = 32 * \frac{\mu v L}{\rho g d^2} * \frac{v}{v} * \frac{2}{2}$$

$$= 32 * 2 * \frac{\mu}{\rho v d} * \frac{L}{d} \frac{v^2}{2g}$$

$$= \frac{64}{R_e} * \frac{L}{d} \frac{v^2}{2g}$$

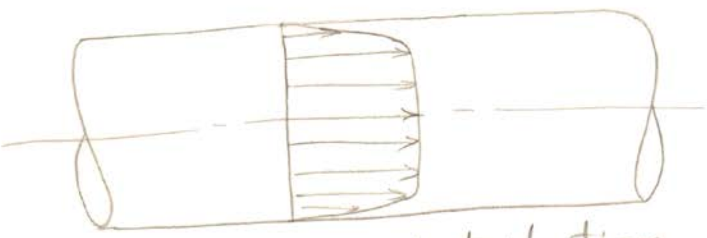
$$h_{f.L} = f \frac{L}{d} \frac{v^2}{2g} \rightarrow$$

Another from $f = \dfrac{64}{R_e}$ where f : coefficient of friction

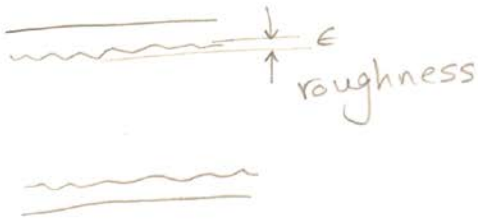
<u>Note :</u>

for laminar flow , the pipe roughness has no effect on friction losses because the velocity of the fluid layer beside the pipe wall is very small and almost stationary.

<u>B – friction losses in turbulent flow Re > 4000</u>



velocity distribution in turbulent flow

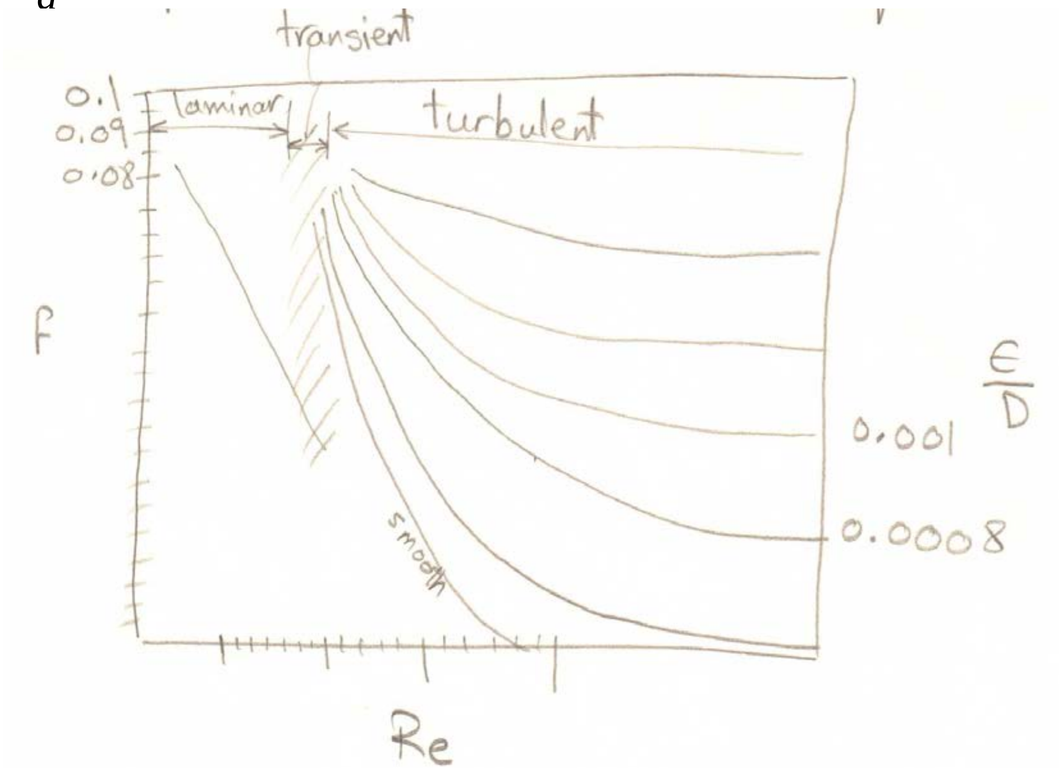


* All the following results are experimental results written as empirical formula

$$h_{f.l.} = f \frac{L}{d} \frac{v^2}{2g}$$

$$h_{f.l.} \alpha v^2$$

$$F \begin{cases} Re = \dfrac{\rho\, v\, d}{\mu} \\ \dfrac{\varepsilon}{d} \end{cases}$$

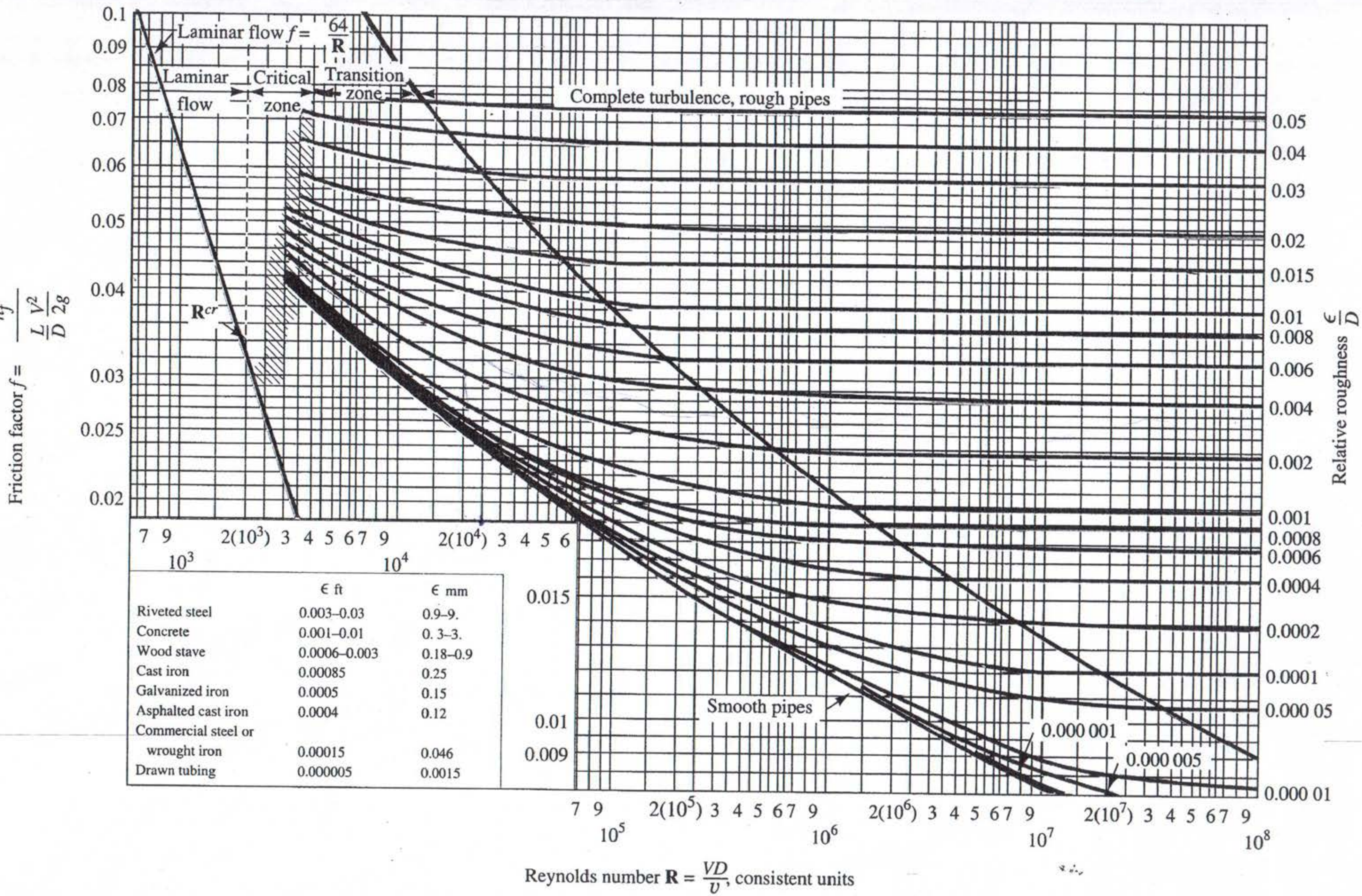


***Eddy losses :** اي تغيير في السرعة مقدار او اتجاه

All eddy losses depend only on kinetic energy ($\frac{v^2}{2g}$) of flow (not type of flow).

I – change in magnitude

II – change in direction



	ϵ ft	ϵ mm
Riveted steel	0.003–0.03	0.9–9.
Concrete	0.001–0.01	0. 3–3.
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015

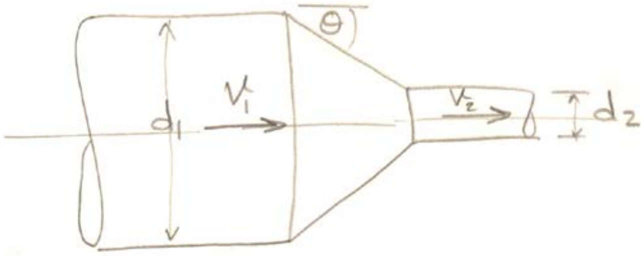
*** contraction**

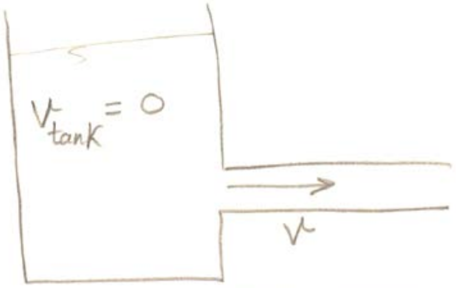
$h_\ell = K \dfrac{V_2^2}{2g}$

K: Coefficient of eddy loss

$k = f^n\left(\dfrac{d_1}{d_2}, \theta\right)$

Kinetic energy

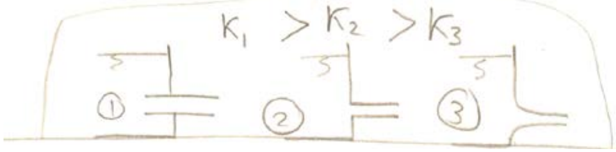




$$at \ \frac{d_1}{d_2} = \infty \quad \& \quad \theta = 90^o$$

$$k_{\max} = 0.5$$

$$h_l = 0.5 \ \frac{v^2}{2g}$$



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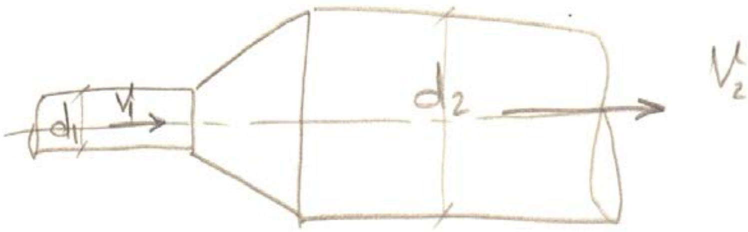
*** Enlargement**



$$h_\ell = k \ \frac{(v_1 - v_2)^2}{2g}$$

k = fn (θ) The effect of $\dfrac{d_2}{d_1}$ is presented in $(v_1 - v_2)$

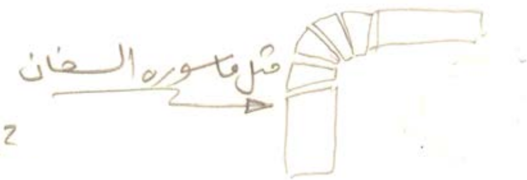
$k_{\max}$ For enlargement at $\theta = 90^\circ$

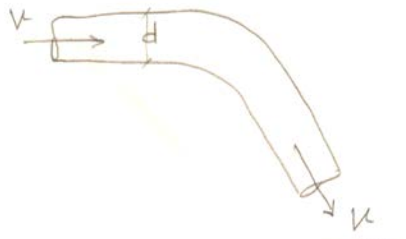
(sudden enlargement)

$$k_{\max} = 1$$

$$h_\ell = \frac{(v_1 - v_2)^2}{2g}$$

<u>* Bend or Elbow</u>





$$h_\ell = k\,\frac{v^2}{2g}$$

k = fn (θ, d, type of bend)

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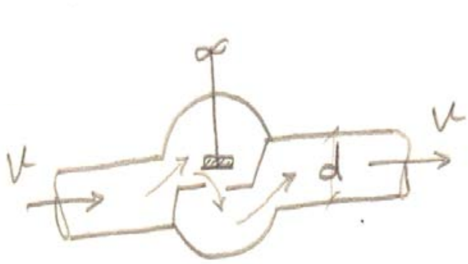
* <u>valves</u>

<u>Globe valve</u>



$$h_\ell = k\,\frac{v^2}{2g}$$

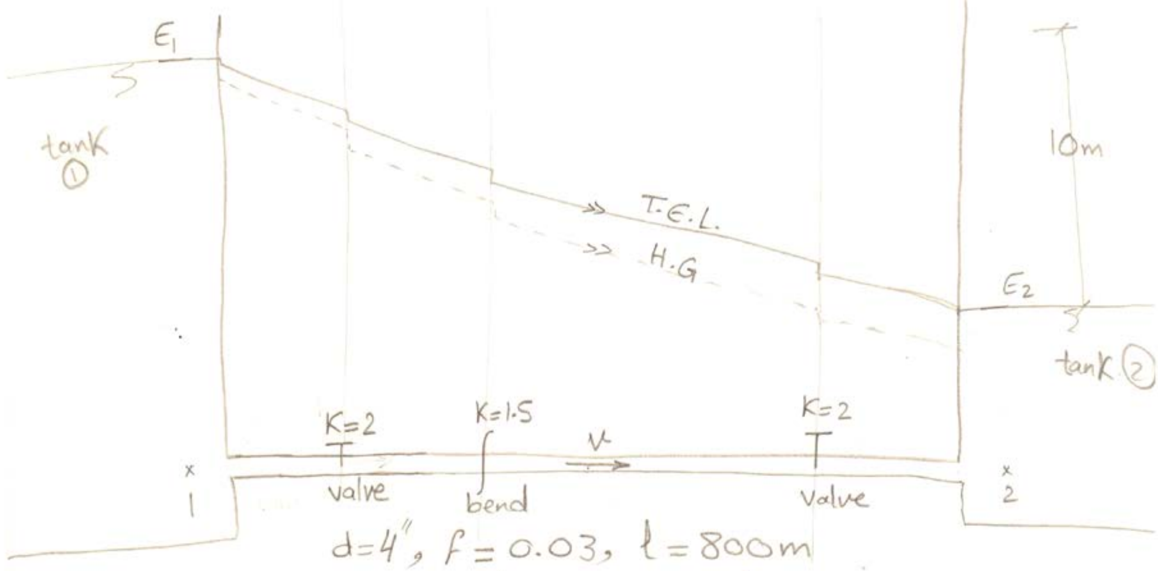
k = fn (d, type of valve)

Only for fully opened valves

-عند غلق المحابس نصف يكون هناك فاقد اكبر في الطاقة

-لذلك الاستخدام الامثل لها ان تكون مفتوحة بالكامل او مغلقة بالكامل

<u>Example :</u>



a) Calculate the discharge between the tanks.
b) One of valves is partially closed. The discharge reduced by 50 % .calculate the losses in the value in this case.

a) $d = \frac{4*2.54}{100} = 0.1m$

$$Q = AV$$

$$E_1 = E_2 + h_{loss}$$

$$10 = f\ \frac{\ell}{d}\frac{v^2}{2g} + k_{ent}\ \frac{v^2}{2g} + K_v\ \frac{v^2}{2g} + K_b\ \frac{v^2}{2g} + K_v\ \frac{v^2}{2g} + K_{ent}\ \frac{(v-o)^2}{2g}$$

$$10 = \frac{v^2}{2*9.8}(0.03*\frac{800}{0.1} + 0.5 + 2 + 1.5 + 2 + 1)$$

$$V = 0.89m/s$$

$$Q = 7 \times 10^{-3}\ m^3/s$$

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b)

$$Q = 0.5Q = Av^1$$

$$\therefore 10^{-3} * 3.5 = \frac{\pi}{4}(0.1)^2 V_1$$

$$V_1 = 0.445m/s$$

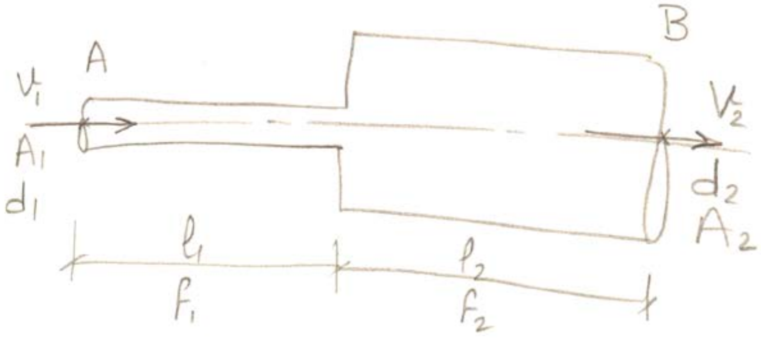
$$E_1 - E_2 = h_{loss} = 10$$

$$10 = f\ \frac{\ell}{d}\frac{v_1^2}{2g} + k_{ent}\ \frac{v_1^2}{2g} + h_{loss} + K_b\ \frac{v_1^2}{2g} + K_v\ \frac{v_1^2}{2g} + K_{\max}\ \frac{v_1^2}{2g}$$

$$10 = 0.03*\frac{800}{0.1} + \frac{(0.445)^2}{2*9.8}0.5\frac{(0.445)^2}{2*9.8} + h_{loss}\ 2 + 1.5 + \frac{(0.445)^2}{2*9.8} + 2 + \frac{(0.445)^2}{2*9.8} + \frac{1*(0.445-o)^2}{2*9.8}$$

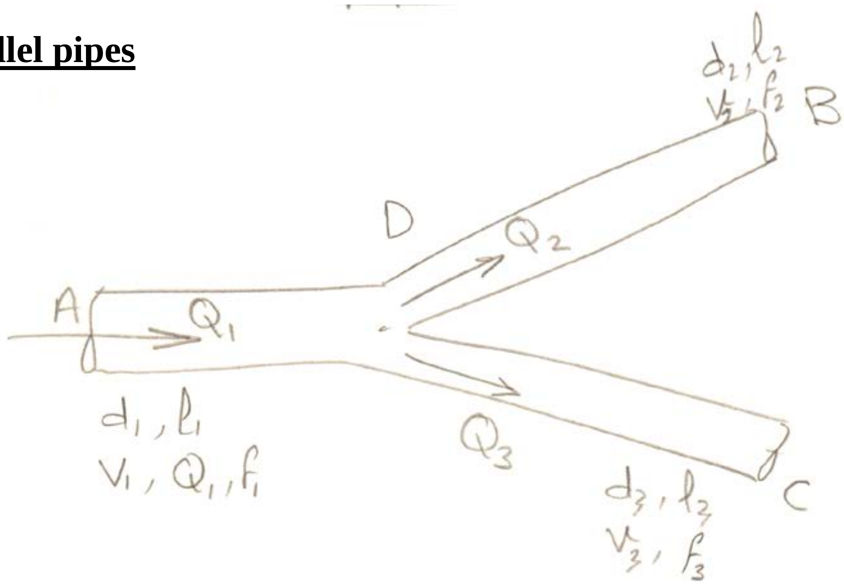
H_{loss} = 7.5 m of liquid

*** losses in series pipes :**



$$h_\ell = f_1 \frac{\ell_1}{d_1}\frac{v_1^2}{2g} + \frac{K(v_1 - v_2)^2}{2g} + f_2 \frac{\ell_2}{d_2}\frac{v_2^2}{2g}$$

*** losses in parallel pipes**



$$h_{\ell_{A\to B}} = f_1 \frac{\ell_1}{d_1}\frac{v_1^2}{2g} + h\ell_{branching} + f_2 \frac{\ell_2}{d_2}\frac{v_2^2}{2g}$$

$$h_{\ell_{A\to C}} = f_1 \frac{\ell_1}{d_1}\frac{v_1^2}{2g} + h\ell_{branching} + f_3 \frac{\ell_3}{d_3}\frac{v_3^2}{2g}$$

$$Q_1 = Q_2 + Q_3$$

$$A_1V_1 = A_2V_2 + A_3V_3$$